

Set-membership state estimation by solving data association

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Perceiving underwater landmarks with a side-scan sonar

Detection of **unidentifiable/indistinguishable** rocks on the seafloor:



Some rocks are lying on the seabed.

Localization with data association: assumptions

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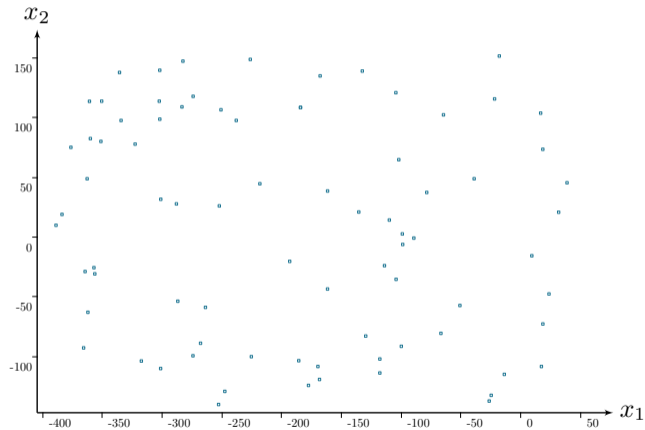
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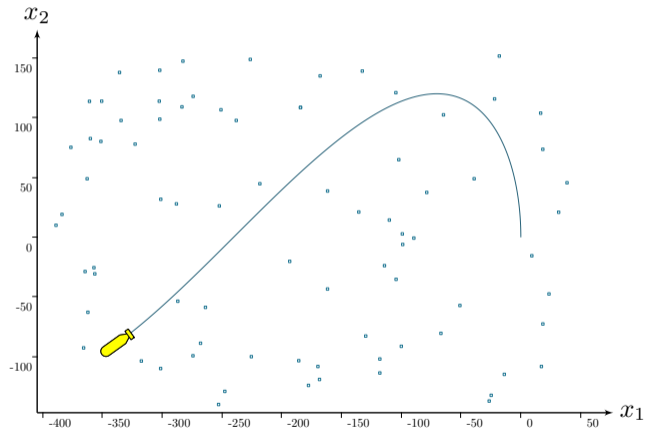
state estimation \Leftrightarrow data association

Localization with data association: formalization



$\mathbf{m}^i \in \mathbb{M}$
(map constraint)

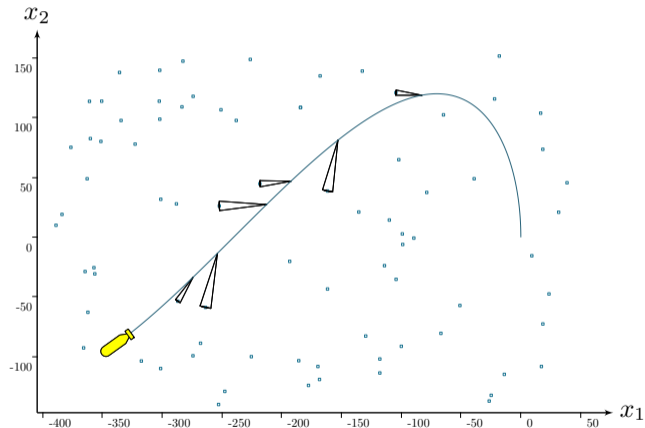
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$$\left\{ \begin{array}{l} \mathbf{m}^i \in \mathbb{M} \\ \text{(map constraint)} \end{array} \right.$$

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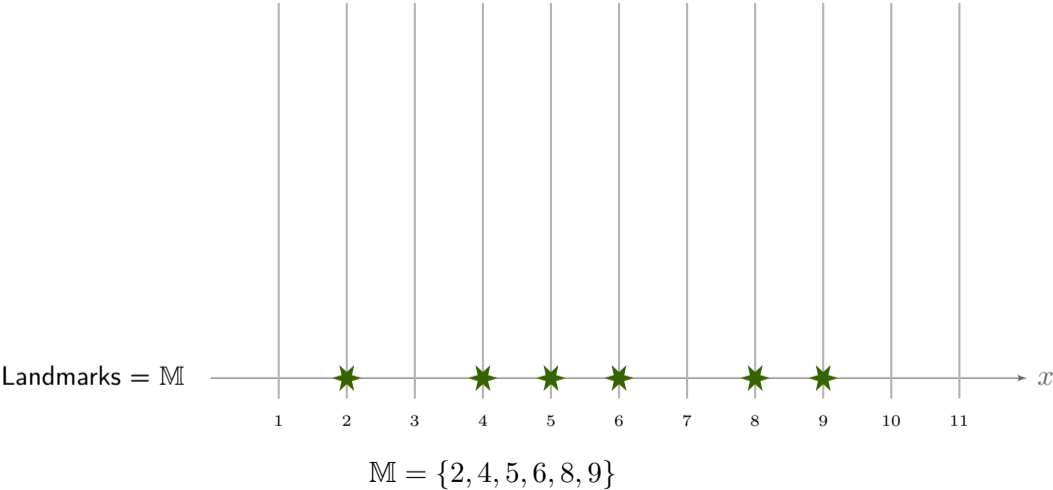
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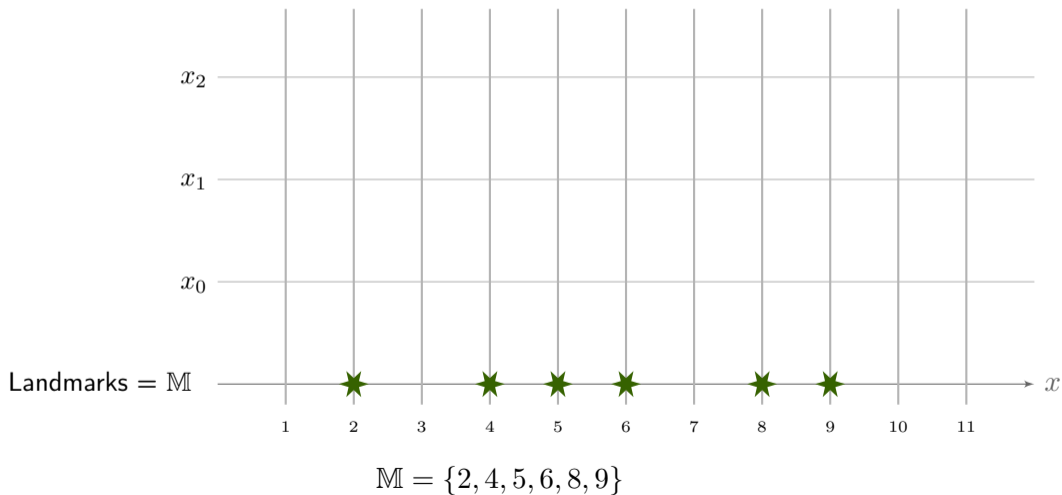
$$\mathbf{g}(\mathbf{x}(t_i), \mathbf{y}^i, \mathbf{m}^i) = \mathbf{0}$$

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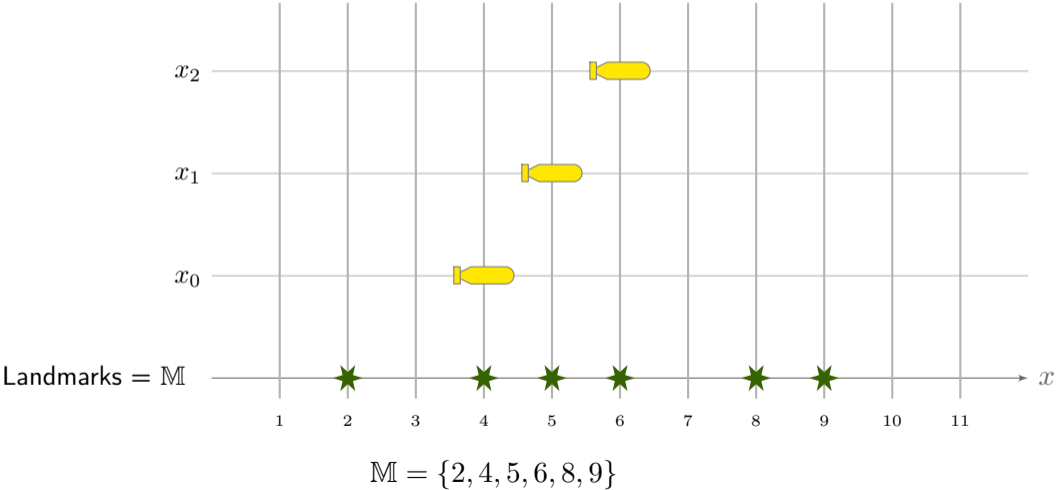
State estimation with indistinguishable landmarks



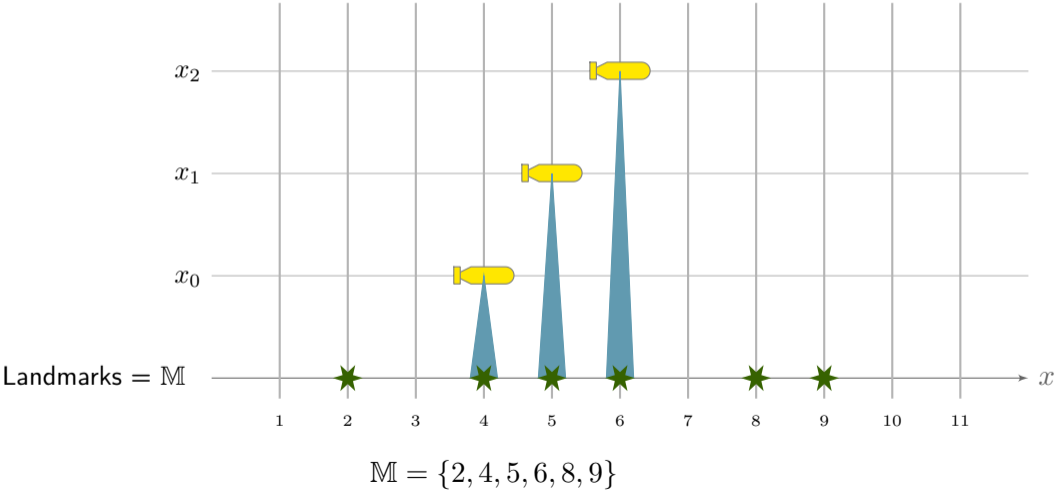
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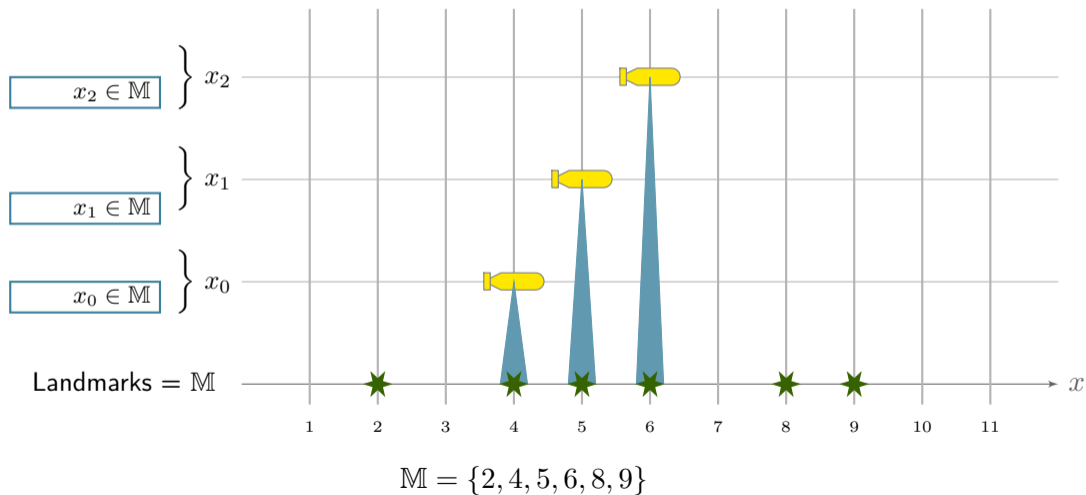
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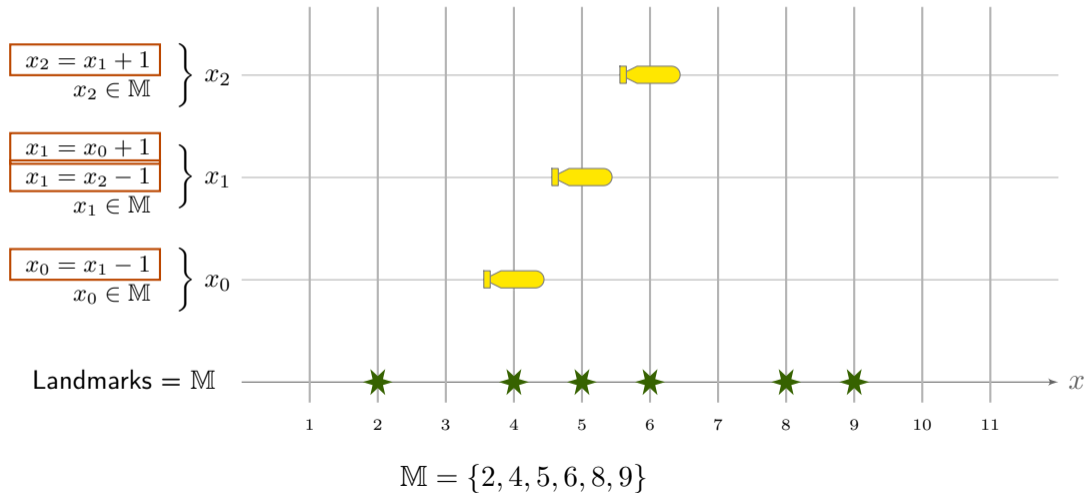
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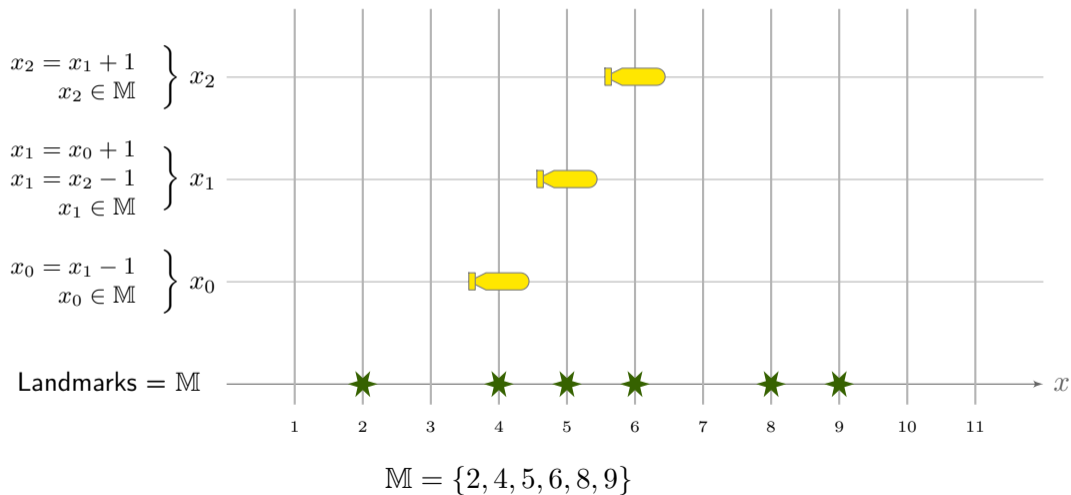
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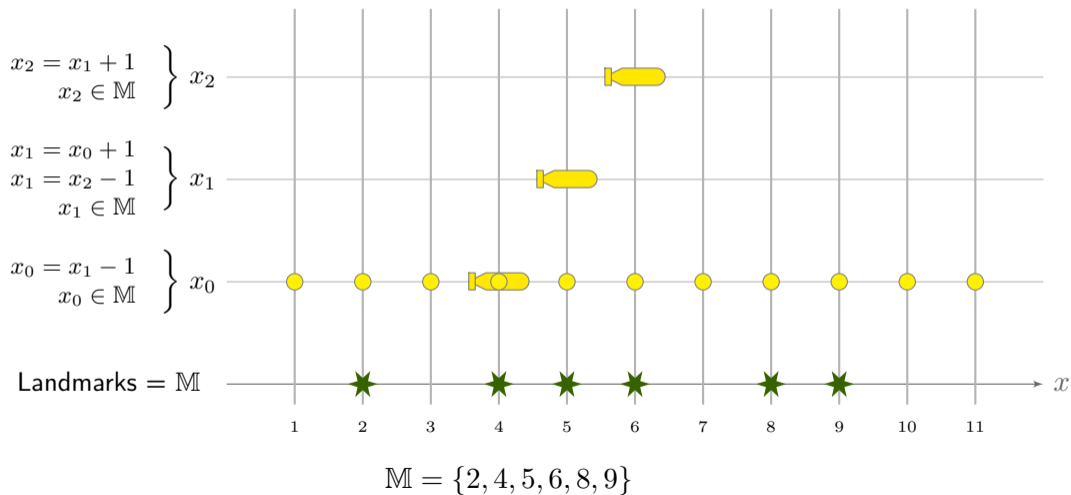
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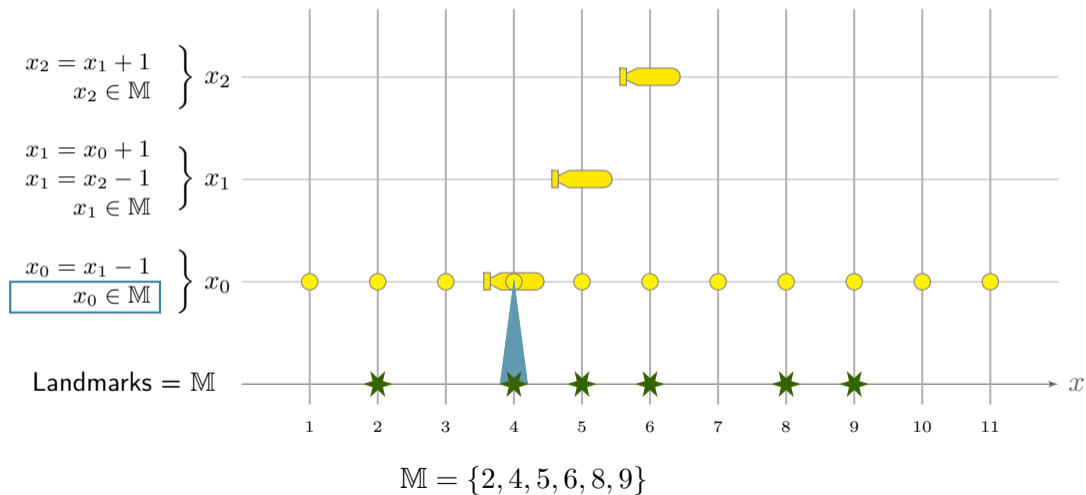
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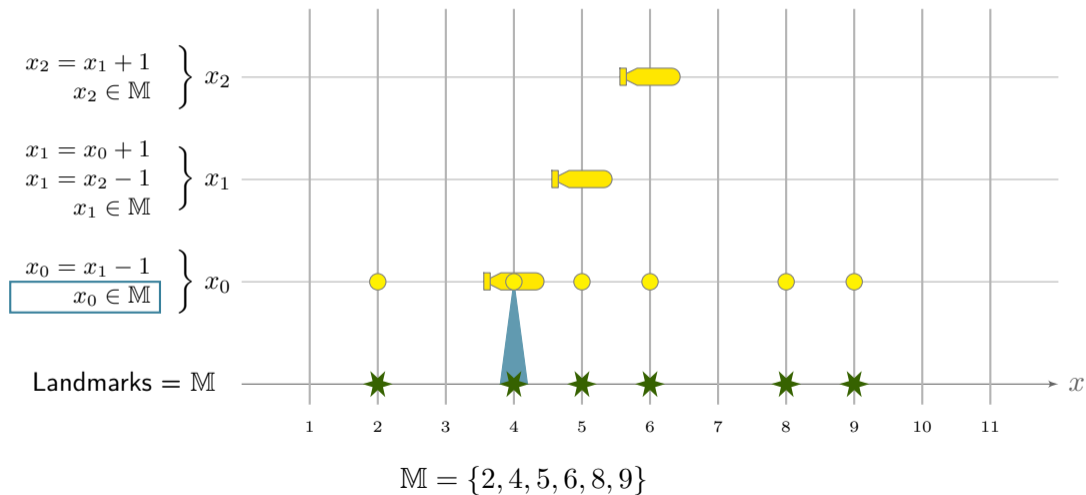
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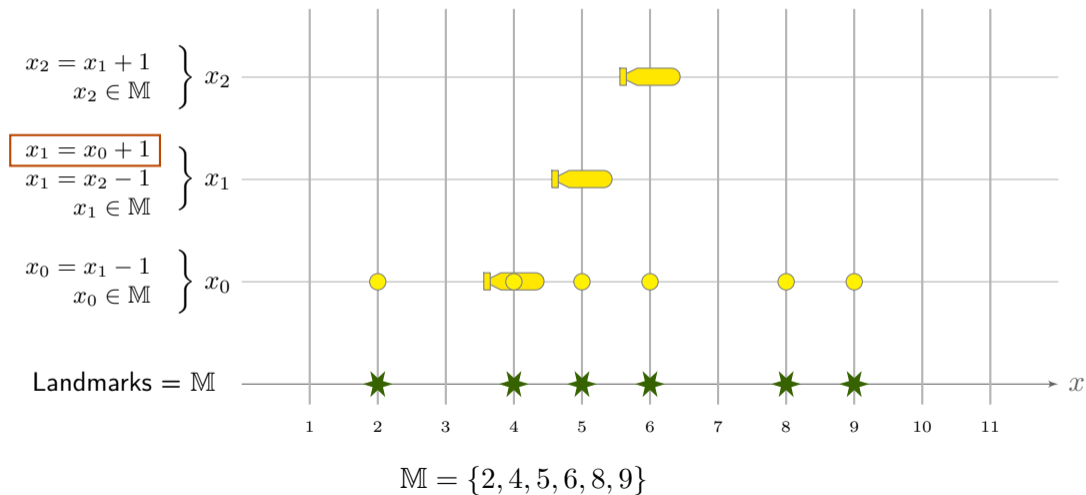
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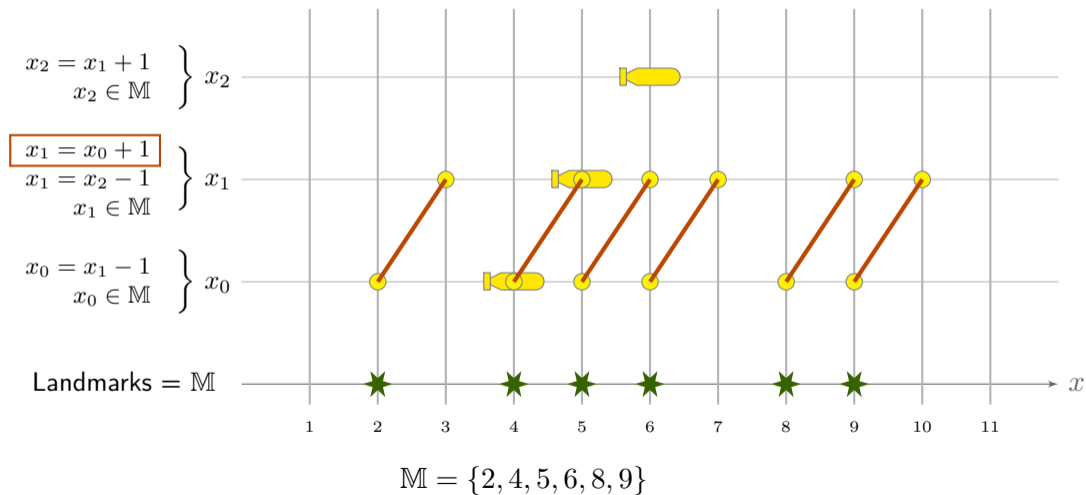
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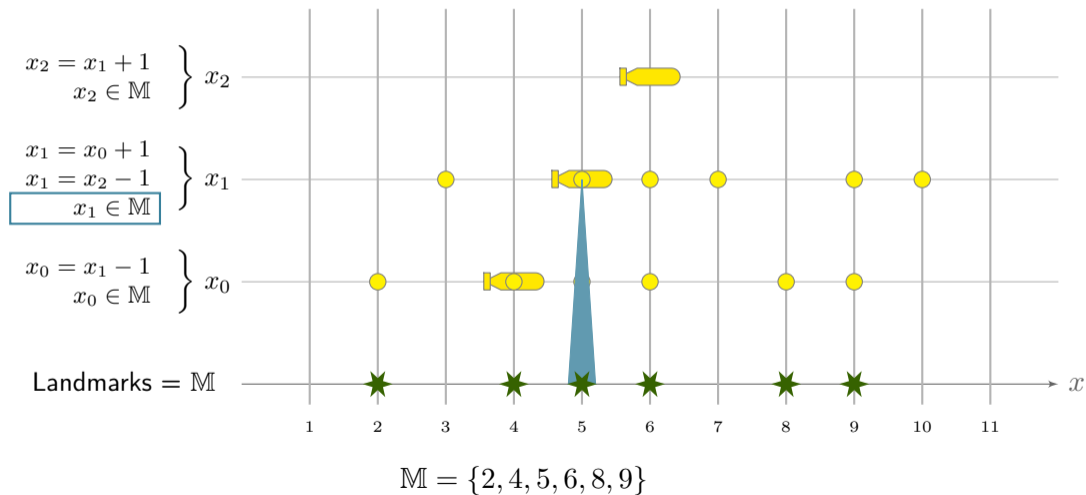
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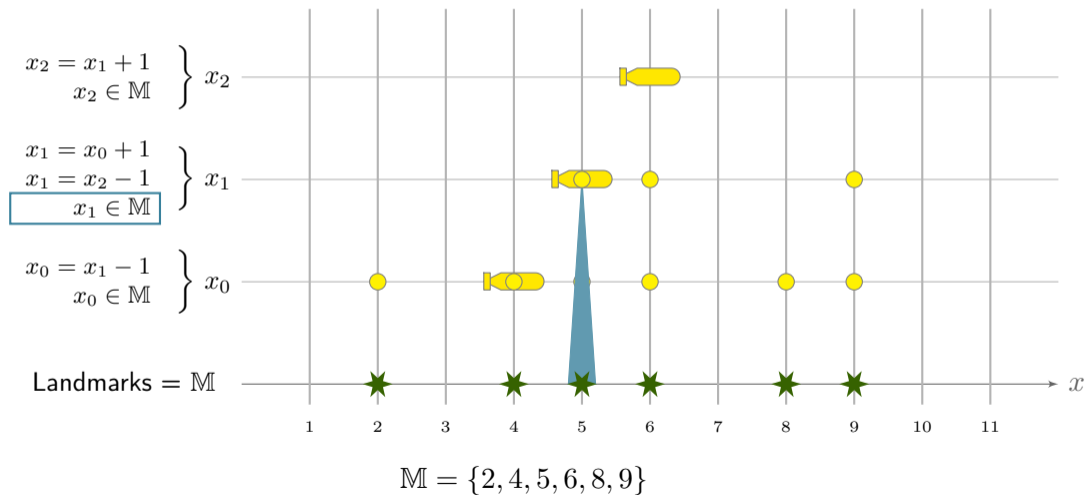
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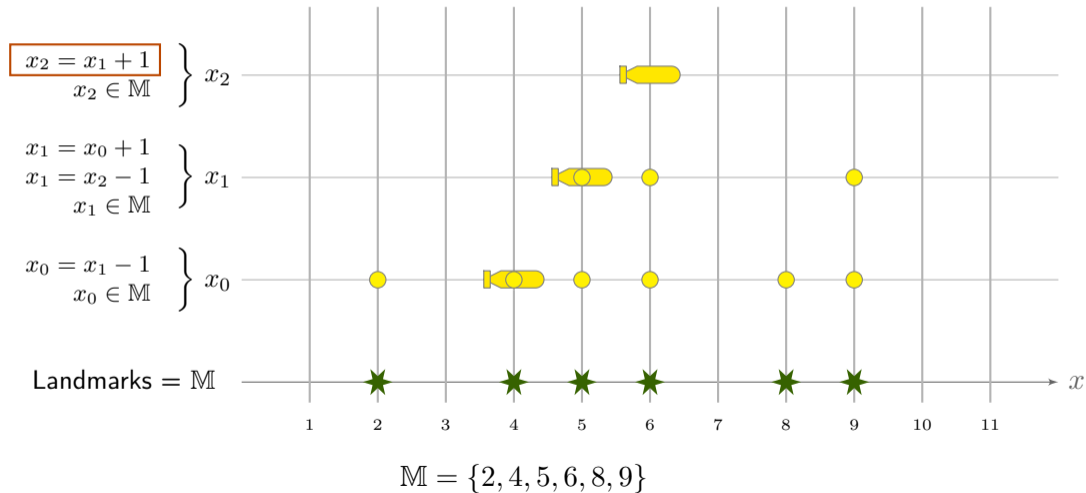
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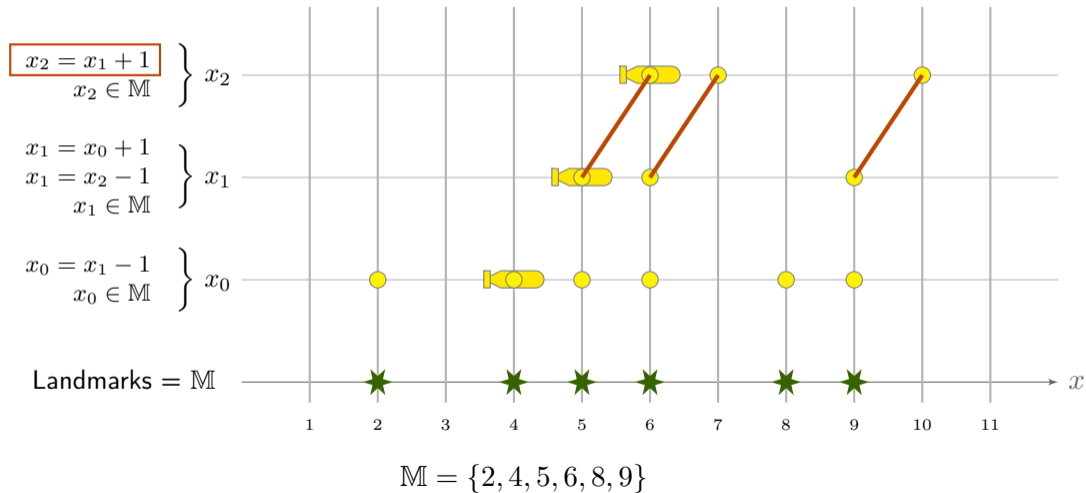
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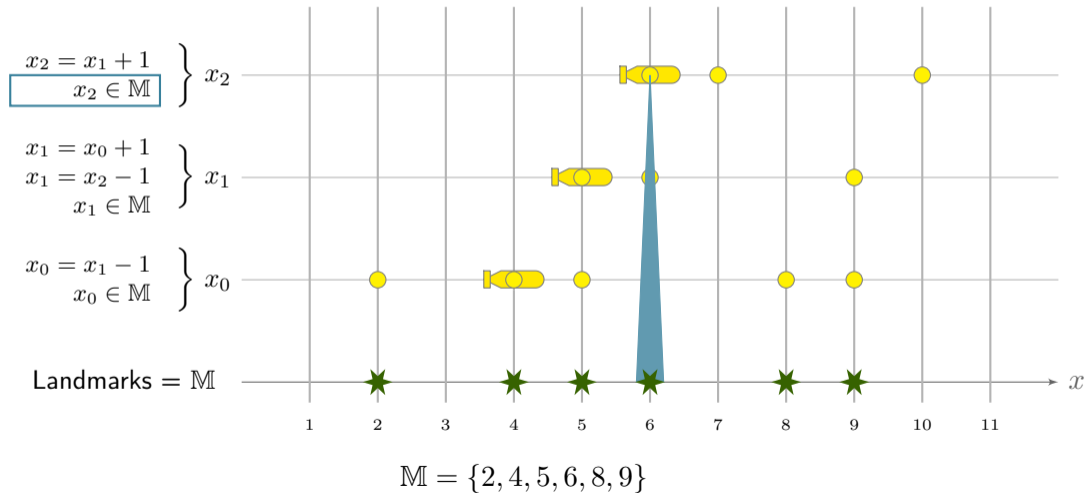
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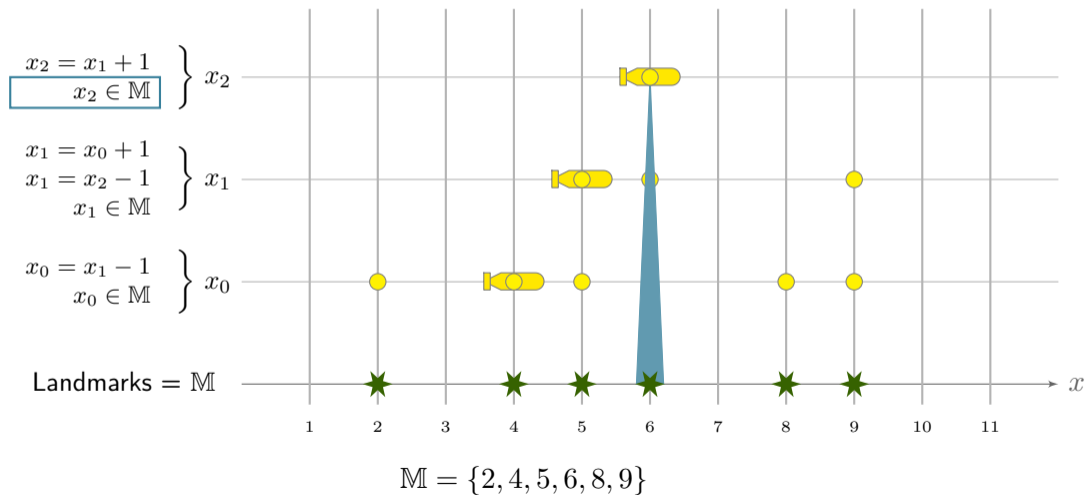
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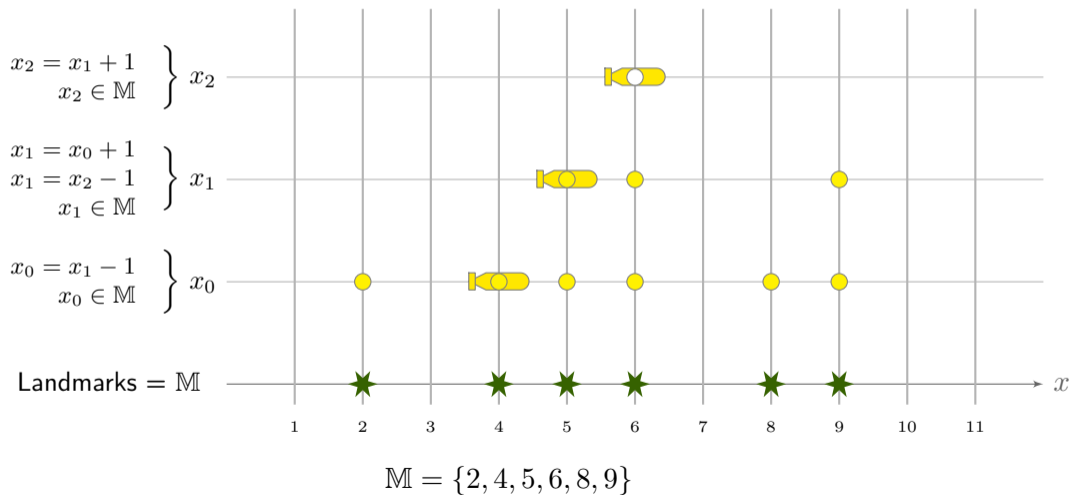
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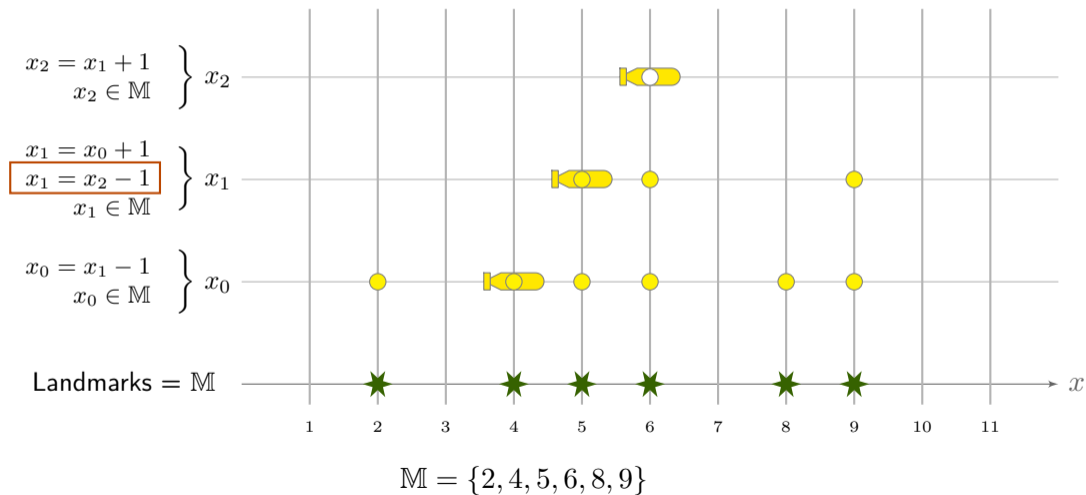
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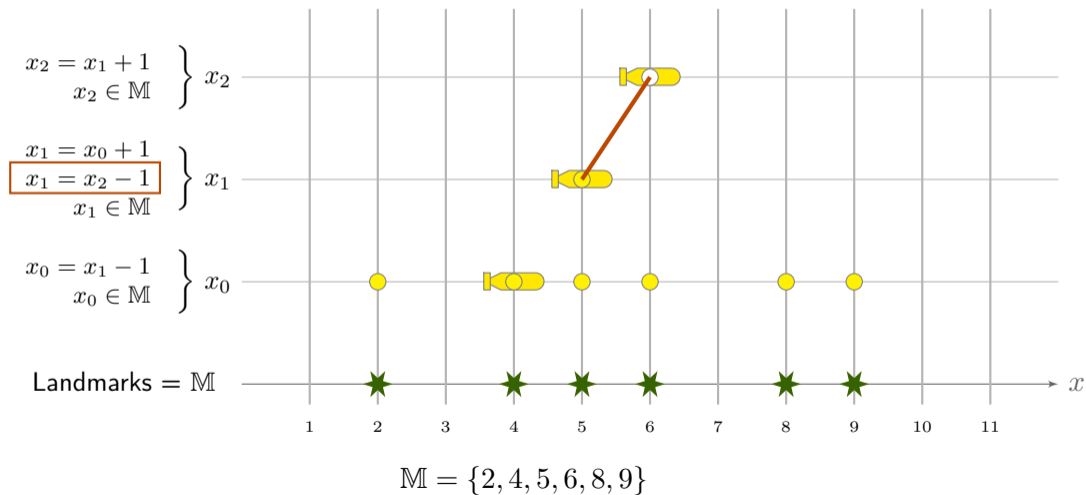
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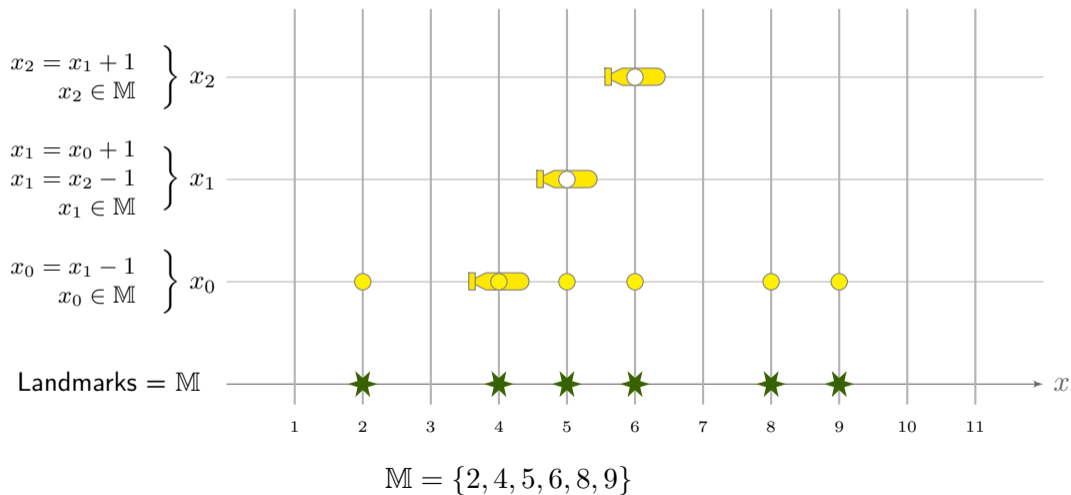
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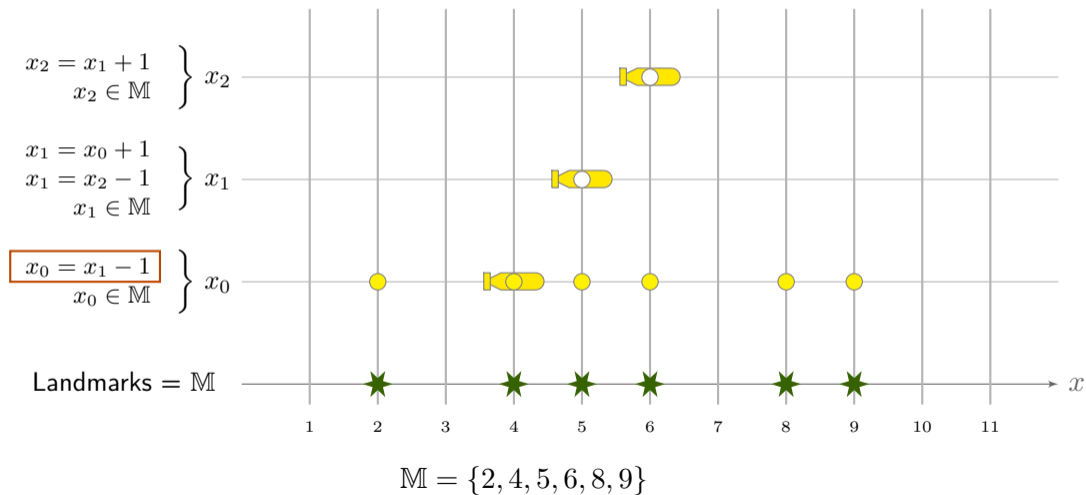
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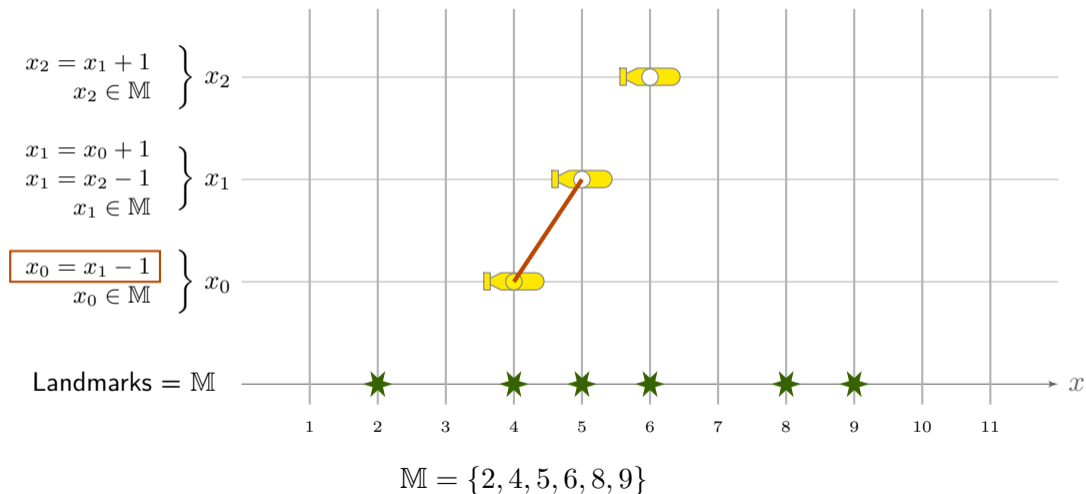
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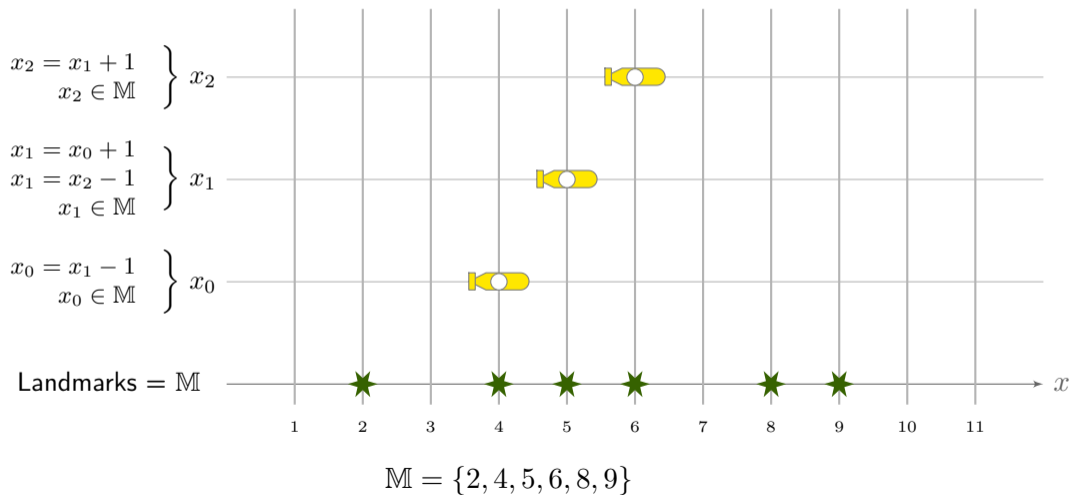
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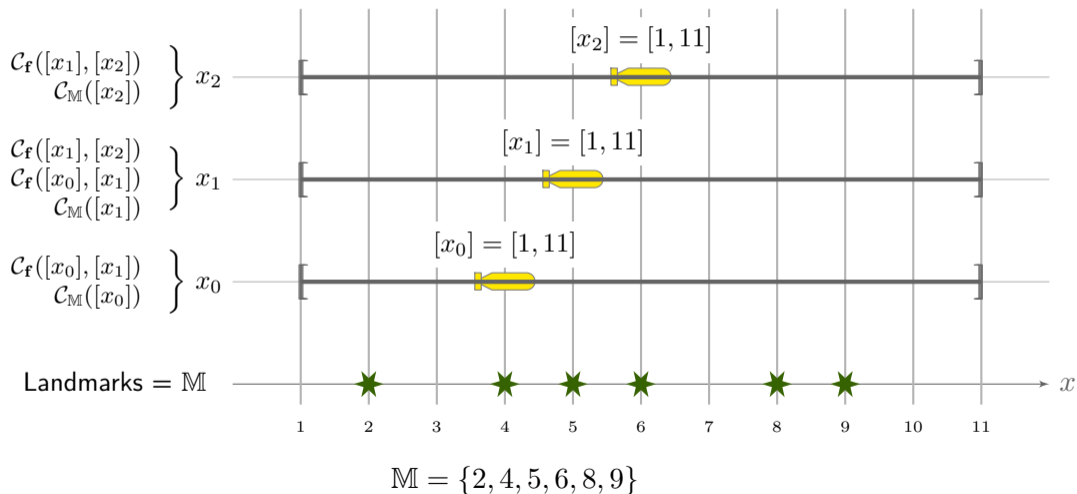


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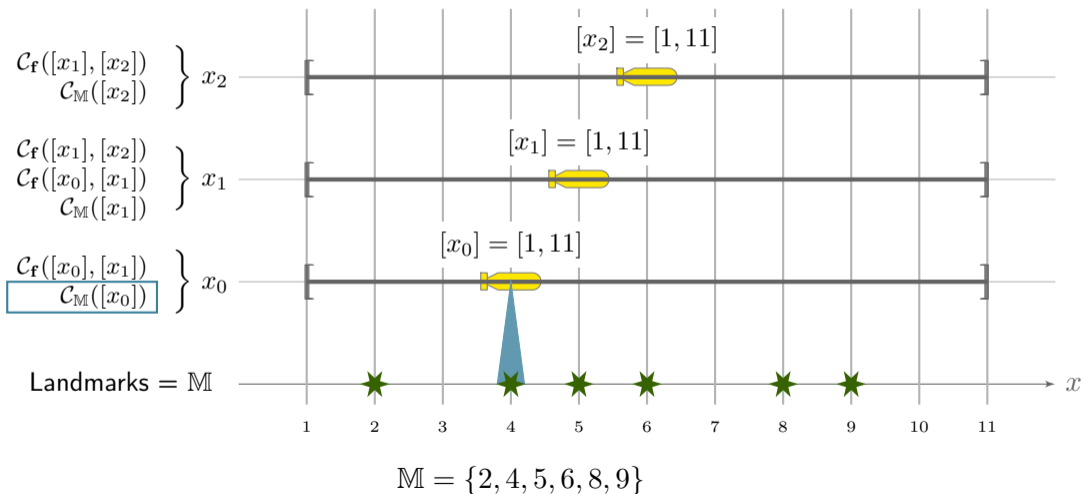
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Set-membership approach



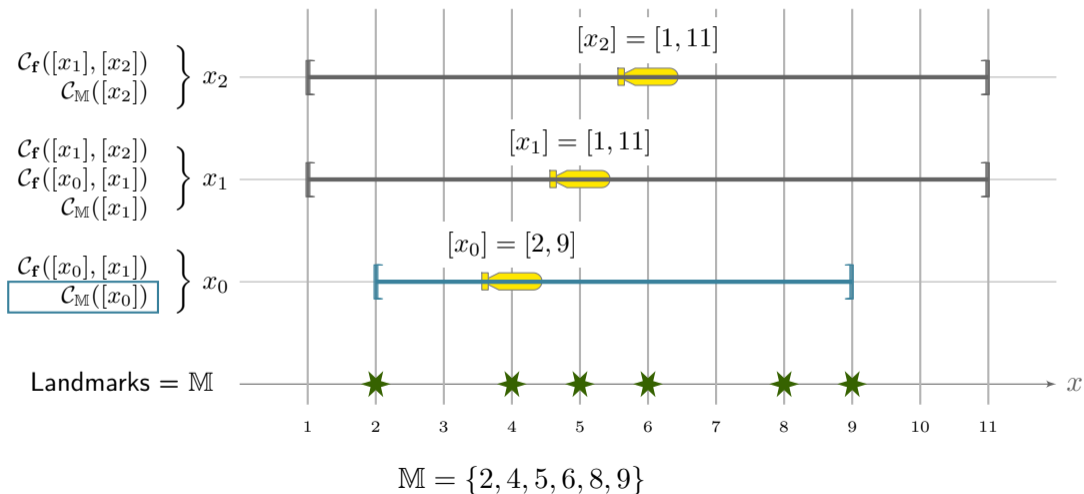
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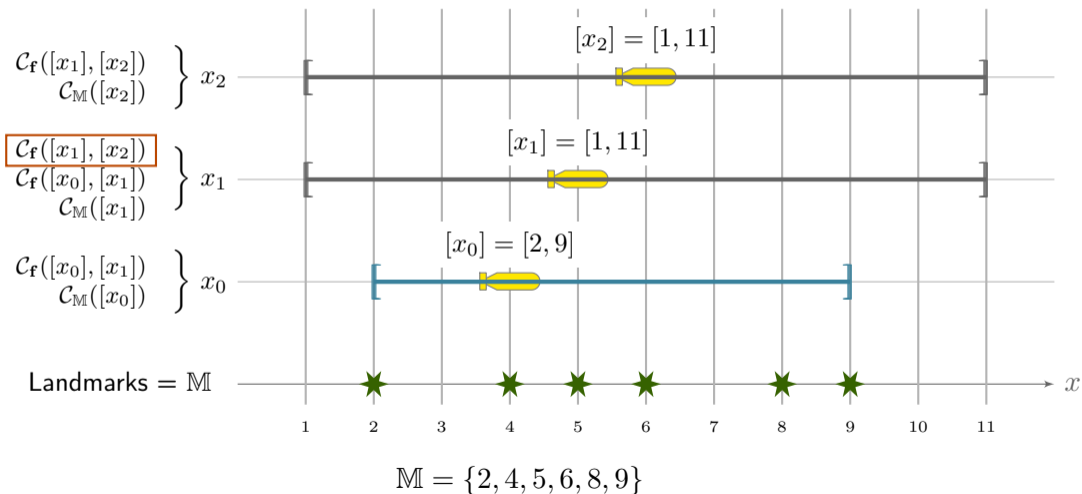
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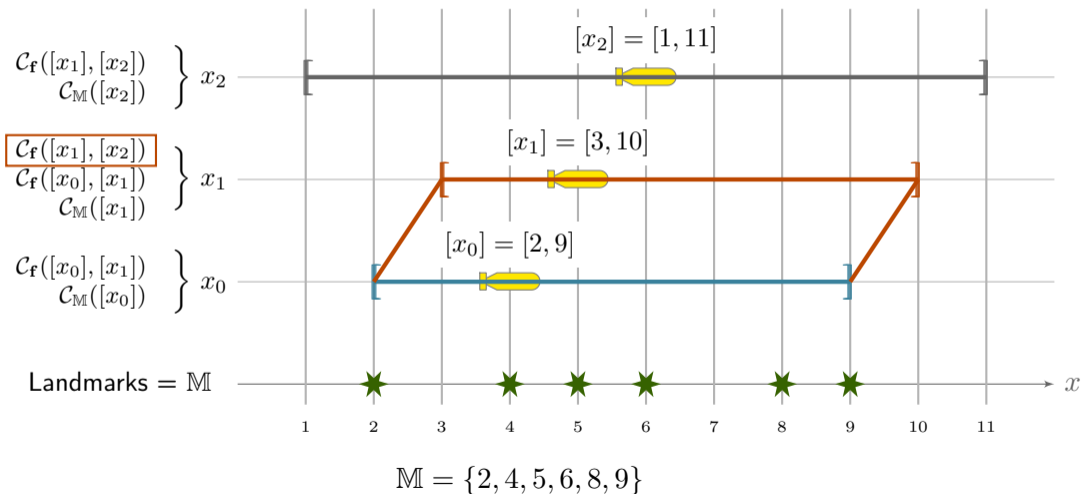
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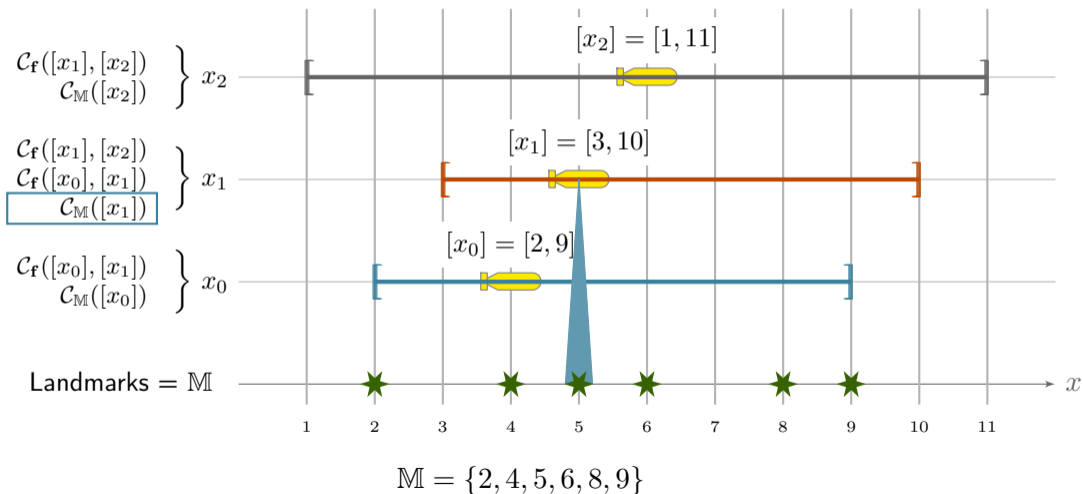
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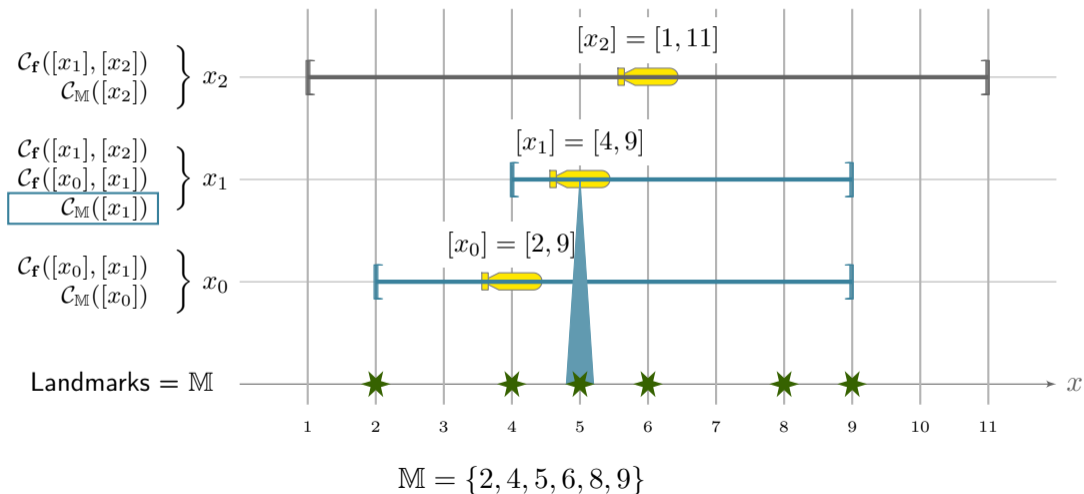
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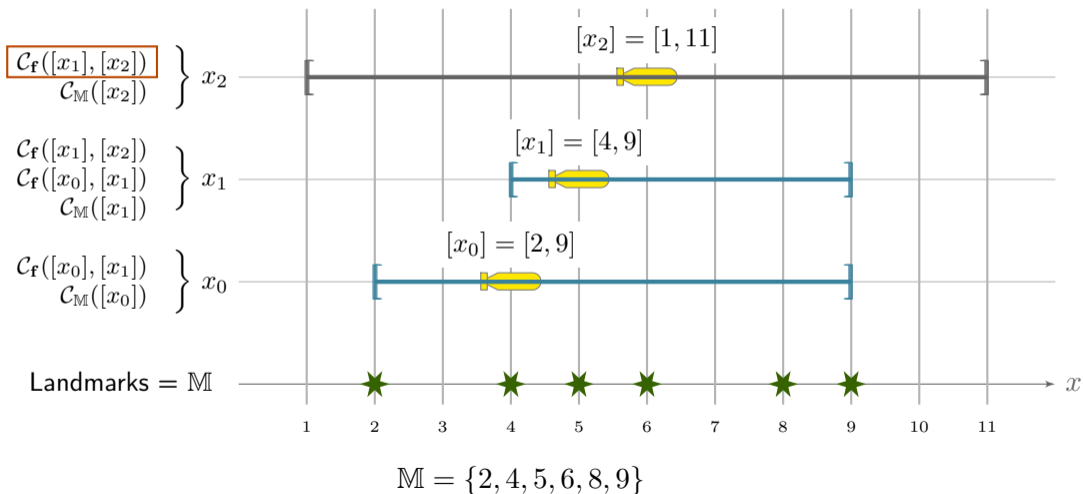
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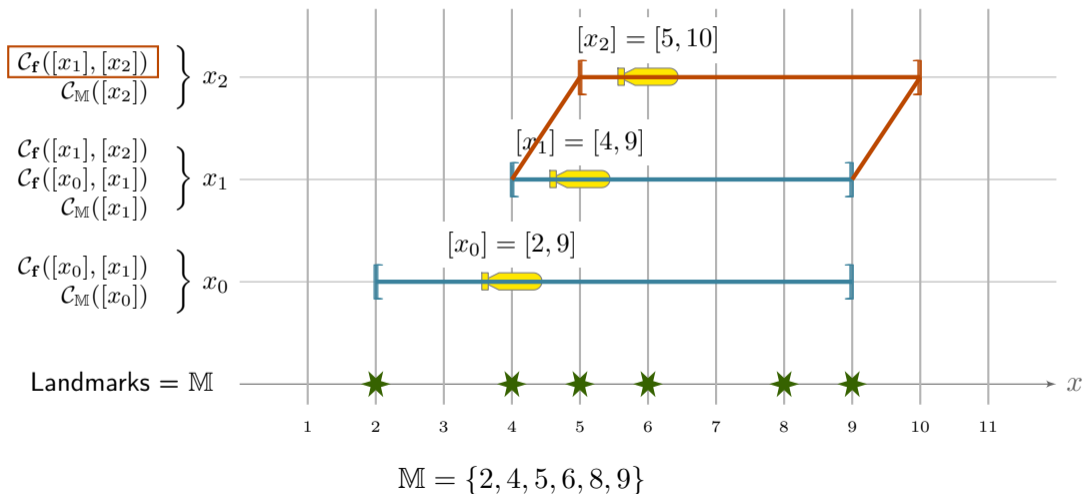
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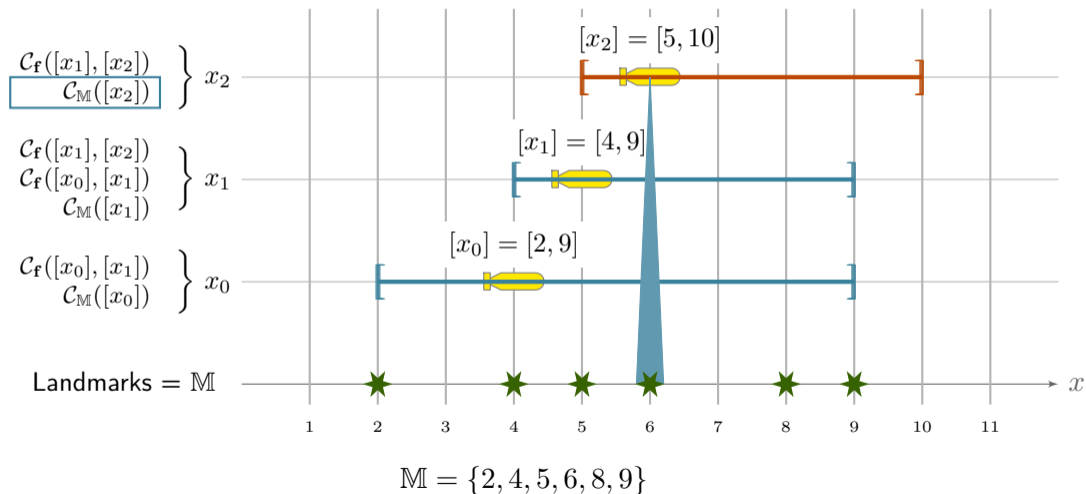
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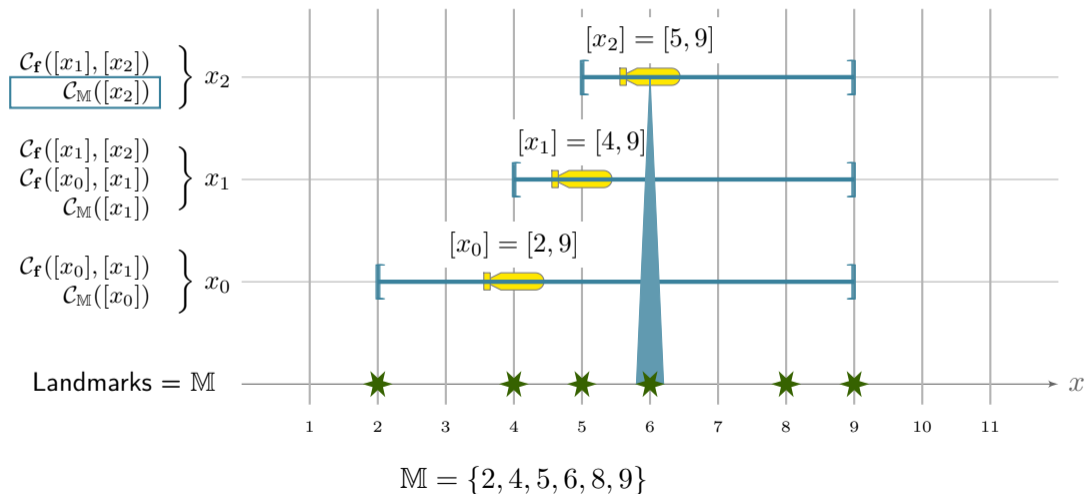
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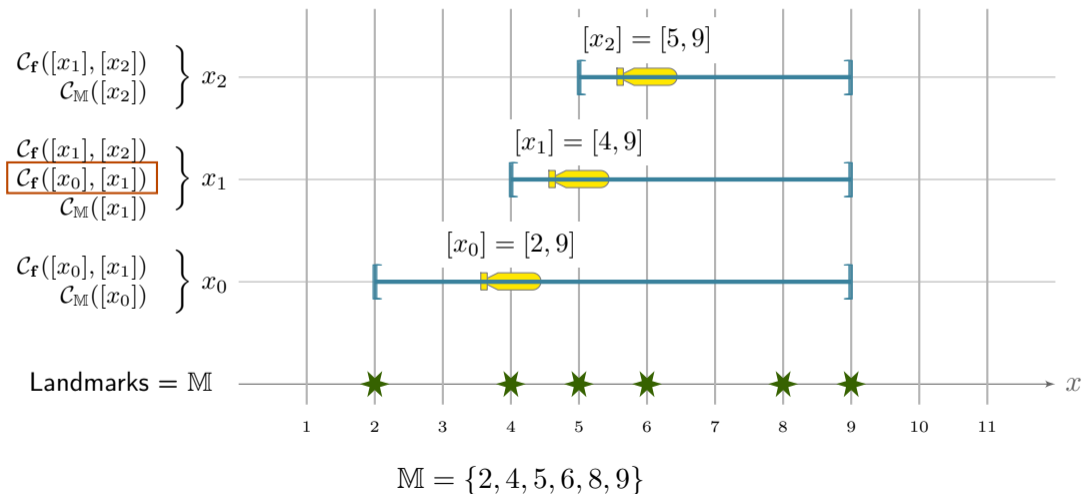
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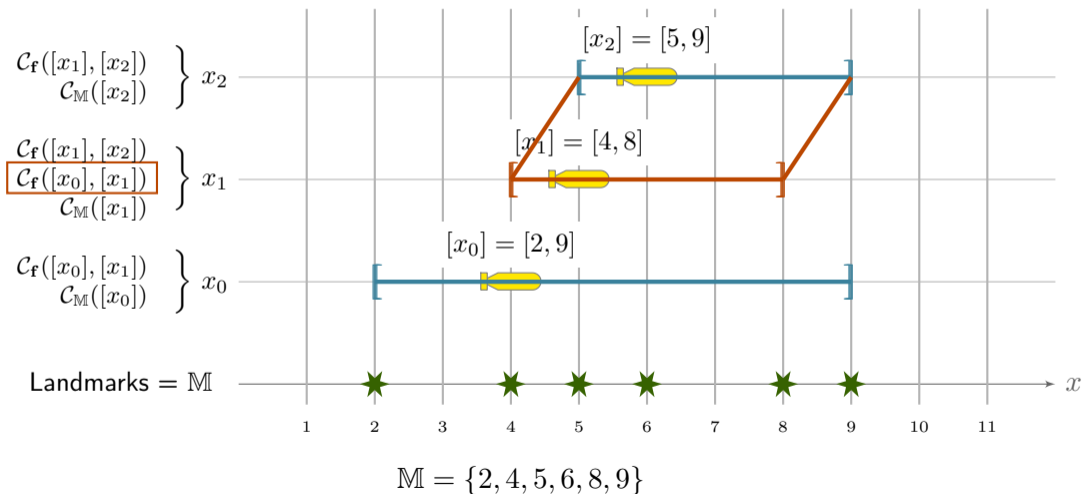
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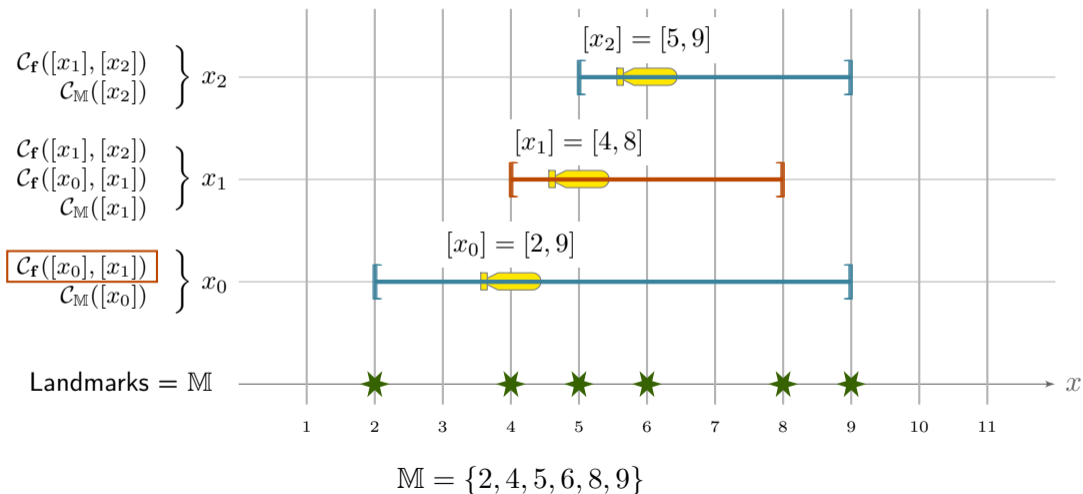
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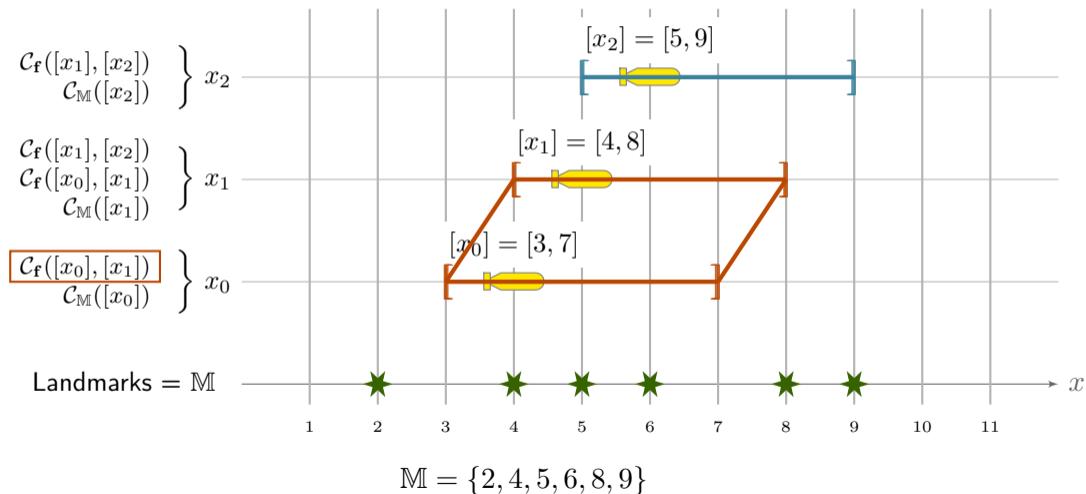
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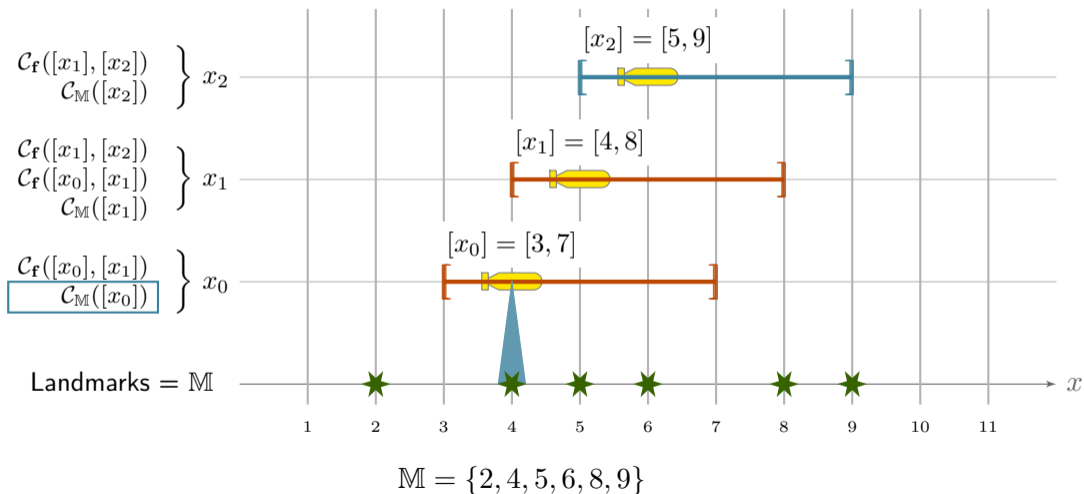
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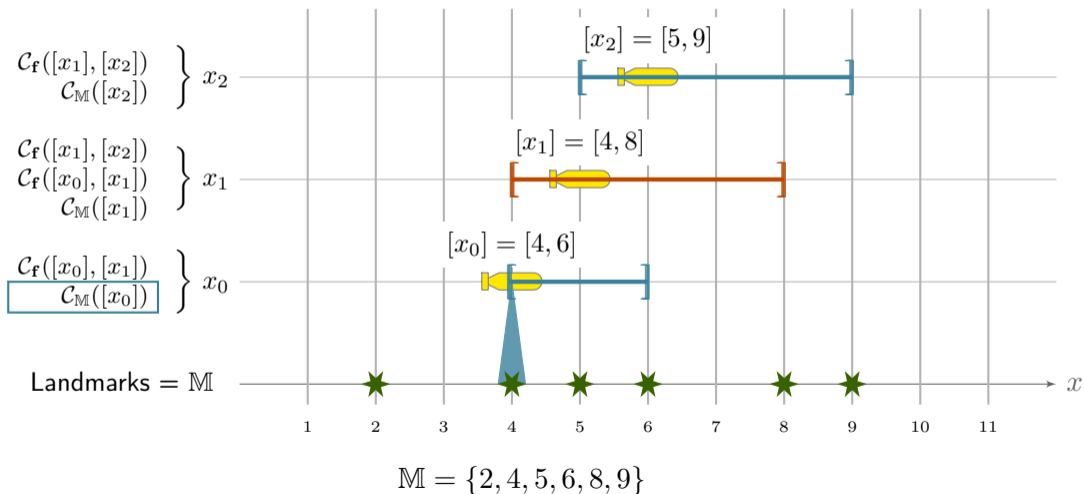
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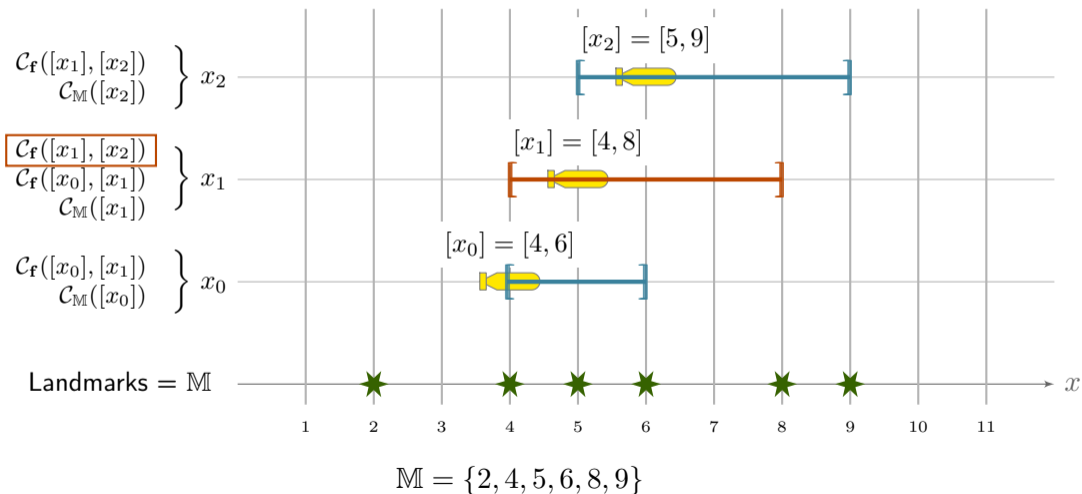
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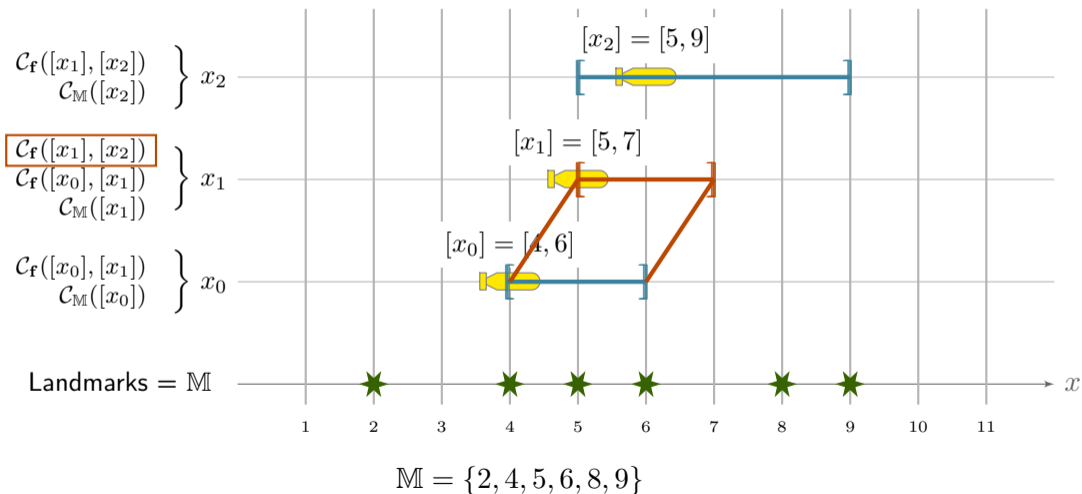
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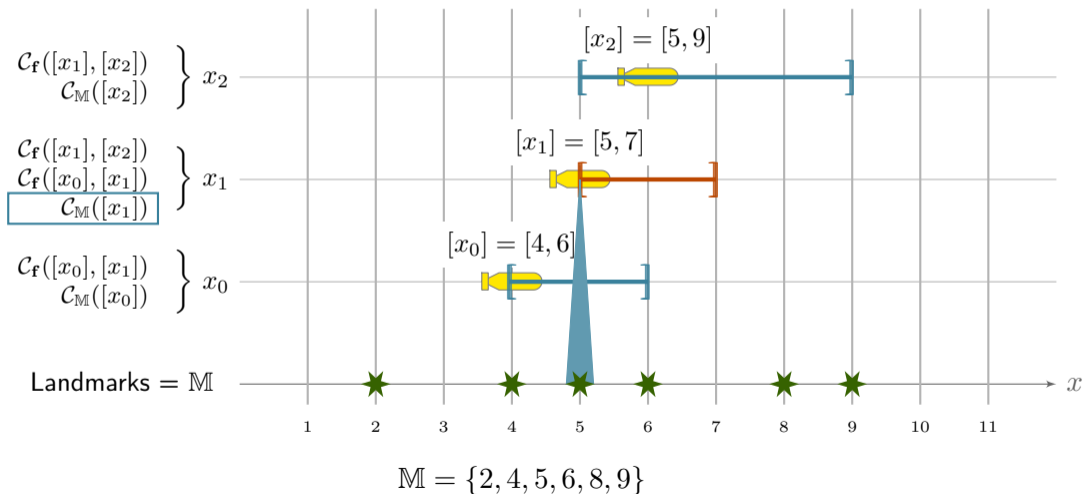
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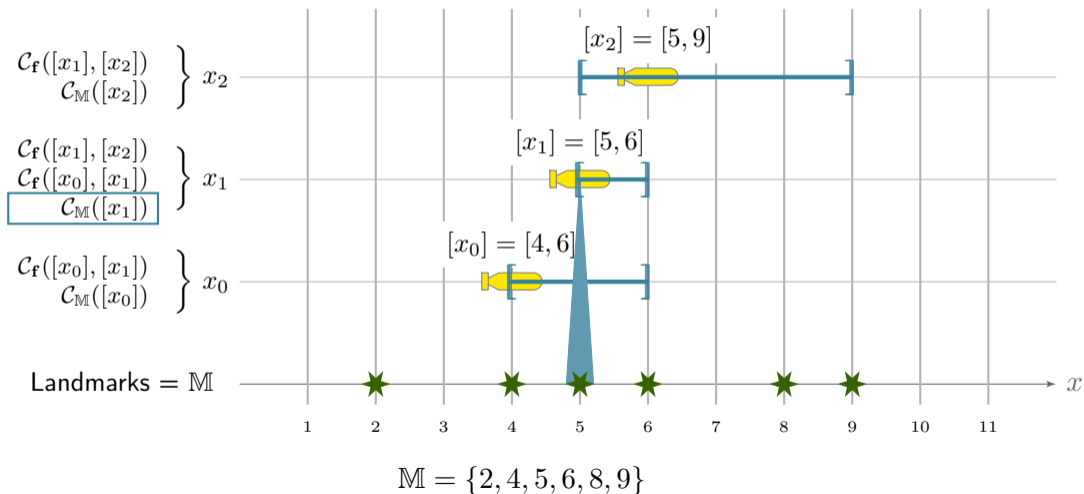
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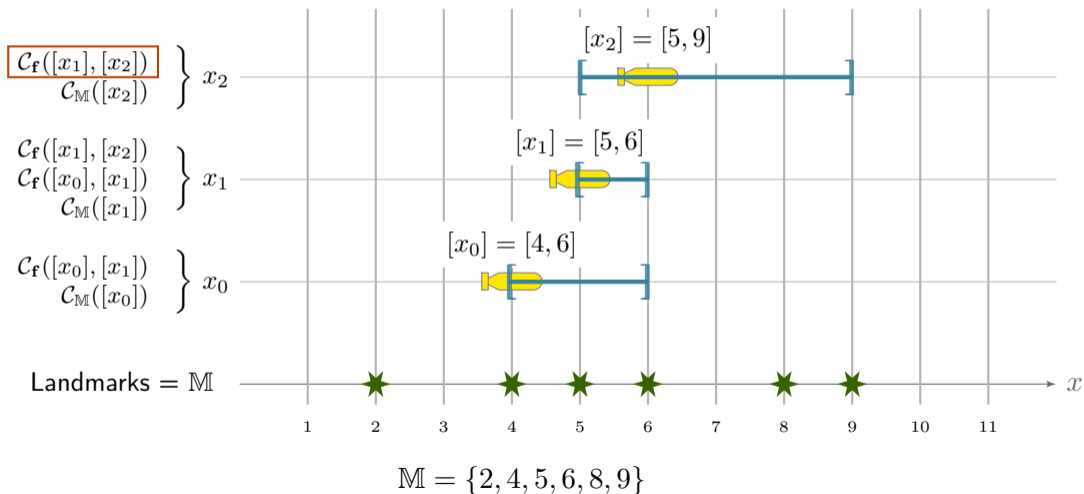
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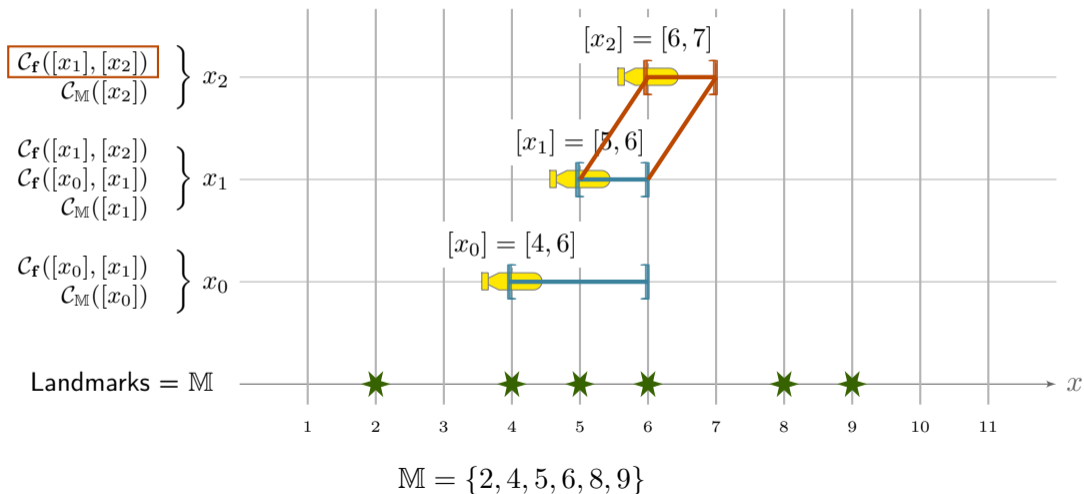
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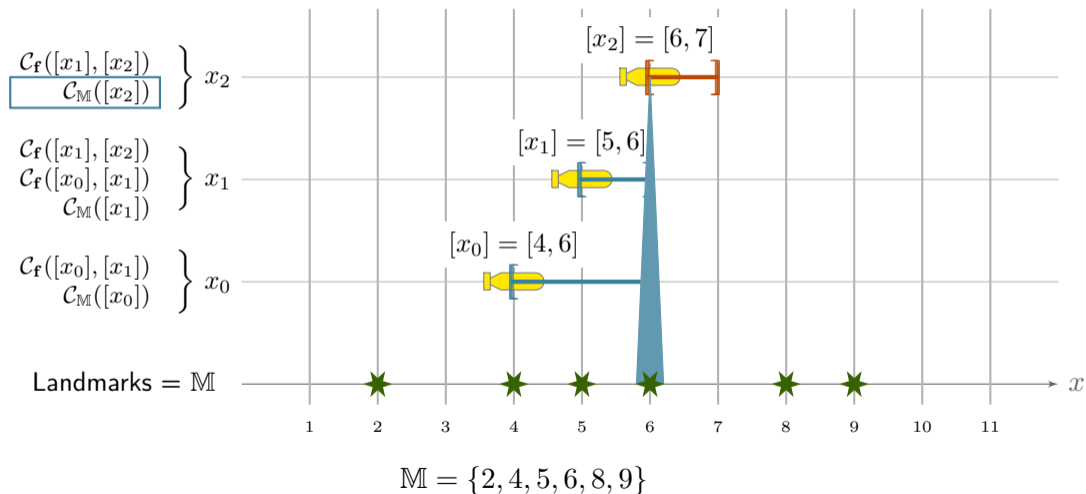
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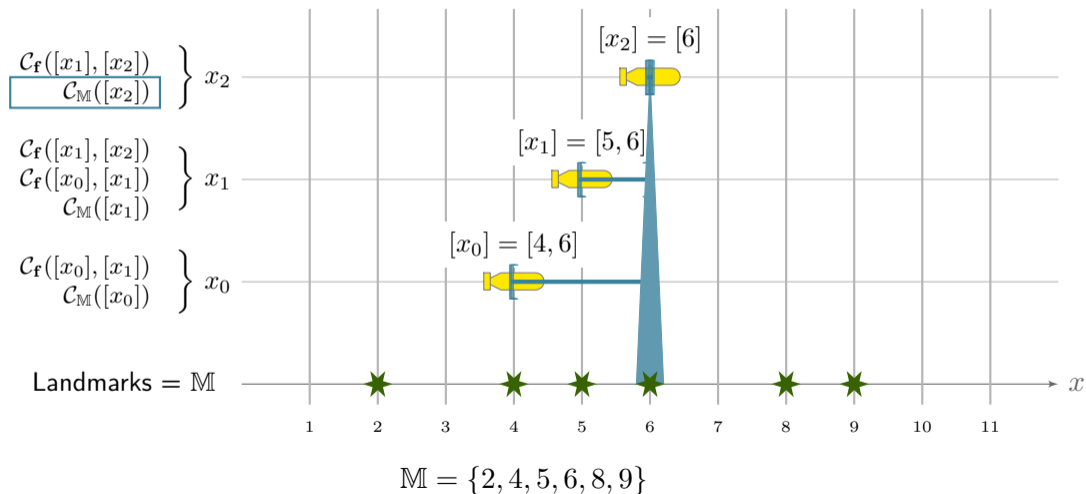
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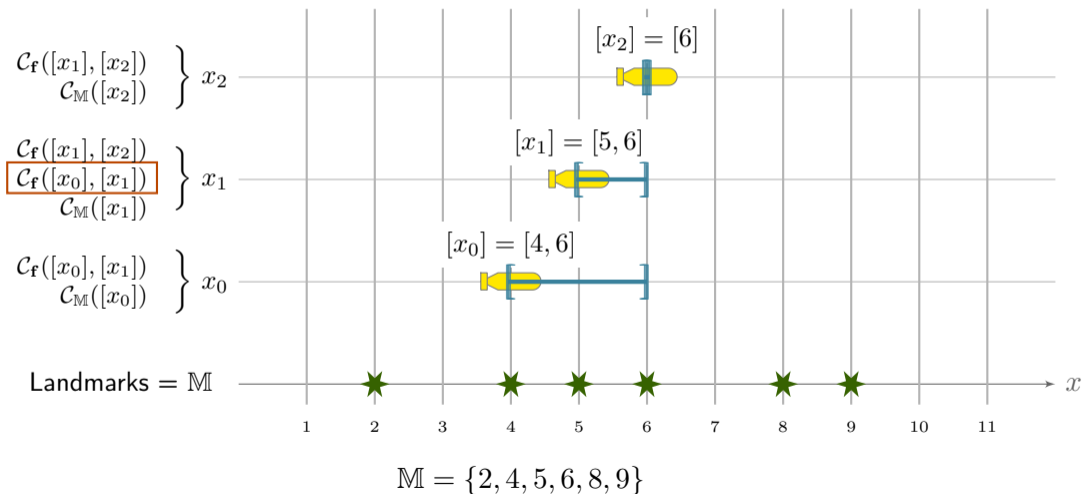
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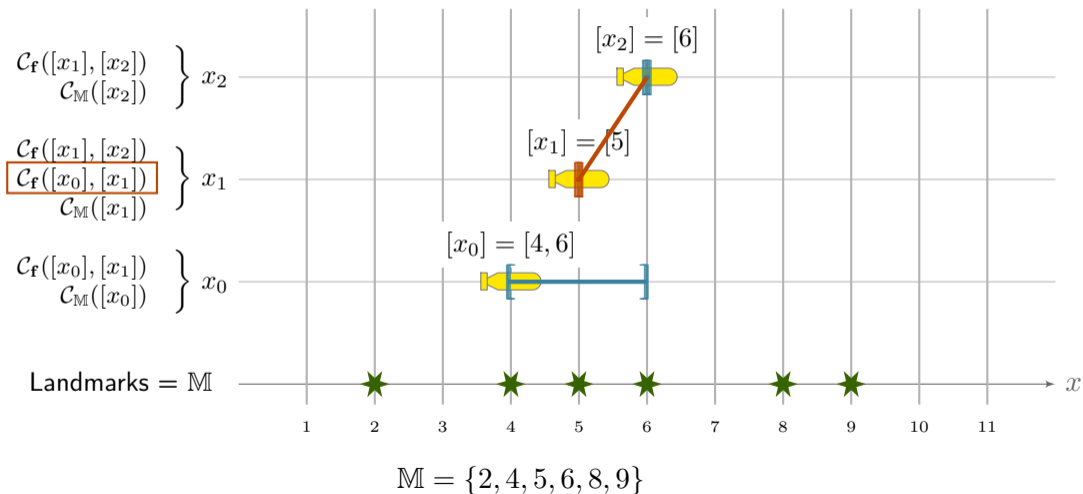
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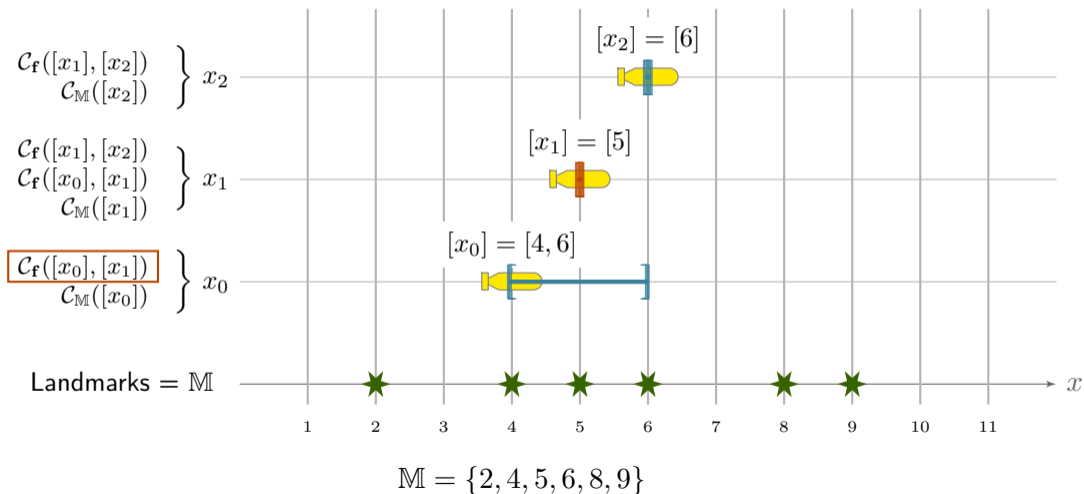
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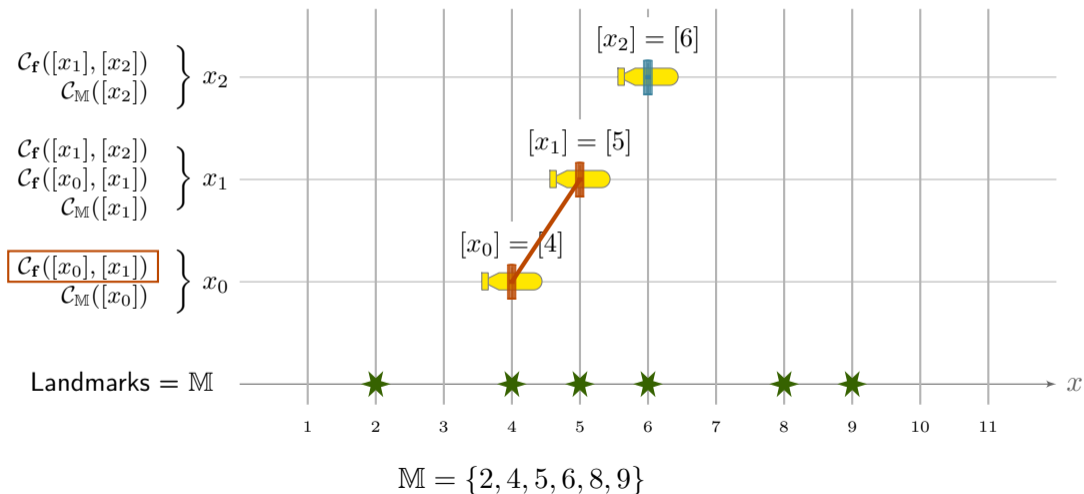
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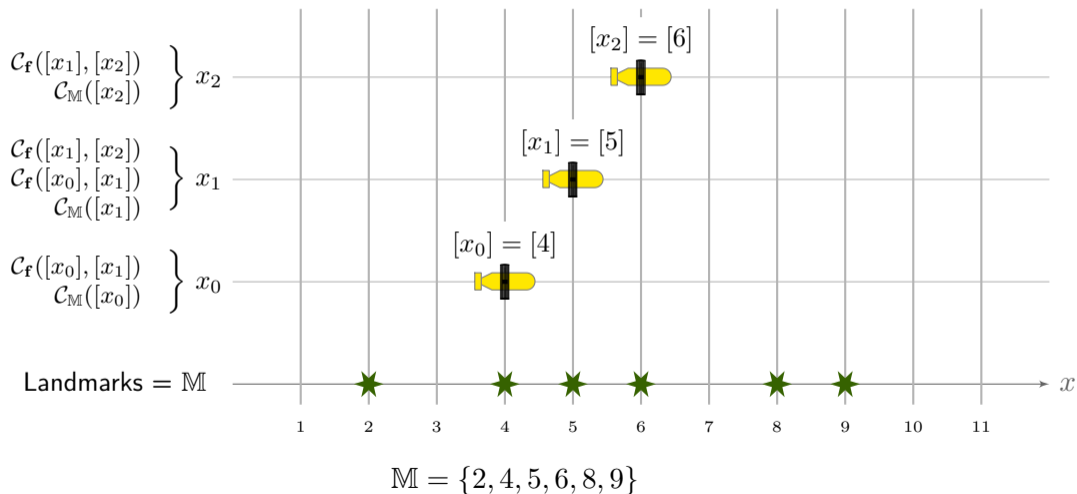
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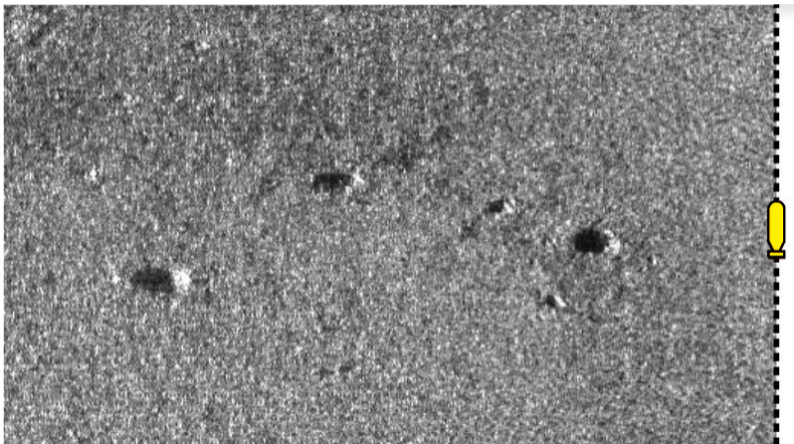
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Constellation contractor for a 2d map of landmarks

Constraint $\mathbf{m}^i \in \mathbb{M}$:

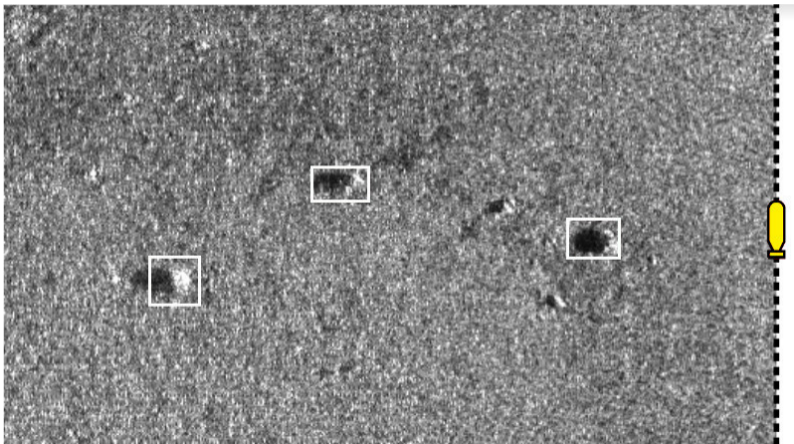
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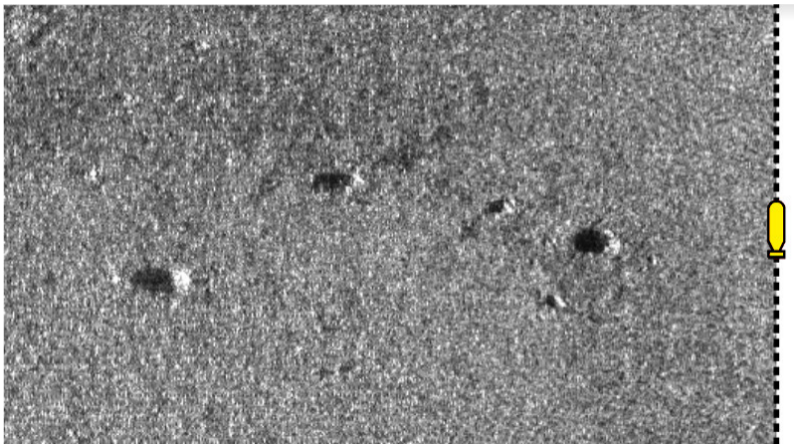
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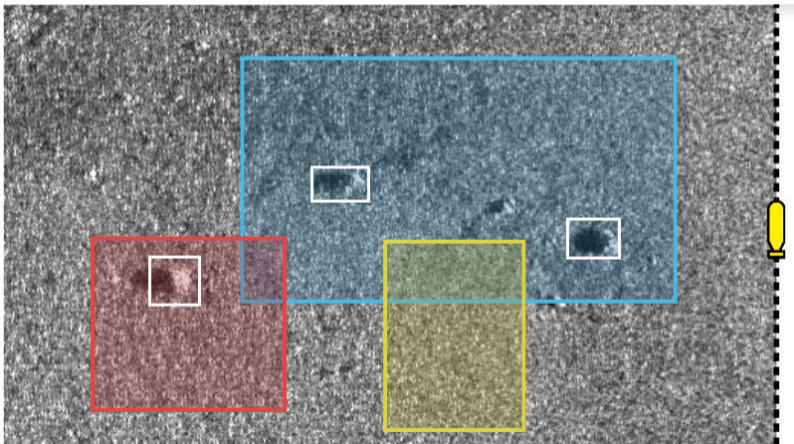
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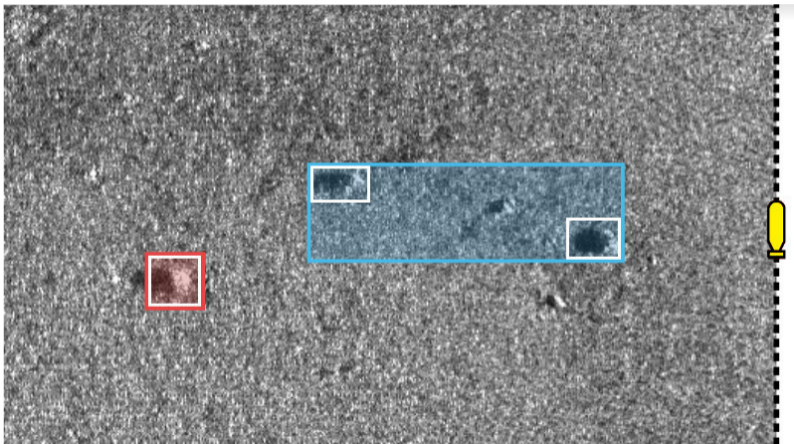
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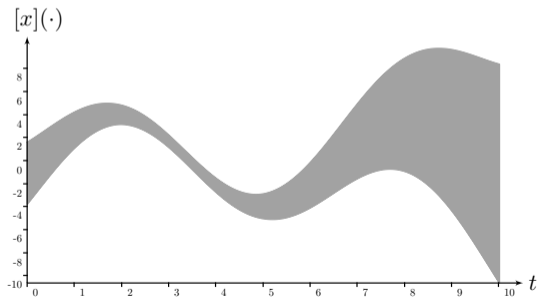
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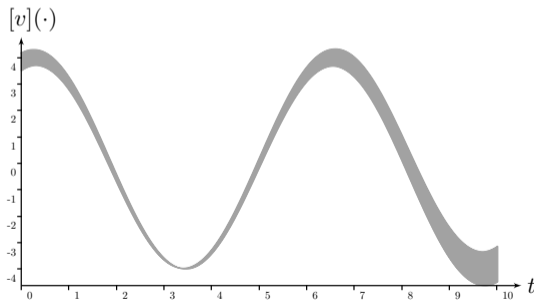
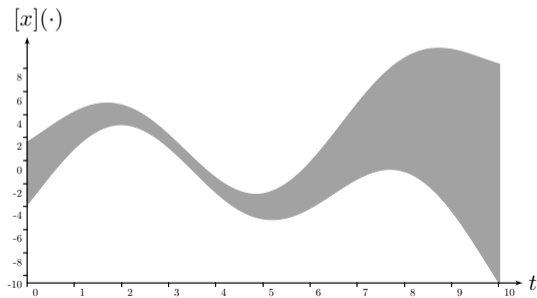
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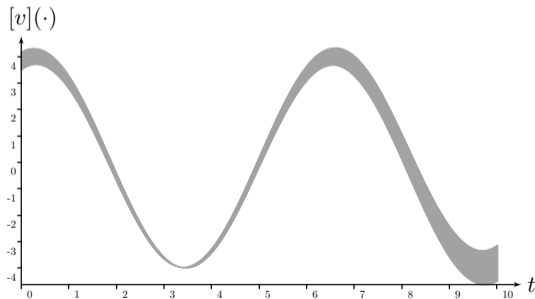
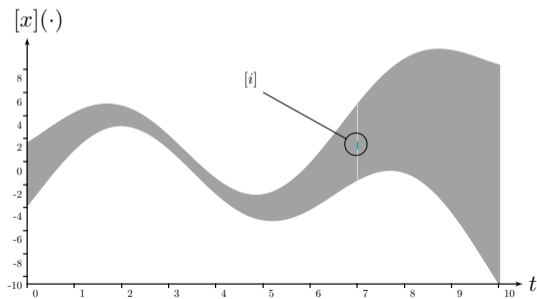
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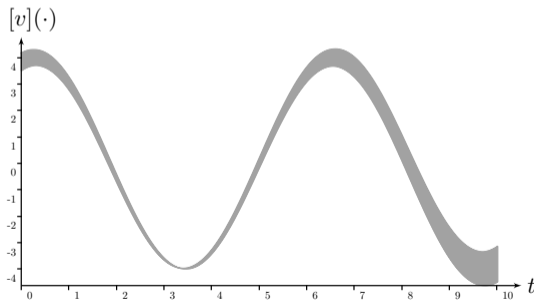
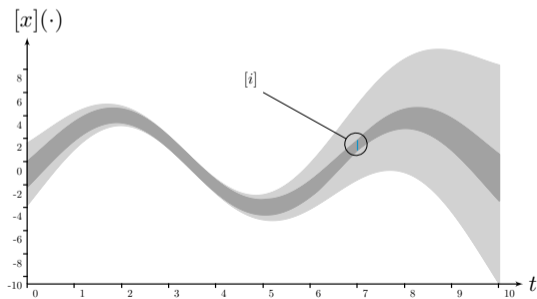
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Applying contractors to solve the problem

A solver is then created as a **list of contractors** \mathcal{C} related to the state equations:

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Fixed point resolution method

Results with the Autonomous Underwater Vehicle (AUV) Daurade



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