Contractor programming for mobile robotics

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AID, Palaiseau
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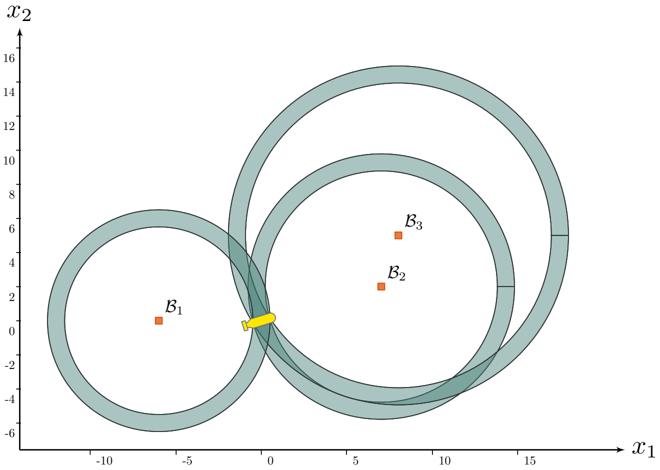
Mobile robotics

- Daurade: Autonomous Underwater Vehicle (AUV)
- weight: 1010kg – length: 5m – max depth: 300m

Special thanks to DGA-TN Brest (formerly GESMA)

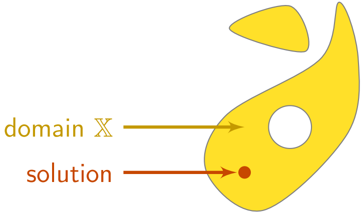
Uncertainties as sets

Example of **range-only** robot localization (three beacons):



Constraint programming: overall concept

- system described by a *constraint network*
- **variables** belonging to **domains** $\mathbb{X}$



Constraint network:

$$\begin{cases}\textbf{Variables: } \mathbf{x} \\ \textbf{Constraints:} \\ \\ \\ \\ \textbf{Domains: } \mathbb{X}\end{cases}$$

Contractor Programming
Chabert, Jaulin *Artifical Intelligence*, 2009

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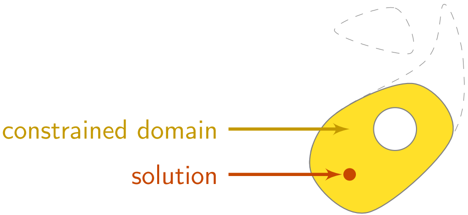

Constraint network:

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Constraints:

1. $\mathcal{L}_1(\mathbf{x})$

Domains: $\mathbb{X}$

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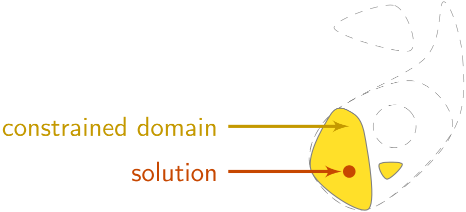

Constraint network:

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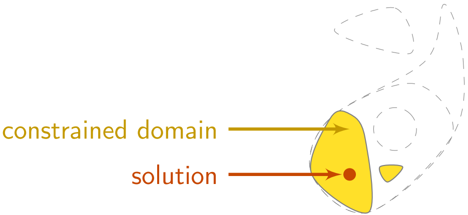

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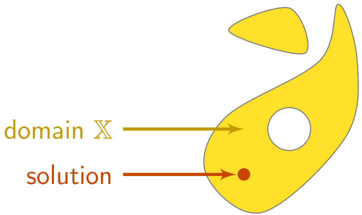

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■ Contractor Programming
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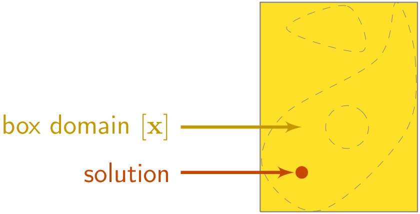

Constraint network:

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3. ...

Domains: $[\mathbf{x}]$

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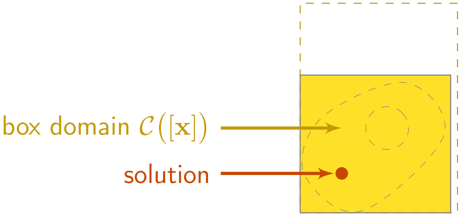
- resolution by **contractors**, $\mathcal{C}_{\mathcal{L}}([\mathbf{x}])$



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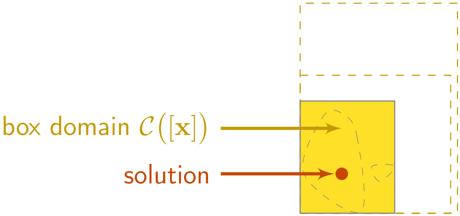

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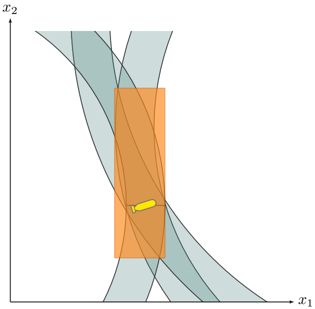
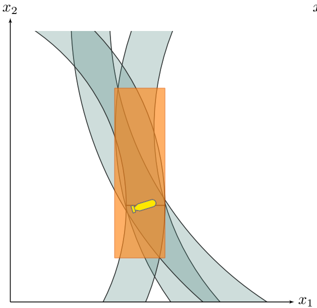
Wrappers

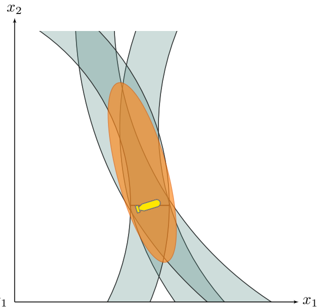
- box



Wrappers

- box
- ellipse





Wrappers

- box
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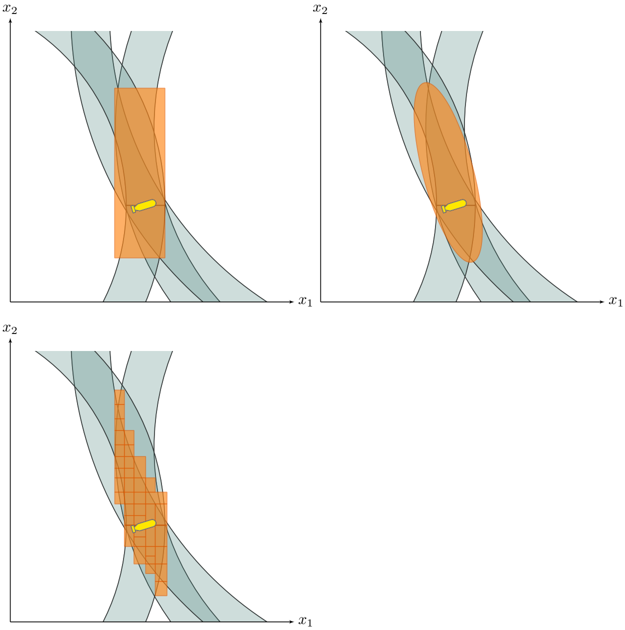
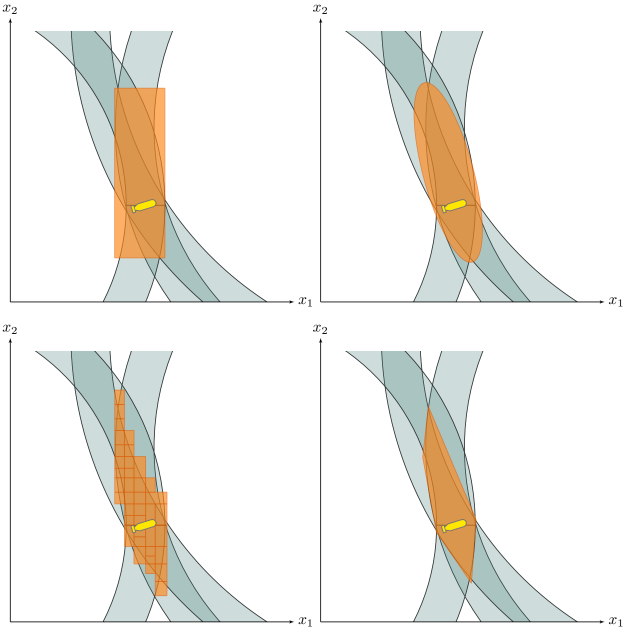


Wrappers

- box
- ellipse
- paving
- polygon



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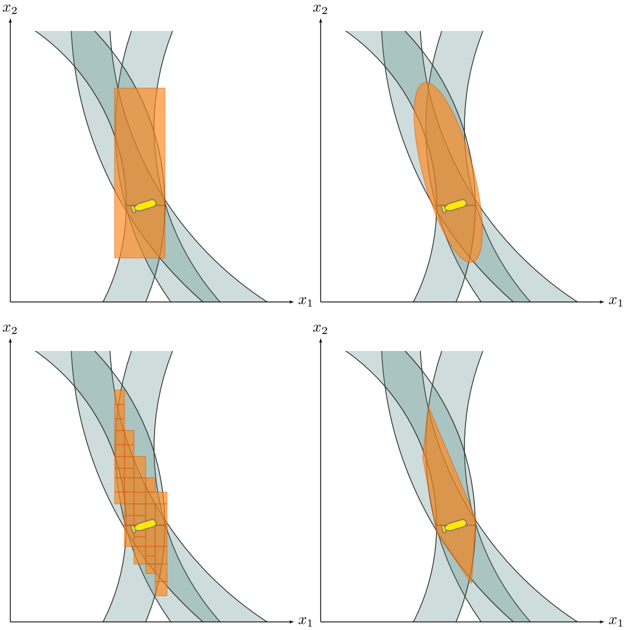
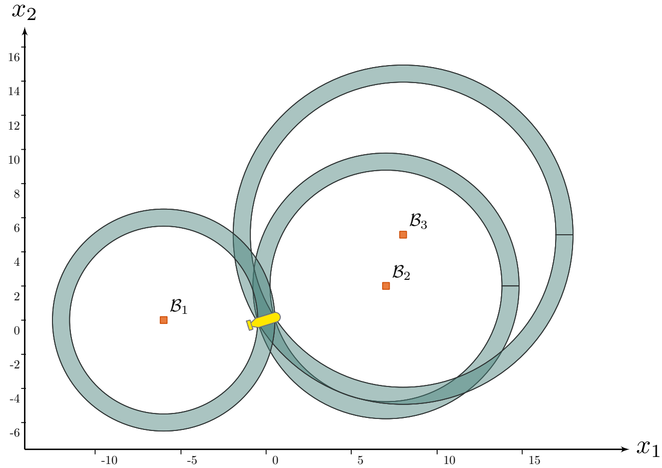


Set-membership state estimation

Three observations $\rho^{(k)}$ from three beacons $\mathcal{B}^{(k)}$:



Constraints

Observation constraint, links a measurement $\rho^{(k)}$ to the state $\mathbf{x}$:

$$\rho^{(k)} = \sqrt{\left(x_1 - \mathcal{B}_1^{(k)}\right)^2 + \left(x_2 - \mathcal{B}_2^{(k)}\right)^2}.$$

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Problem synthesized as a **constraint network**:

Variables: $\mathbf{x}$, $\rho^{(1)}$, $\rho^{(2)}$, $\rho^{(3)}$

Constraints:

1. $\mathcal{L}_g^{(1)}\left(\mathbf{x}, \rho^{(1)}\right)$
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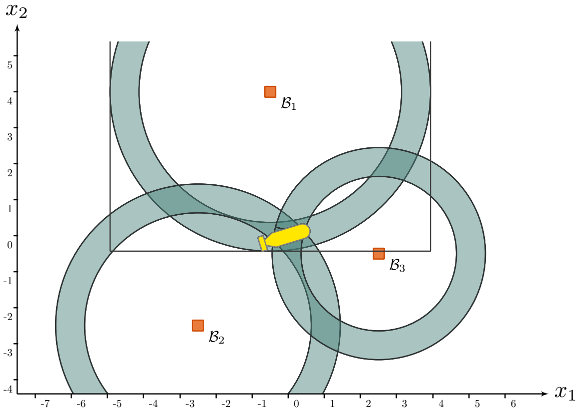
Fixed point propagations



Applying $\mathcal{L}_g^{(1)}$

Study of robust set estimation methods for a high integrity multi-sensor localization.

Vincent Drevelle *Thesis*, 2011

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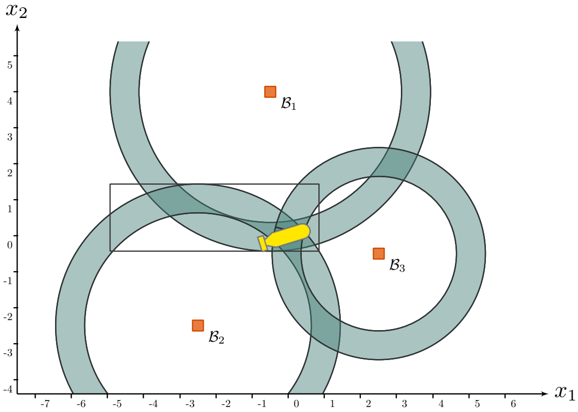

Applying $\mathcal{L}_g^{(2)}$

Study of robust set estimation methods for a high integrity multi-sensor localization.

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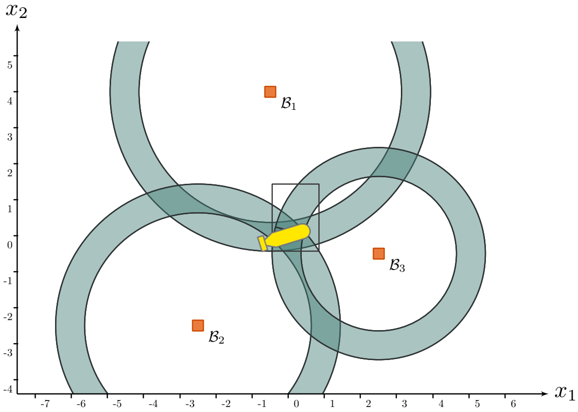
Fixed point propagations



Applying $\mathcal{L}_g^{(3)}$

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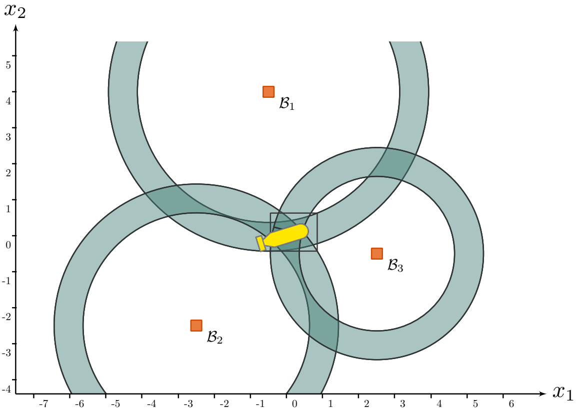

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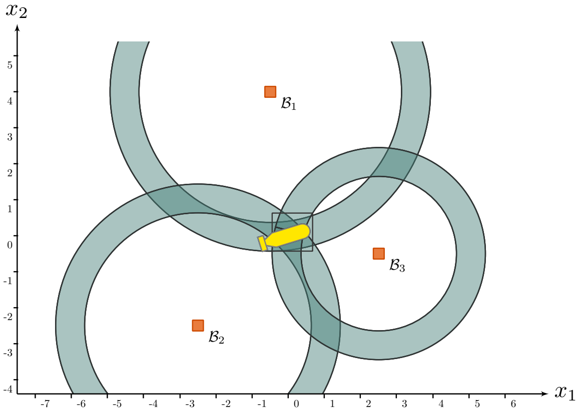

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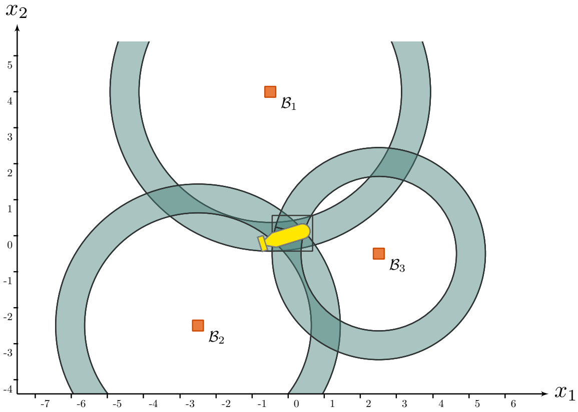

Fixed point reached.

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Constraint programming coupled with **mobile robotics**:

- robot's state vector $\mathbf{x}$ to be estimated
- several proprioceptive/exteroceptive measurements

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Drawbacks:

- unwanted pessimism
- sets as outputs

Sets from sensor data

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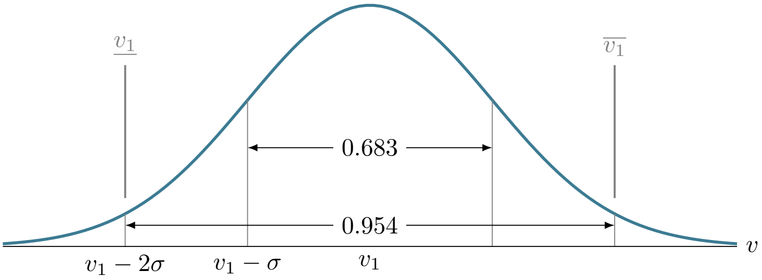
Video

Sets from sensor data

Uncertainties:

- datasheets $\implies$ standard deviation σ for each sensor
- 95% confidence rate: $v_1^* \in [v_1] = [v_1 - 2\sigma, v_1 + 2\sigma]$



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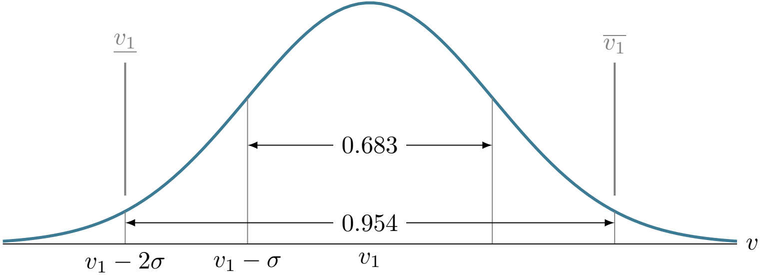
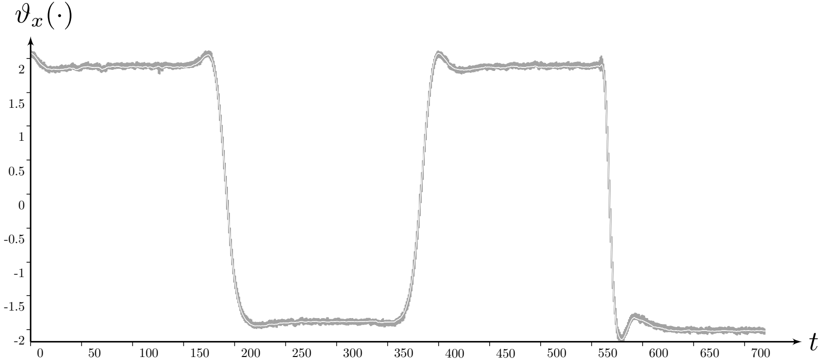


- uncertainties then reliably propagated in the system

  ex: $[x] + [y] = \left[\underline{x} + \underline{y}, \overline{x} + \overline{y}\right]$

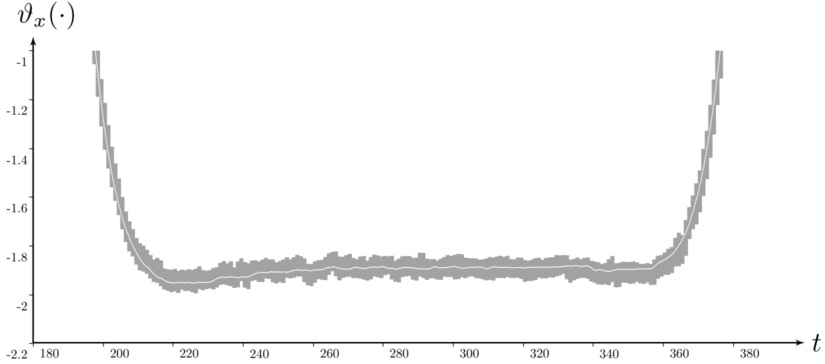
Example: velocity sensing

East velocity given by DVL + IMU:



Example: velocity sensing

East velocity given by DVL + IMU (zoom):



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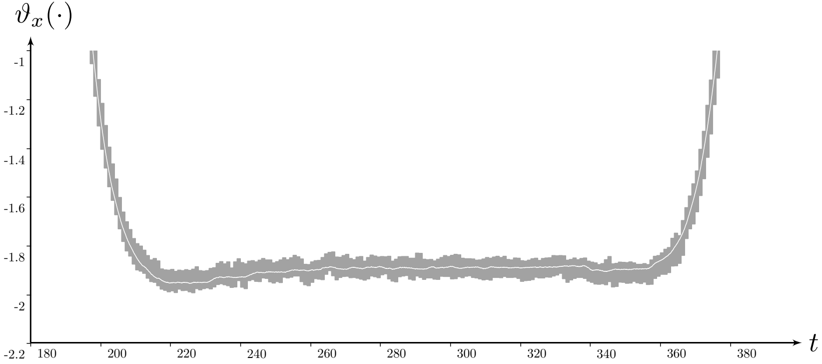
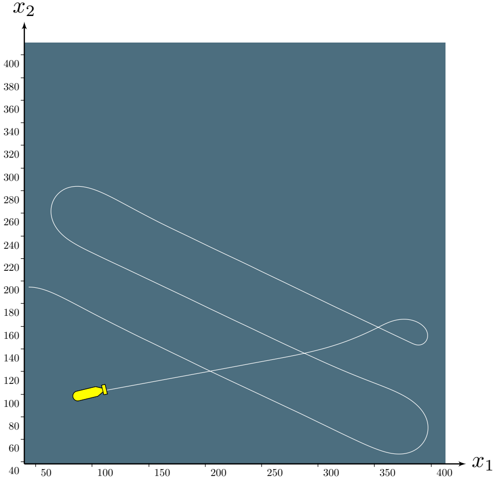


- new variable: **trajectory** $x(\cdot)$
- new domain (set): **tube** $[x](\cdot)$, interval of trajectories

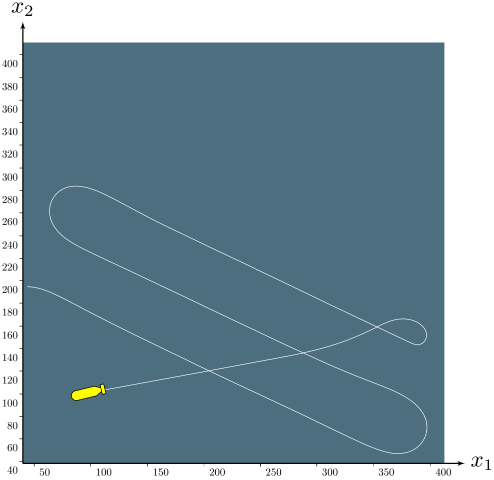
Dynamic state estimation



State estimation:

$$\left\{ \begin{array}{l} \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) \\ \\ \end{array} \right.$$

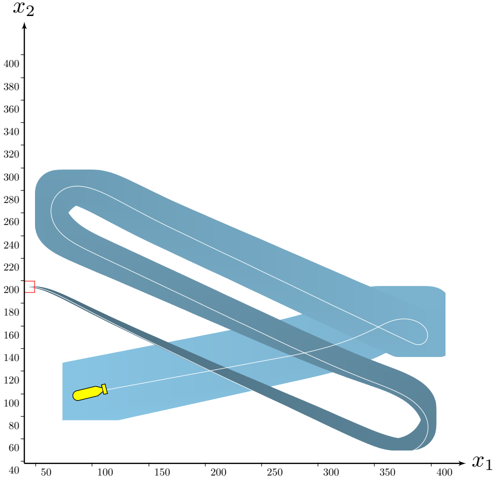
Dynamic state estimation



State estimation:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) \\ \dot{\mathbf{x}}(t) = \mathbf{v}(t) \\ \\ \end{cases}$$

Dynamic state estimation



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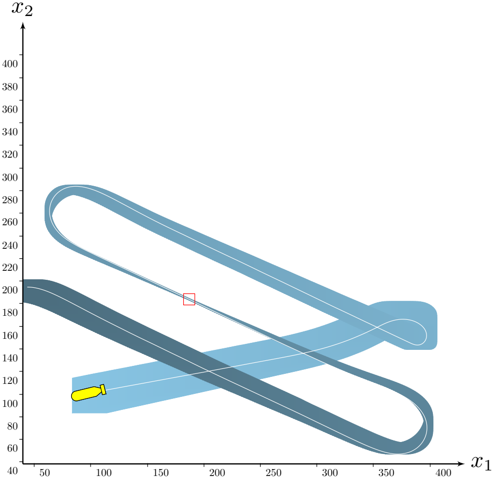
Dynamic state estimation



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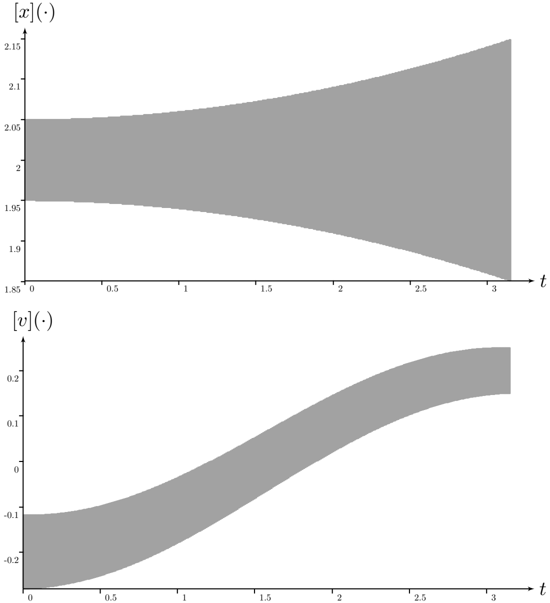
Derivative constraint

Differential constraint:

- $\dot{\mathbf{x}}(\cdot) = \mathbf{v}(\cdot)$
- one trajectory and its derivative



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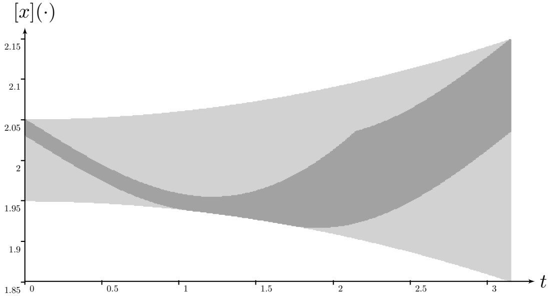
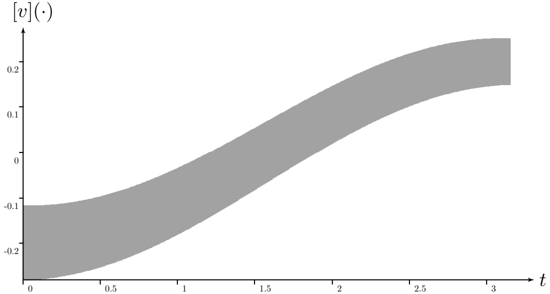
Contractor programming:

$$\mathcal{C}_{\frac{d}{dt}}\big([\mathbf{x}](\cdot), [\mathbf{v}](\cdot)\big)$$





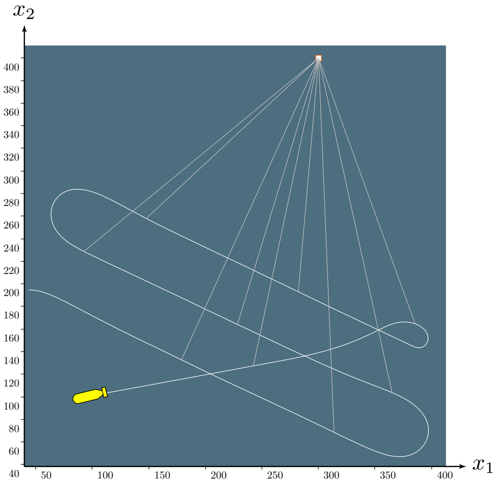
Guaranteed computation of robot trajectories

Rohou, Jaulin, Mihaylova, Le Bars, Veres

Robotics and Autonomous Systems, 2017

Dynamic state estimation

Considering **range-only measurements** from a known beacon.

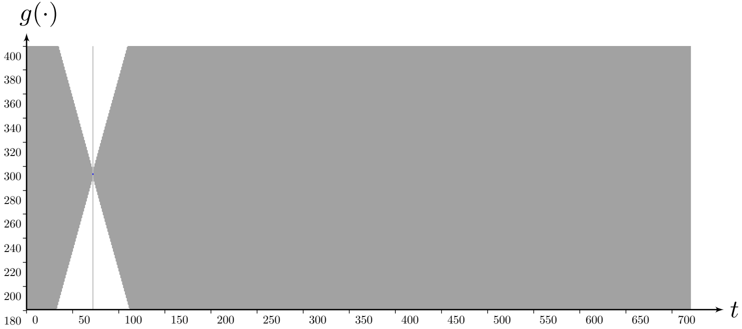


Non-linear state estimation:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) \\ y_i = g\big(\mathbf{x}(t_i)\big) \end{cases}$$

Exteroceptive measurements

Creating another tube $[g](\cdot)$ that will be **constrained by measurements**.



Observation tube, considering 1 range-only measurement from the beacon.

Exteroceptive measurements

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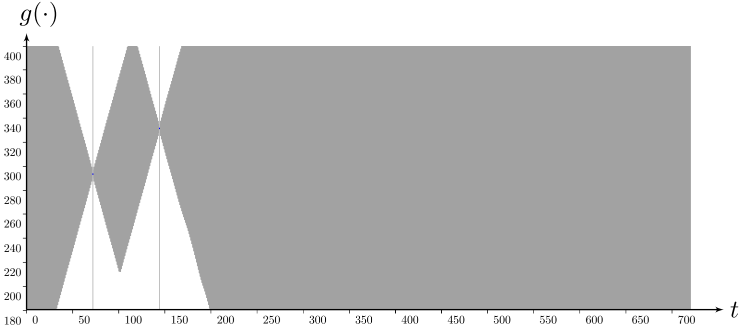

Observation tube, considering 2 range-only measurements from the beacon.

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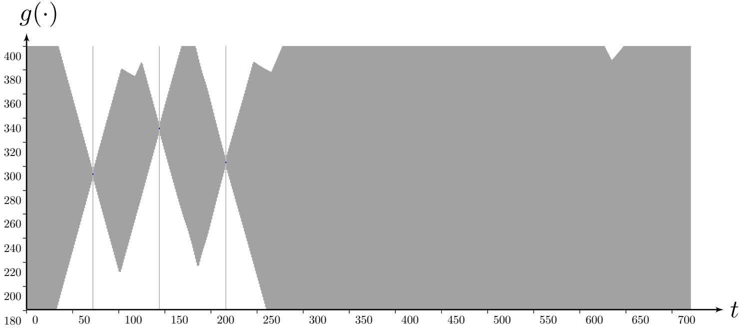

Observation tube, considering 3 range-only measurements from the beacon.

Exteroceptive measurements

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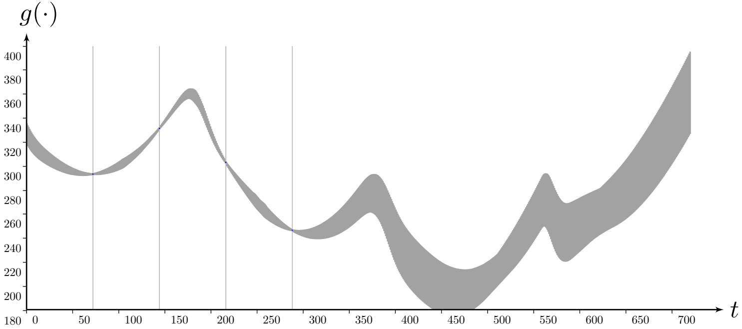

Observation tube, considering 4 range-only measurements from the beacon.

Exteroceptive measurements

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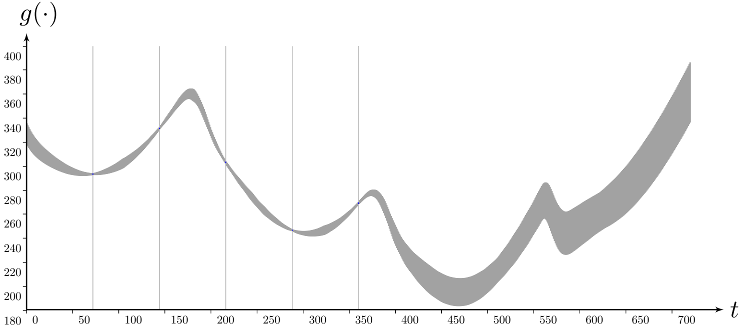

Observation tube, considering 5 range-only measurements from the beacon.

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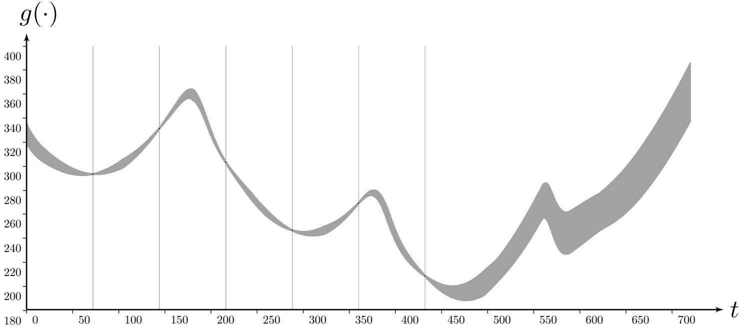

Observation tube, considering 6 range-only measurements from the beacon.

Exteroceptive measurements

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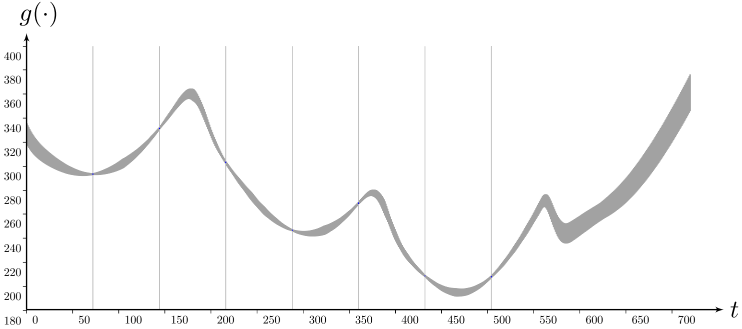
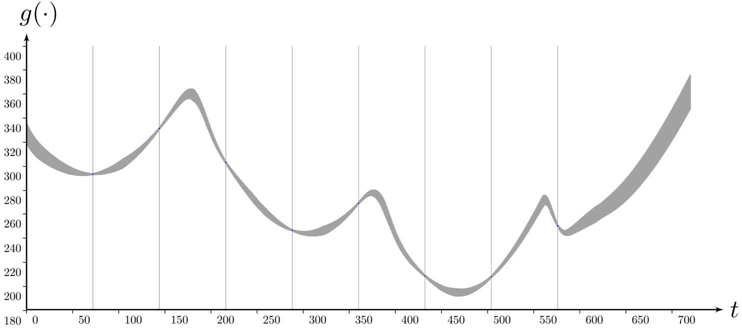


Observation tube, considering 7 range-only measurements from the beacon.

Exteroceptive measurements

Creating another tube $[g](\cdot)$ that will be **constrained by measurements**.



Observation tube, considering 8 range-only measurements from the beacon.

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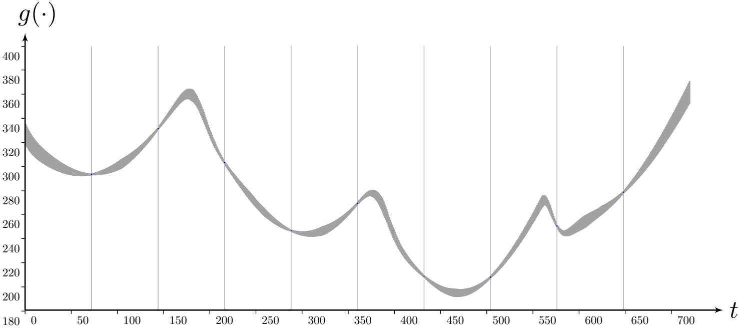

Observation tube, considering 9 range-only measurements from the beacon.

Exteroceptive measurements

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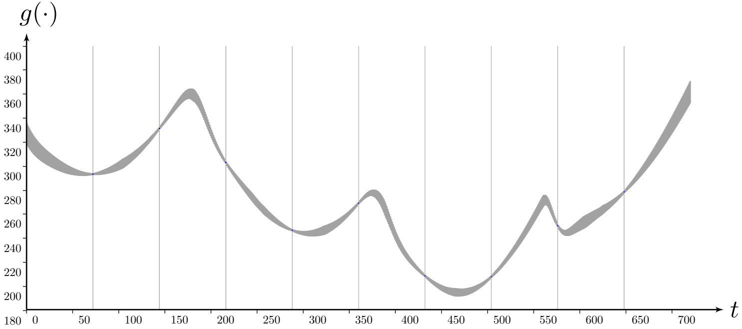


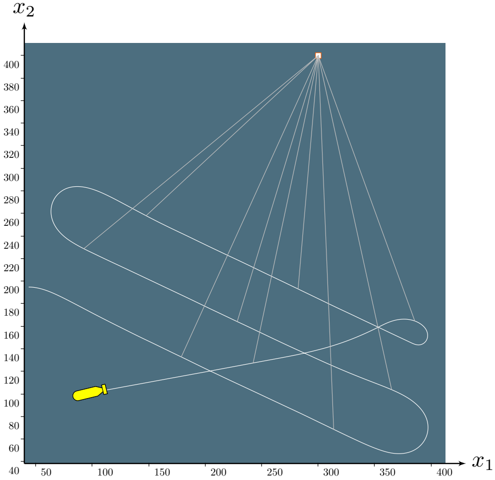
Observation tube, considering 9 range-only measurements from the beacon.

Then the state tube $[\mathbf{x}](\cdot)$ will be constrained by $[g](\cdot)$.

$$\mathcal{L}_g : \quad g(\cdot) = \sqrt{\big(x_1(\cdot) - \mathcal{B}_1\big)^2 + \big(x_2(\cdot) - \mathcal{B}_2\big)^2}.$$

Dynamic state estimation

Considering **range-only measurements** from a known beacon.



State estimation:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) \\ y_i = g\big(\mathbf{x}(t_i)\big) \end{cases}$$

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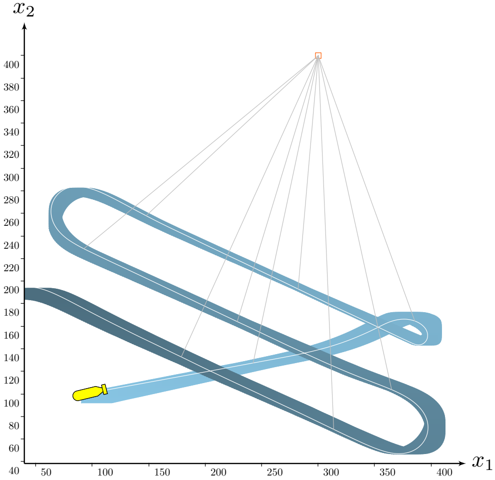

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Trajectory evaluation constraint

$$\text{Trajectory evaluation}\left\{\begin{array}{l}\mathbf{z}=\mathbf{y}(t)\\ \\ \\ \end{array}\right.$$

Reliable non-linear state estimation involving time uncertainties
Rohou, Jaulin, Mihaylova, Le Bars, Veres *Automatica*, 2018

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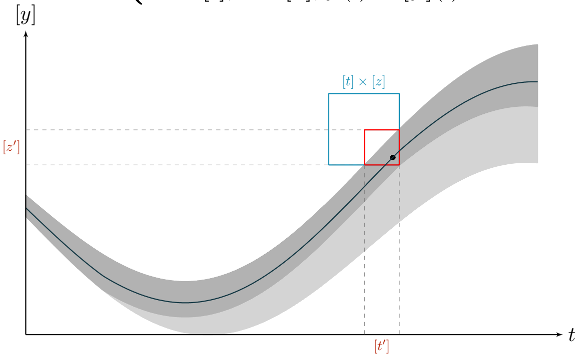

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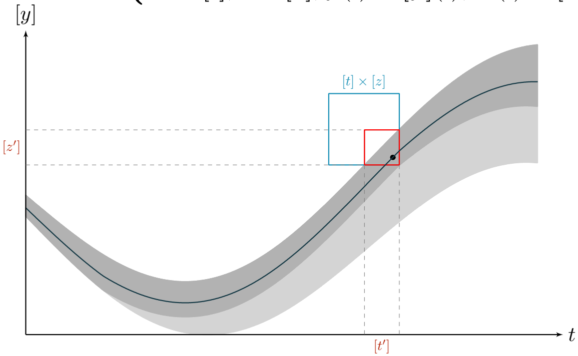
Trajectory evaluation constraint

Trajectory evaluation $\begin{cases} \mathbf{z} = \mathbf{y}(t) \\ \dot{\mathbf{y}}(\cdot) = \mathbf{w}(\cdot) \\ t \in [t], \mathbf{z} \in [\mathbf{z}], \mathbf{y}(\cdot) \in [\mathbf{y}](\cdot), \mathbf{w}(\cdot) \in [\mathbf{w}](\cdot) \end{cases}$



Contractor programming: $\mathcal{C}_{\text{eval}}\big([t], [\mathbf{z}], [\mathbf{y}](\cdot), [\mathbf{w}](\cdot)\big)$

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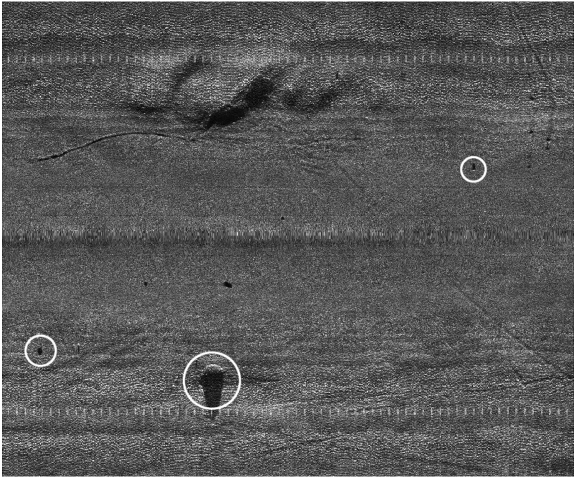
Tubex library: open-source library providing tools for constraint programming over dynamical systems

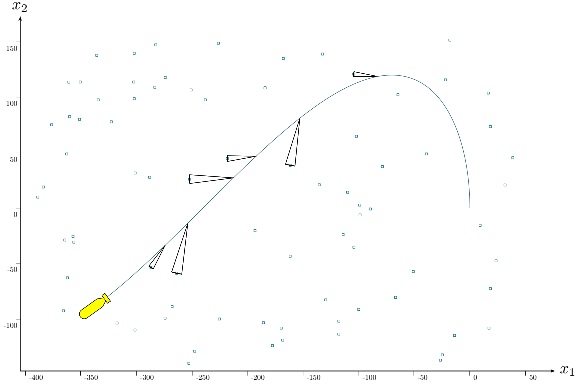
http://www.simon-rohou.fr/research/tubex-lib

Towards more applications...



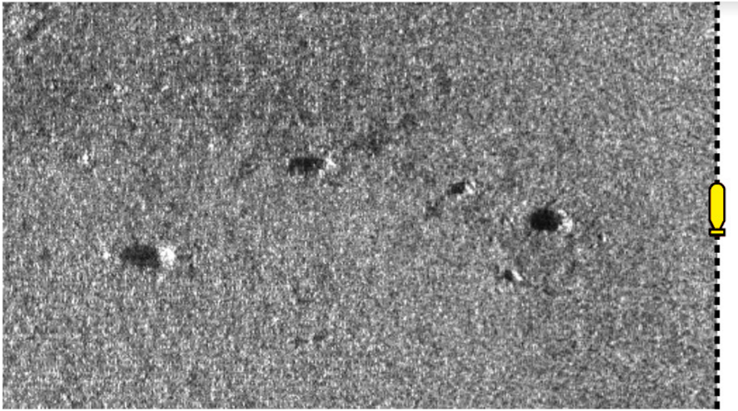
Localization with data association

$$
\begin{cases}
\dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big), & \text{(evolution equation)} \\
\mathbf{g}\big(\mathbf{x}(t_i), \mathbf{y}(t_i), \mathbf{m}(t_i)\big) = \mathbf{0}, & \text{(observation equation)} \\
\mathbf{m}(t_i) \in \mathbb{M}. & \text{(mapped landmark constraint)}
\end{cases}
$$



Localization with data association

Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



Perception of the seabed with a side-scan sonar.

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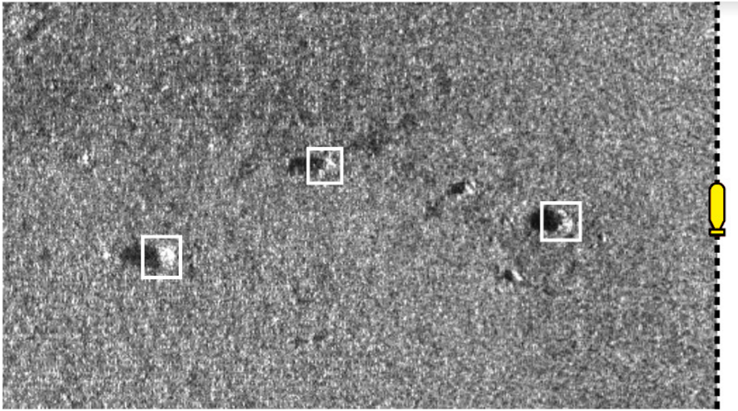

Seamarks are already known with some uncertainty.

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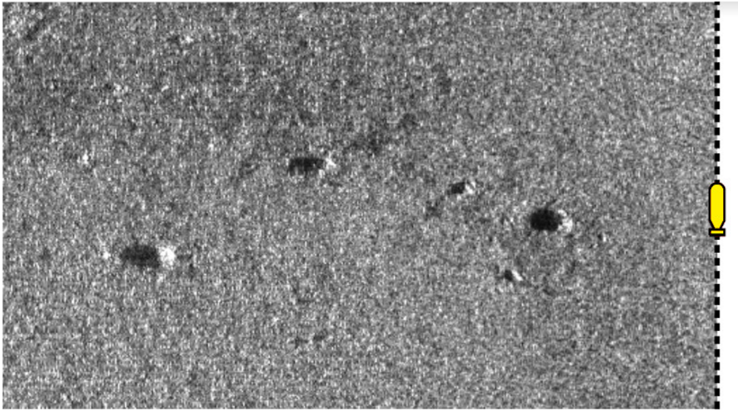

Some of the rocks may be observed by the robot with its sonar.

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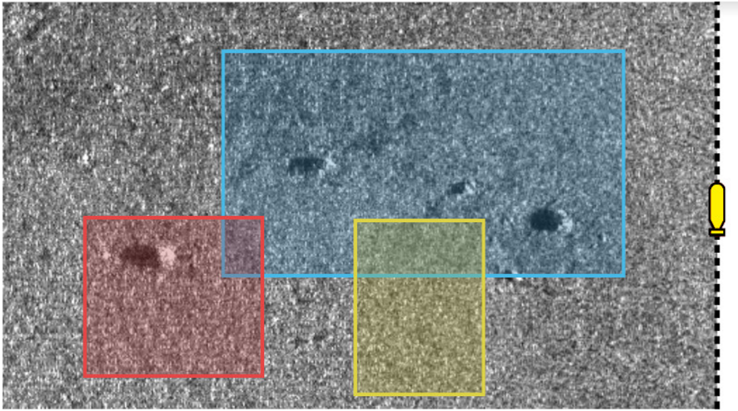

The position of the rock is first estimated from robot's position estimate.

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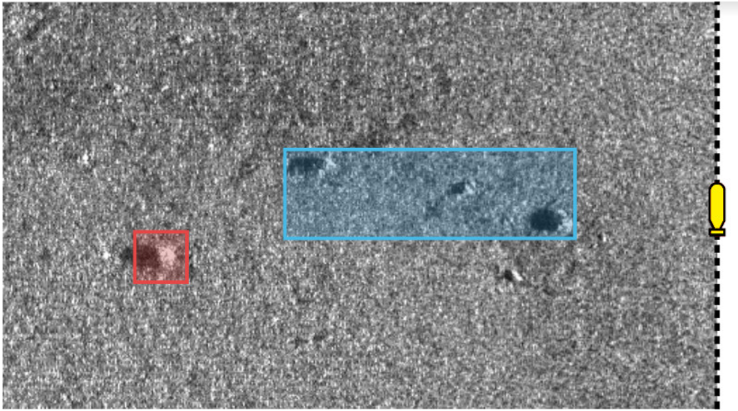

Then the position of the rock is contracted from the known map.

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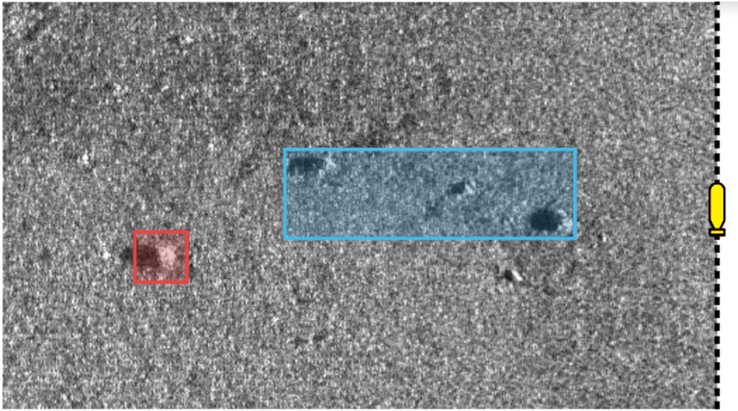

If the boxed-position is a singleton, then the rock is *identified*.

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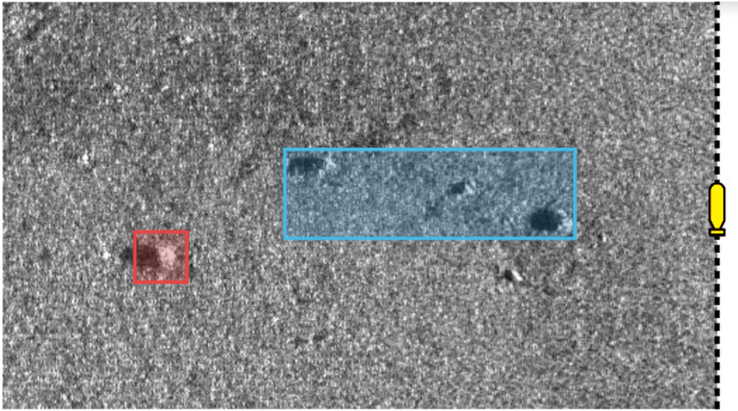

In any cases, the boxed-positions of the rocks allow localization updates.

Localization with data association

Video