From *Tintin, Land of Black Gold.*

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Verifying loops in robot trajectories under uncertainties

Simon Rohou[1], Peter Franek[2], Clément Aubry[3], Luc Jaulin[1]

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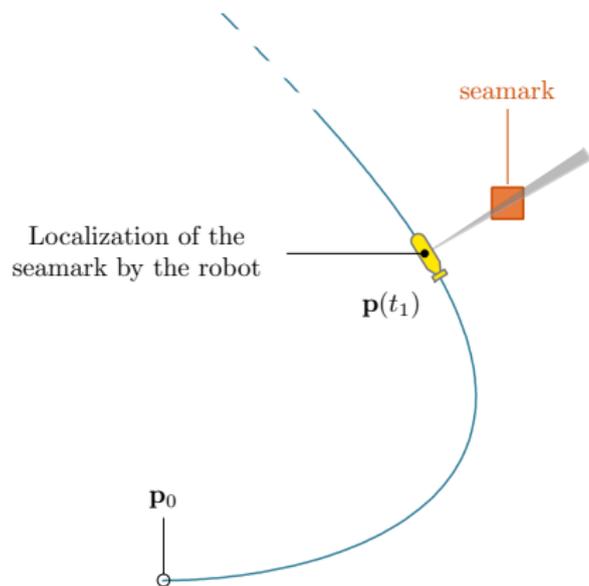
SWIM
26th July 2019, Palaiseau

Section 2

Motivations
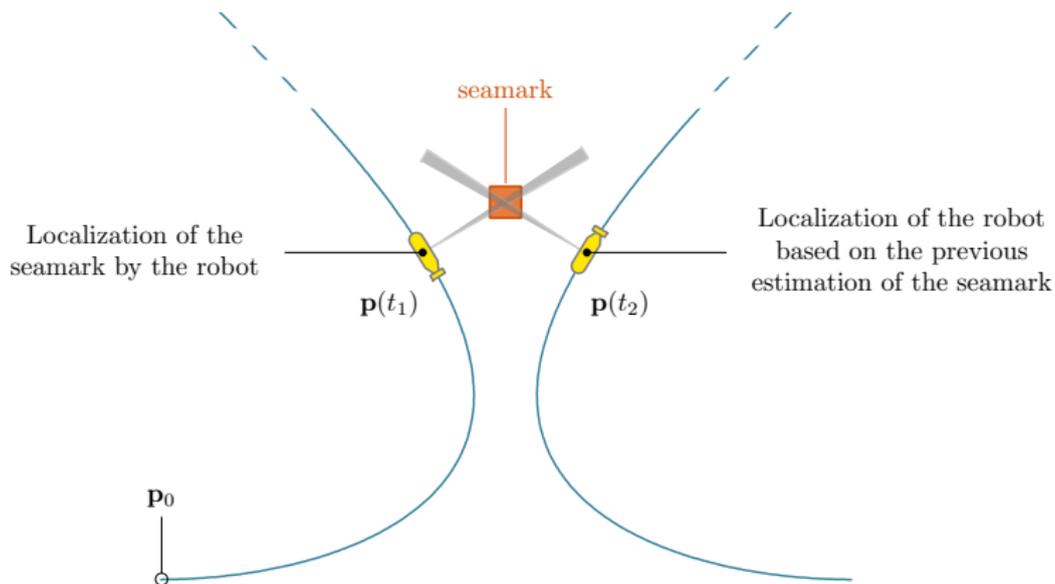
Motivations

Simultaneous Localization and Mapping

▶ **come back** to a previous pose and **recognize** the environment
▶ problem: loop closure detection

seamark

Localization of the
seamark by the robot

$\mathbf{p}(t_1)$

$\mathbf{p}_0$

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seamark

Localization of the robot
based on the previous
estimation of the seamark

$\mathbf{p}(t_1)$ $\mathbf{p}(t_2)$

$\mathbf{p}_0$

Motivations
Simultaneous Localization and Mapping

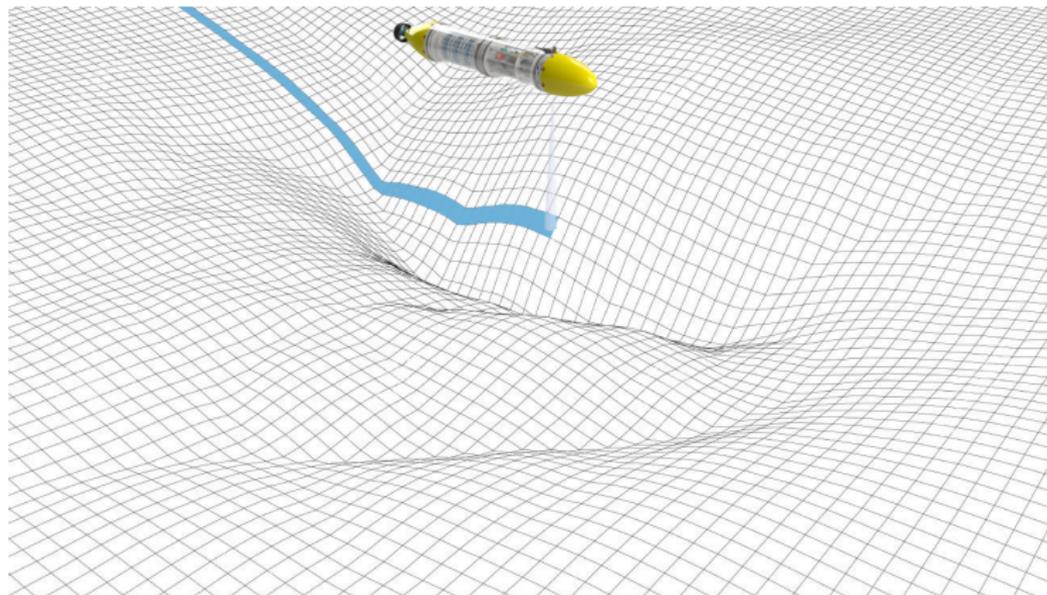The problem of **false loop detections** in similar environments.

In Versailles' gardens: similar places. *Did we really come back to a previous place?*

Motivations
Underwater robot localization

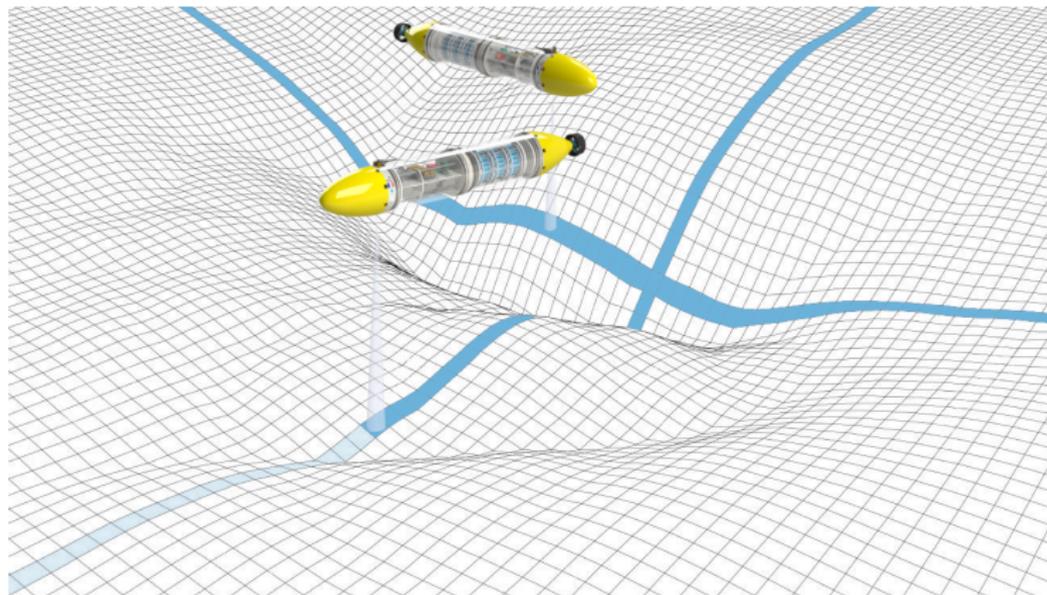An underwater robot performing a loop during an exploration:

Robot's trajectory is projected in blue on the seabed.

Motivations

Underwater robot localization

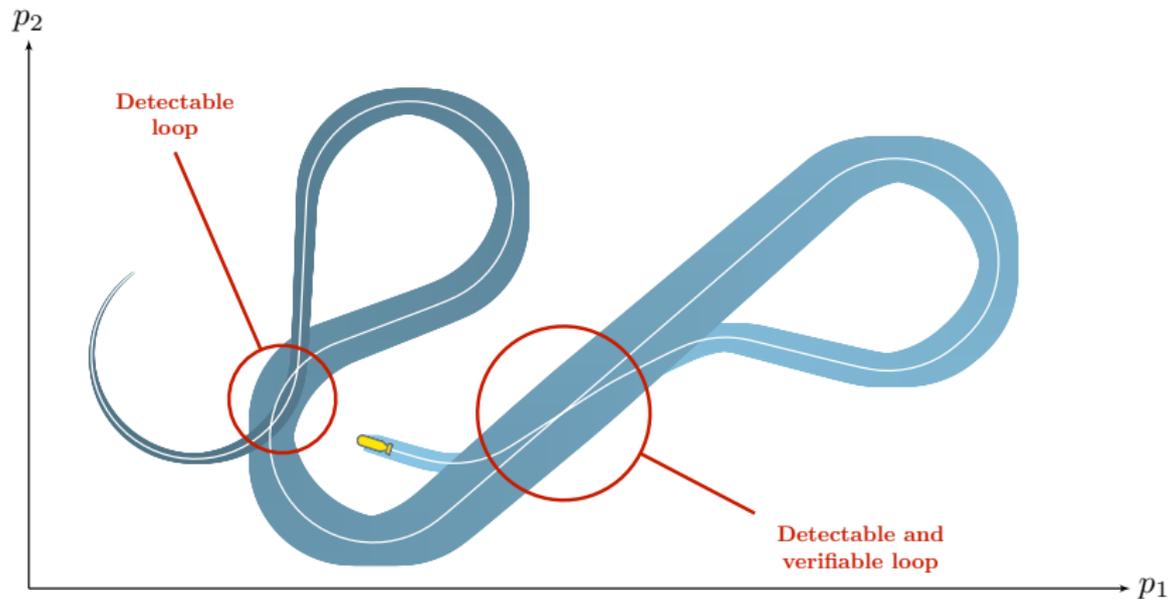An underwater robot performing a loop during an exploration:

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Motivations

Uncertainties: detection vs verification

Uncertain trajectories enclosed by **tubes**.

Only one loop can be verified – at least two feasible loops are detected
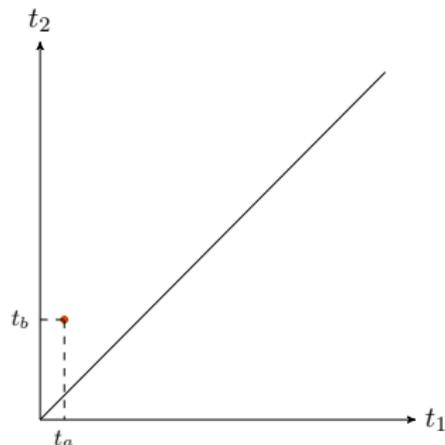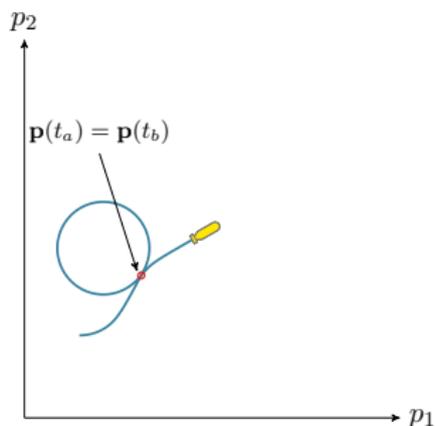
Section 3

Looped trajectories

Looped trajectories
Definitions (Aubry, 2013)

▶ robot position: $\mathbf{p} = (x, y)^{\mathsf{T}} \in \mathbb{R}^2$

▶ 2D robot trajectory: $\mathbf{p}(t) : \mathbb{R} \to \mathbb{R}^2$, $t \in [t_0, t_f]$

▶ looped trajectory ⇔ trajectory that crosses itself
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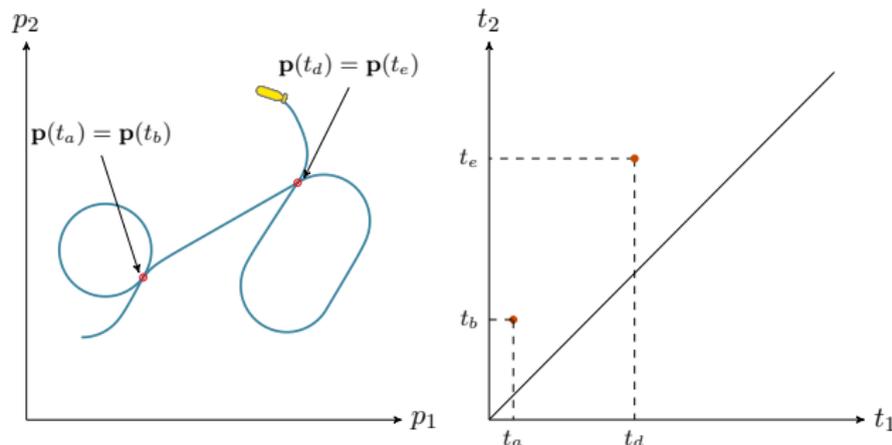
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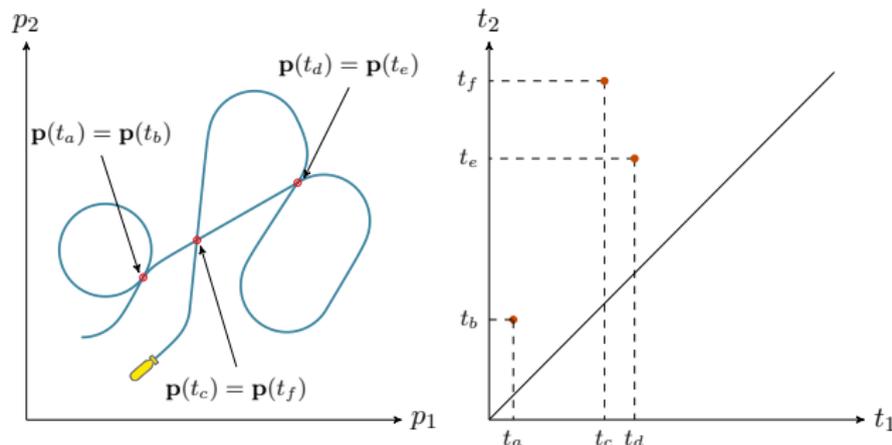
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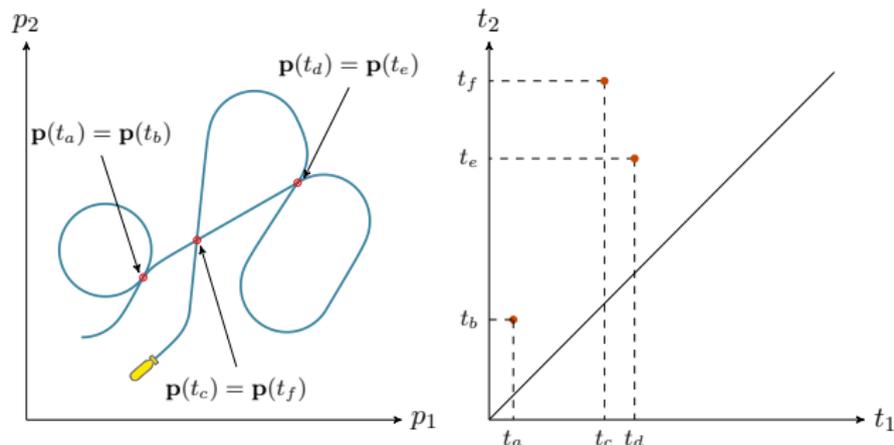
Looped trajectories
Definitions (Aubry, 2013)

▶ t-plane $\Leftrightarrow$ all feasible t-pairs $= [t_0, t_f]^2$

▶ *loop set* $\mathbb{T}^*$:
  ▶ $\mathbb{T}^* = \left\{ (t_1, t_2) \in [t_0, t_f]^2 \mid \mathbf{p}(t_1) = \mathbf{p}(t_2), t_1 < t_2 \right\}$

▶ *loop set* of below example:
  ▶ $\mathbb{T}^* = \{(t_a, t_b), (t_c, t_f), (t_d, t_e)\}$

Looped trajectories

Computing loops from robot sensors

Context: robot trajectory $\mathbf{p}(t)$ cannot be directly sensed.
Computation from speed measurements:

$$\mathbf{p}(t) = \int_{t_0}^{t} \mathbf{v}(\tau)d\tau + \mathbf{p}_0, \tag{1}$$

with $\mathbf{v}(t) \in \mathbb{R}^2$: robot velocity vector at time $t \in [t_0, t_f]$.

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Loop-set from velocity:

$$
\begin{aligned}
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&= \left\{ (t_1, t_2) \in [t_0, t_f]^2 \mid \int_{t_1}^{t_2} \mathbf{v}(\tau)d\tau = \mathbf{0}, t_1 < t_2 \right\} \tag{3}
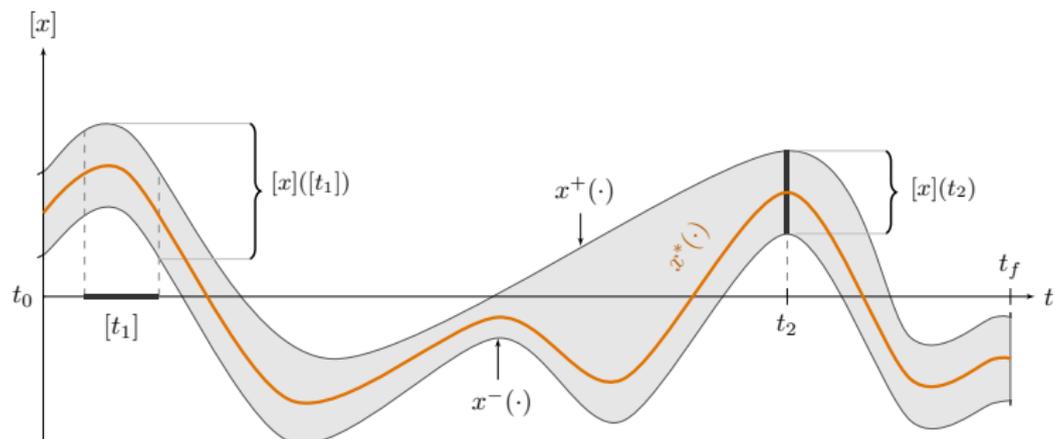\end{aligned}
$$

Section 4

Loop detection

Loop detection
Bounded-error context

Tubes: sets of trajectories

$[x](\cdot)$, interval of trajectories $[x^-(\cdot), x^+(\cdot)]$
such that $\forall t \in \mathbb{R}, \ x^-(t) \leqslant x^+(t)$

Tube $[x](\cdot)$ enclosing an uncertain trajectory $x^*(\cdot)$

Loop detection
Bounded-error context

Actual loop-set $\mathbb{T}^*$ (error free):

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Set-membership approach:

$$\mathbb{T}^* \subset \mathbb{T} \subset [t_0, t_f]^2 \tag{6}$$

Loop detection
Inclusion function

Simplification:
defining the actual but unknown function $\mathbf{f}^* : \mathbb{R}^2 \to \mathbb{R}^2$

$$\mathbf{f}^*(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau \tag{7}$$

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Assessed knowledge:
$[\mathbf{f}] : \mathbb{R}^2 \to \mathbb{IR}^2$ is an *interval function* of $\mathbf{f}^*$:

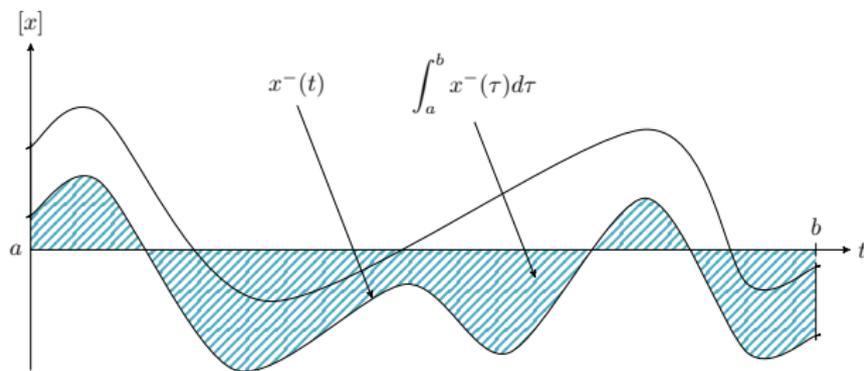$$\mathbf{f}^*(t_1, t_2) \in [\mathbf{f}](t_1, t_2) = \int_{t_1}^{t_2} [\mathbf{v}](\tau) d\tau \tag{8}$$

Loop detection
Integral of tubes

Definition: the integral of a tube $[x](\cdot) = [x^-, x^+]$ is an interval:

$$\int_a^b [x](\tau)d\tau = \left\{ \int_a^b x(\tau)d\tau \mid x(\cdot) \in [x](\cdot) \right\} = \left[\int_a^b x^-(\tau)d\tau, \int_a^b x^+(\tau)d\tau \right]$$

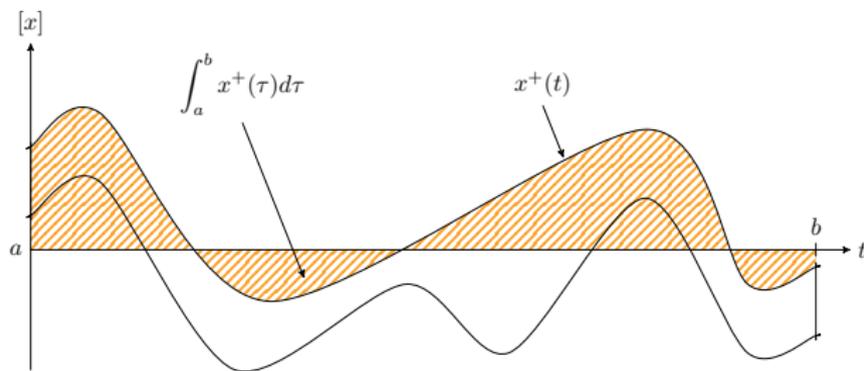[Aubry2013]

blue area: lower bound of the tube's integral

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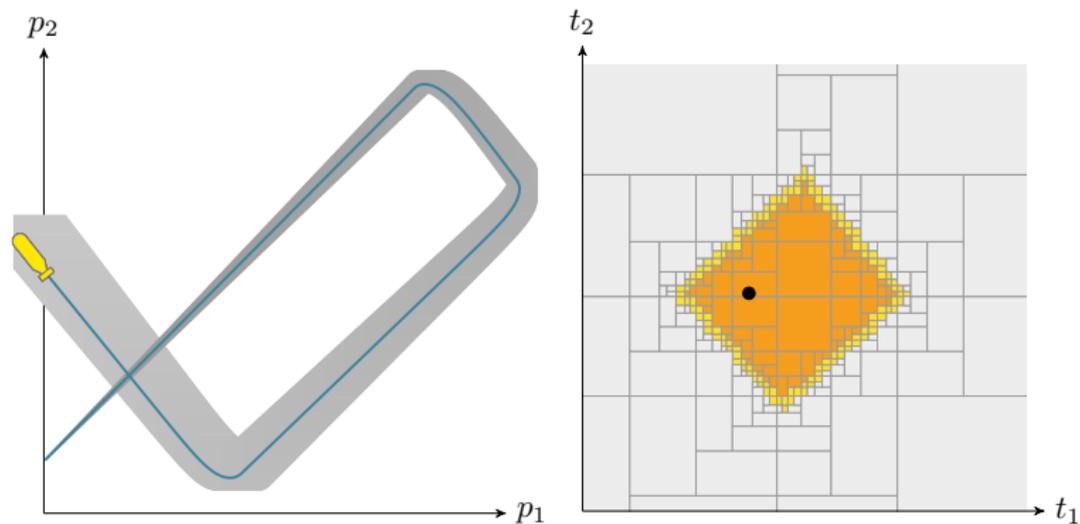
[Aubry2013]

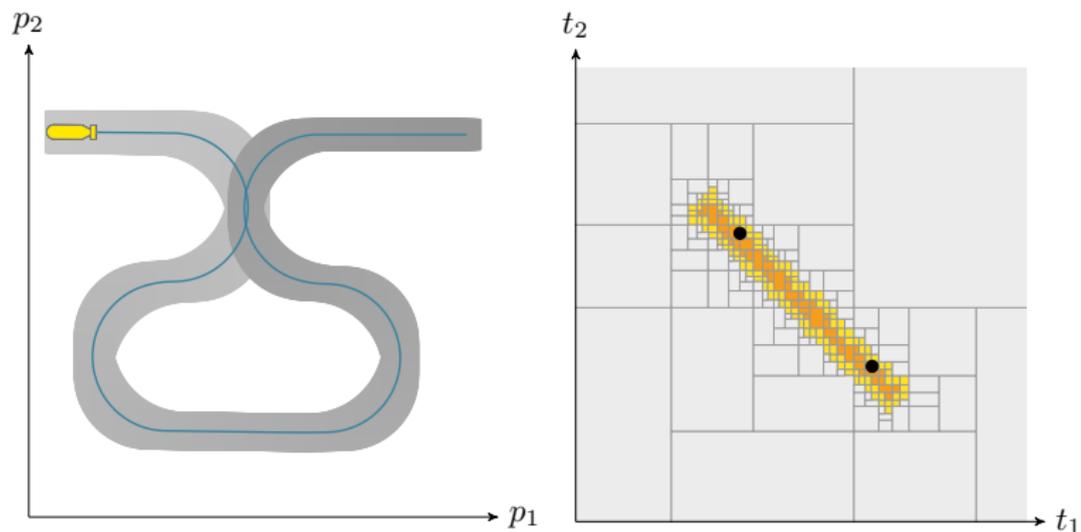orange area: upper bound of the tube's integral

Loop detection
Reliable approximation of a loop set

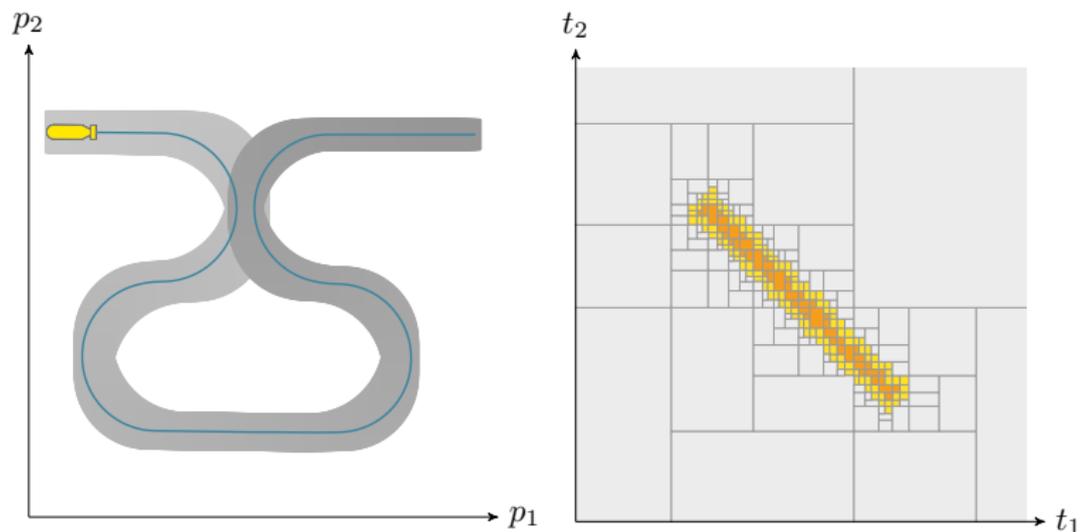Undeniable looped trajectory

Loop detection
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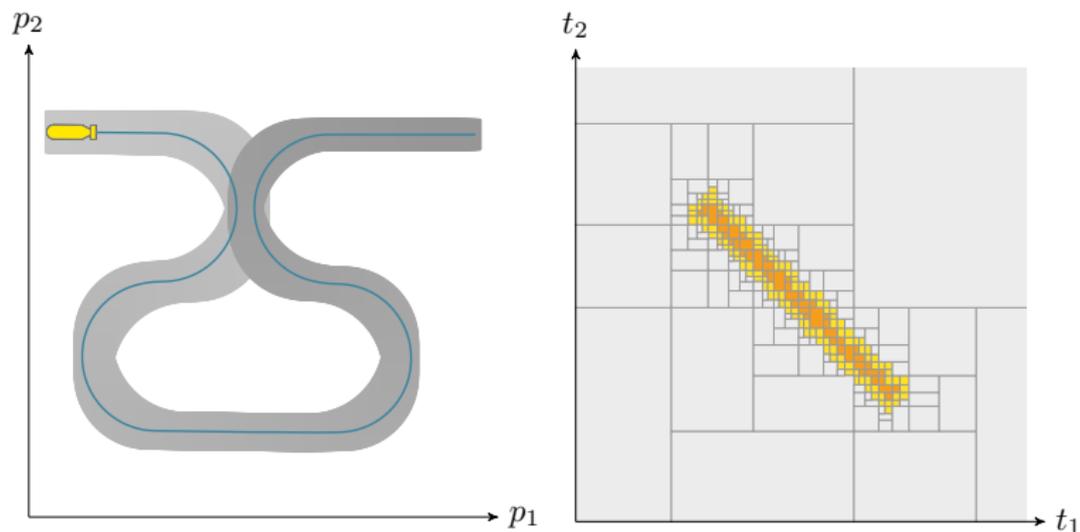Doubtful looped trajectory

Loop detection
Reliable approximation of a loop set

Doubtful looped trajectory

Loop detection
Reliable approximation of a loop set

Doubtful looped trajectory

$$\forall \mathbf{f} \in [\mathbf{f}], \exists \mathbf{t} \in \mathbb{T}_i \mid \mathbf{f}(\mathbf{t}) = \mathbf{0} \implies \underbrace{\exists \mathbf{t} \in \mathbb{T}_i \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0}}_{\text{loop existence proof}} \qquad (9)$$

Section 5

Topological degree for zero verification

Topological degree for zero verification

Problem statement

Usually we cannot find exactly a zero of an uncertain function, but we can prove it exists within some domain Ω.

Assumptions:

- known inclusion function $[\mathbf{f}] : \mathbb{IR}^n \to \mathbb{IR}^m$ of the unknown function $\mathbf{f}^* : \mathbb{R}^n \to \mathbb{R}^m$
- possibly in the form of an algorithm for computing $[\mathbf{f}]([\mathbf{t}])$
- $n = m = 2$
- Ω possibly made of a union of finitely many boxes in $\mathbb{IR}^2$

$\longrightarrow$ need to isolate and verify zeros of $\mathbf{f}^*$

Topological degree for zero verification
Powerful topological degree

Topological degree $\deg(\mathbf{f}^*, \Omega)$:

▶ unique integer assigned to $\mathbf{f}^*$ and a compact set $\Omega \subset \mathbb{R}^n$ such that $\mathbf{f}^*(\mathbf{t}) \neq \mathbf{0}$ for all $\mathbf{t} \in \partial\Omega$

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Most important property of it:

$$\deg(\mathbf{f}^*, \Omega) \neq 0 \quad \implies \quad \exists \mathbf{t} \in \Omega \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0} \qquad (10)$$

■ Topological degree theory and applications
Y. J. Cho, Y. Q. Chen *Mathematical Analysis and Applications*, 2006

■ Degree theory in analysis and applications
I. Fonseca, W. Gangbo *Oxford lecture series*, 1995

■ A set of axioms for the degree of a tangent vector field on differentiable manifolds
M. Furi, M. P. Pera, M. Spadini *Fixed Point Theory and Applications*, 2010

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Assets of topological degree:

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  ■ Effective topological degree computation based on interval arithmetic
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■ Quasi-decidability of a fragment of the first-order theory of real numbers
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▶ useful to count the number of $\mathbf{0}$

Topological degree for zero verification

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Our application for loop detection:

- ▶ deals with the relatively easy case $n = 2$ (t_1, t_2)
- ▶ nice geometric interpretation

Topological degree for zero verification
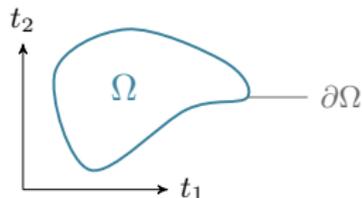
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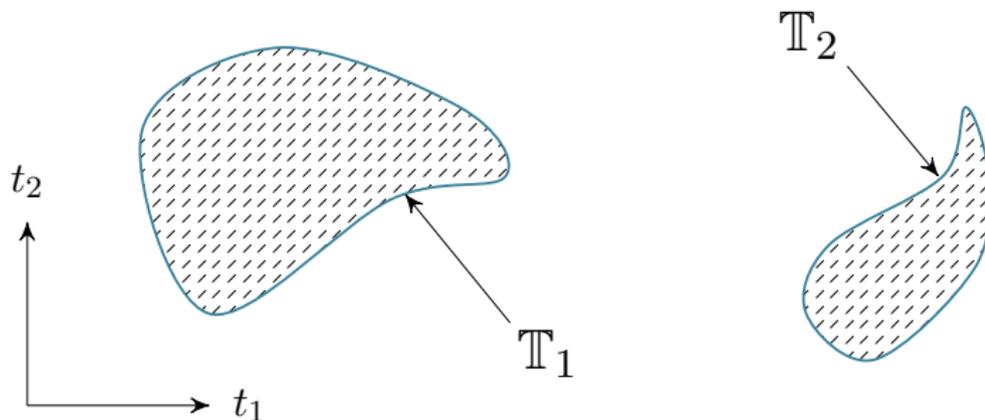
Our application for loop detection:

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  - ▶ *winding number* of the curve $\partial\Omega \xrightarrow{\mathbf{f}^*} \mathbb{R}^2 \setminus \{\mathbf{0}\}$ around $\mathbf{0}$

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Our application for loop detection:

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$$\deg = 0 \qquad\qquad \deg = 1 \qquad\qquad \deg = 2 \qquad\qquad \deg = 3$$

Topological degree for zero verification

Outer approximation of a set $\mathbb{T}$ with SIVIA

Consider $\mathbb{T} \subset \mathbb{R}^n$ in which we want to find zeros of $\mathbf{f}^*$.

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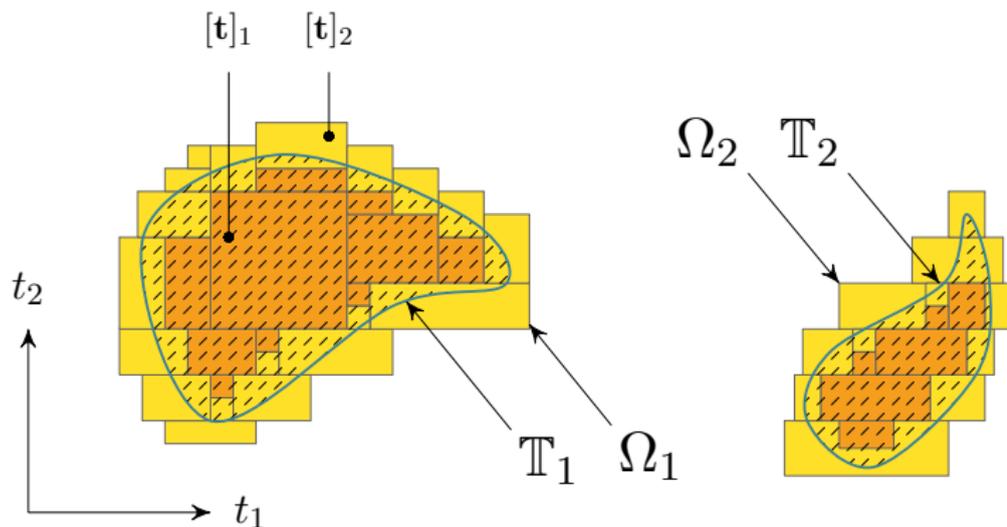
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Outer set has the properties required for Ω: $\mathbf{f}^*(\mathbf{t}) \neq \mathbf{0}, \ \forall \mathbf{t} \in \partial\Omega$

Section 6

Application

Application
Redermor mission

2 hours experimental mission in Brittany (France)
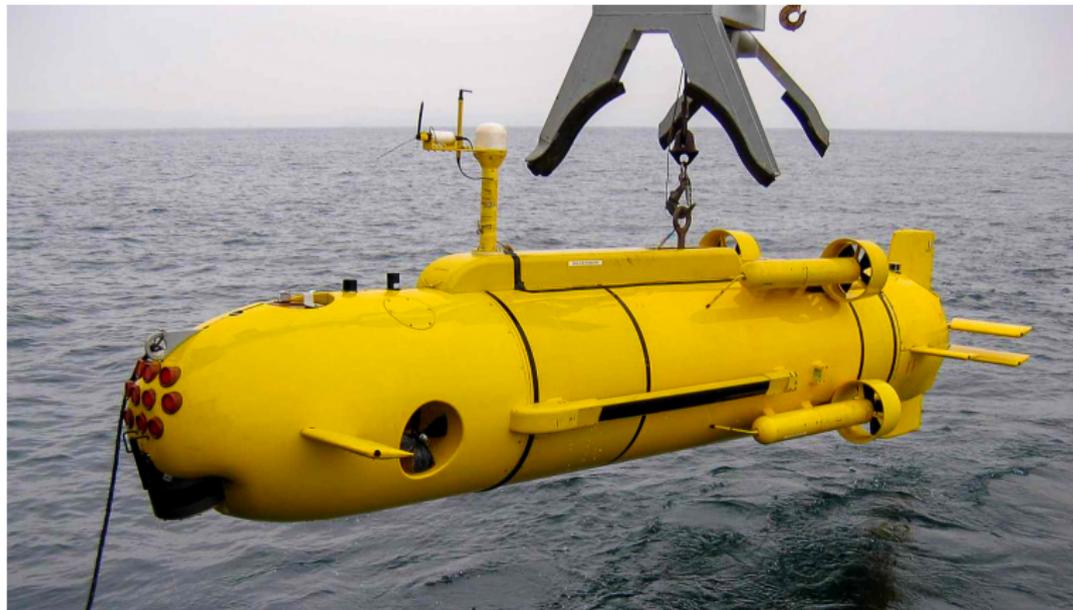
The *Redermor* Autonomous Underwater Vehicle (AUV)

Application
Redermor mission

Tube of proprioceptive measurements $[\mathbf{v}](\cdot)$:
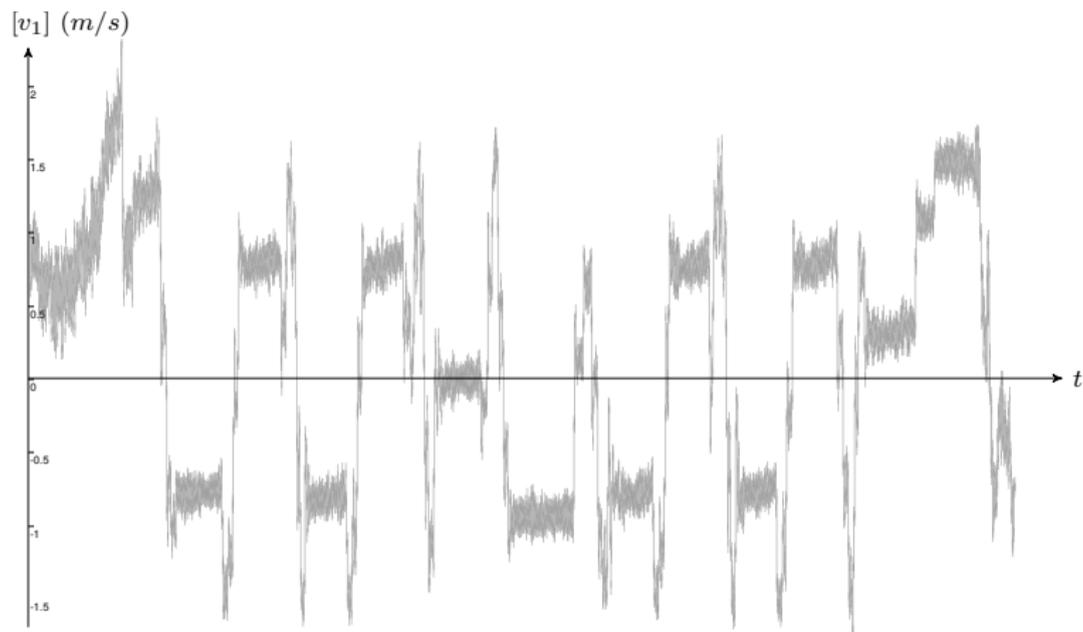
East speed velocity tube $[v_1](\cdot)$

Application
Guaranteed computation of the trajectory
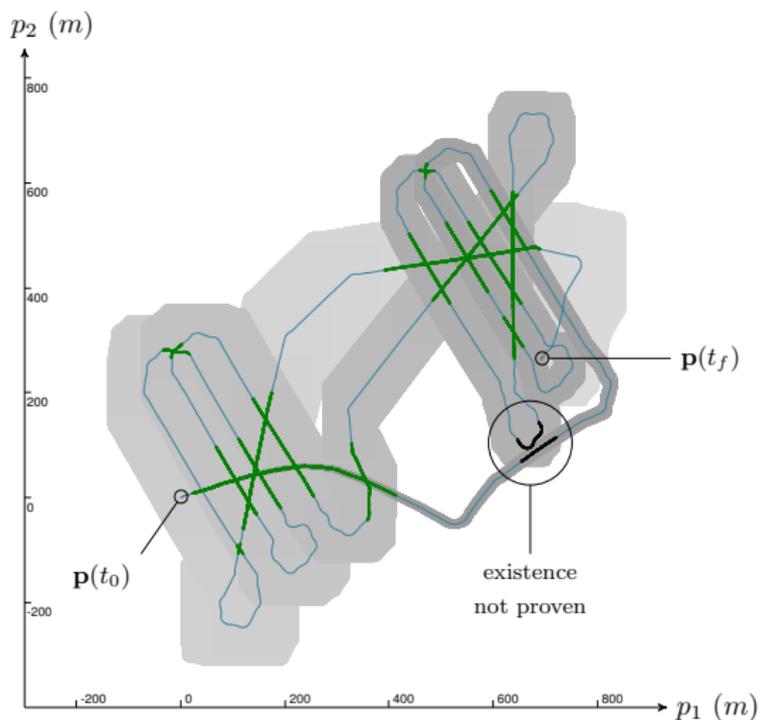
2d trace of *Redermor* AUV

Application

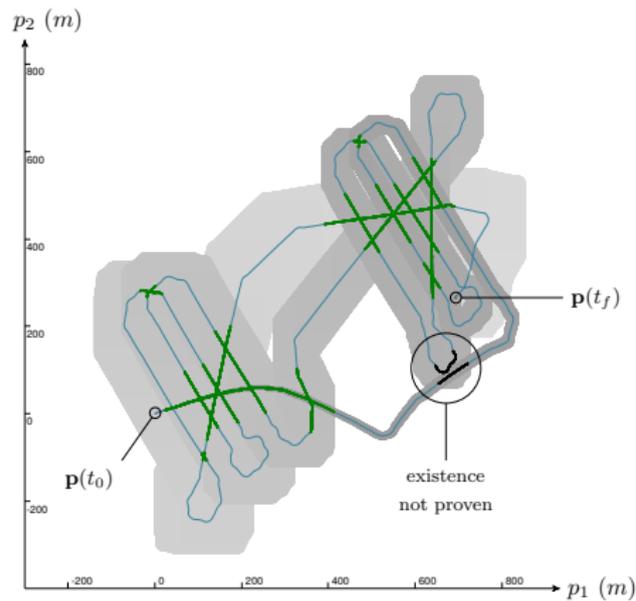t-plane of the mission: $\mathbb{T} = \{(t_1, t_2) \mid \mathbf{0} \in [\mathbf{f}](t_1, t_2), t_1 < t_2\}$
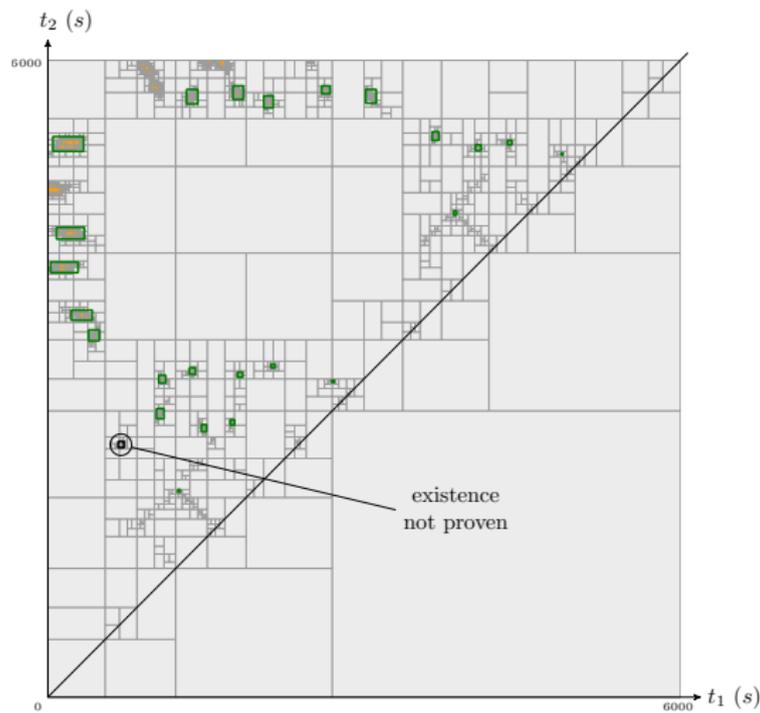
t-plane corresponding to *Redermor*'s mission

Application
Overview and results
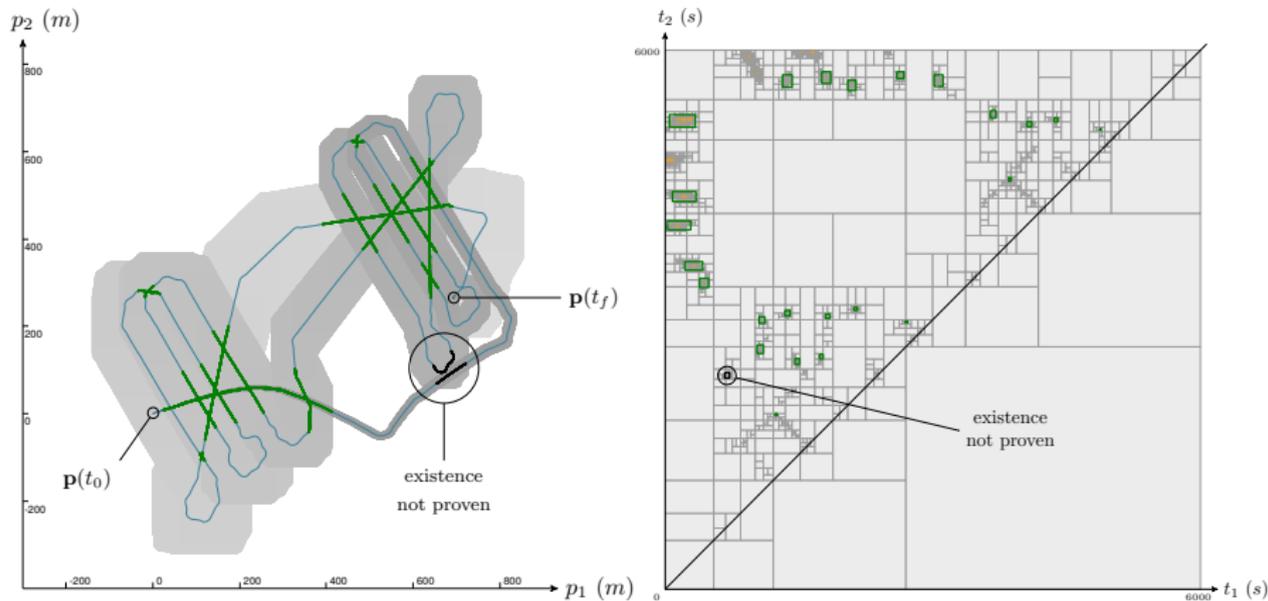
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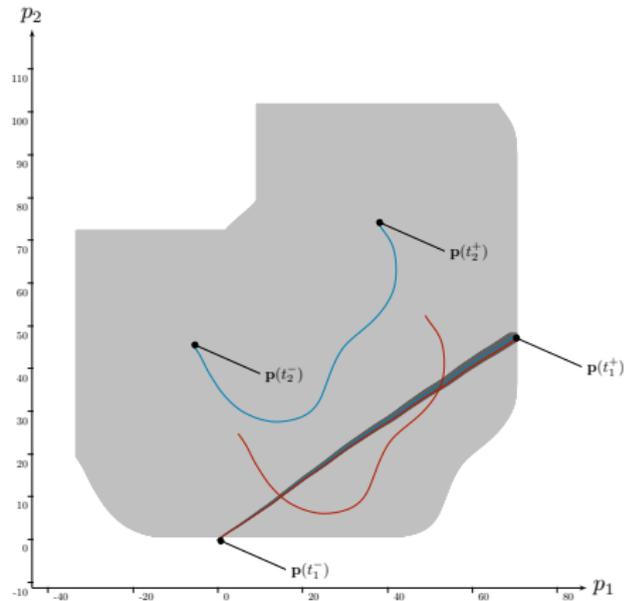
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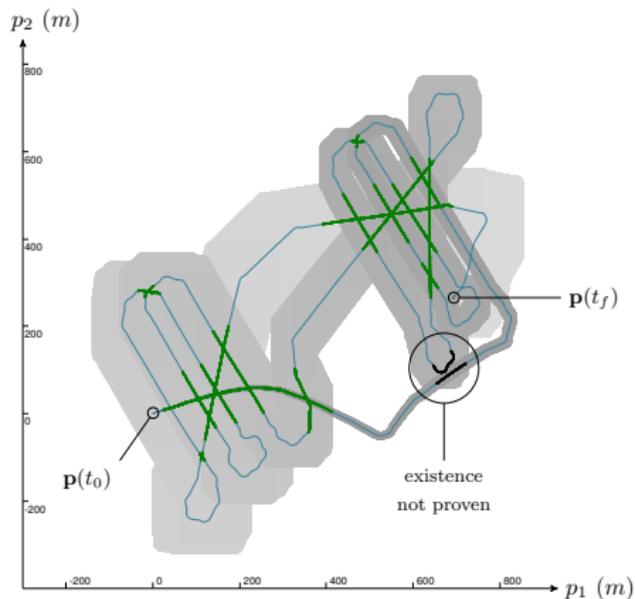
Loop proof number

Without uncertainties:

$$\lambda^* = \#\big\{\mathbf{t} \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0}, t_1 < t_2\big\}$$

Application
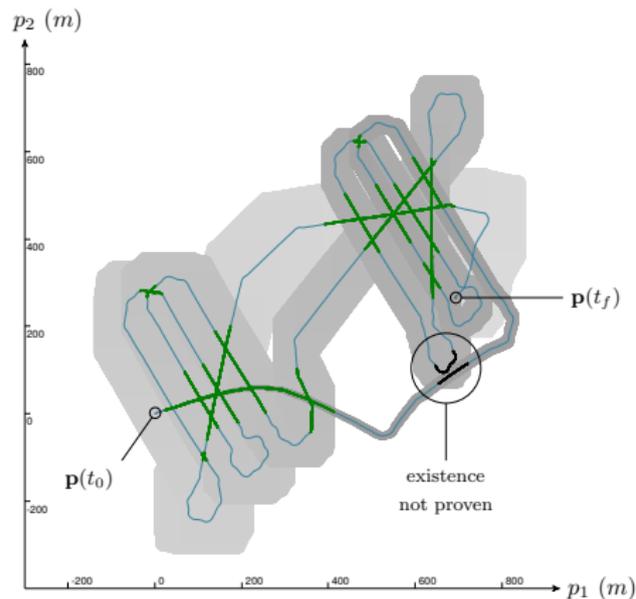# Overview and results
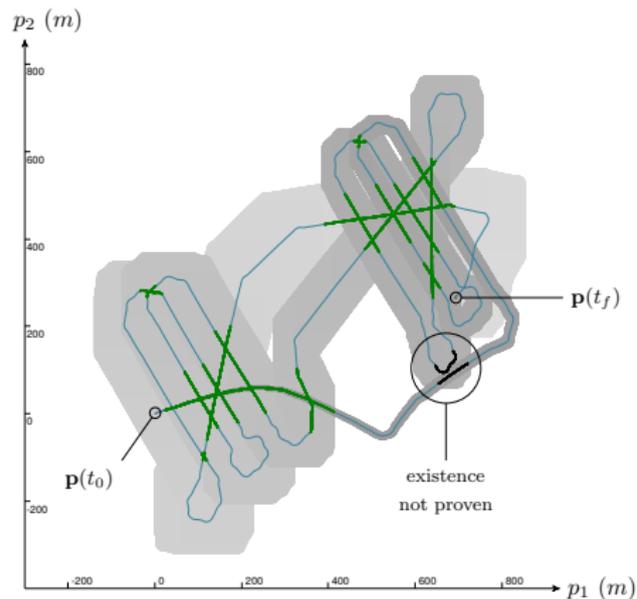
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Results:

Newton operator test: $\lambda_{\mathcal{N}} = 14$

Application
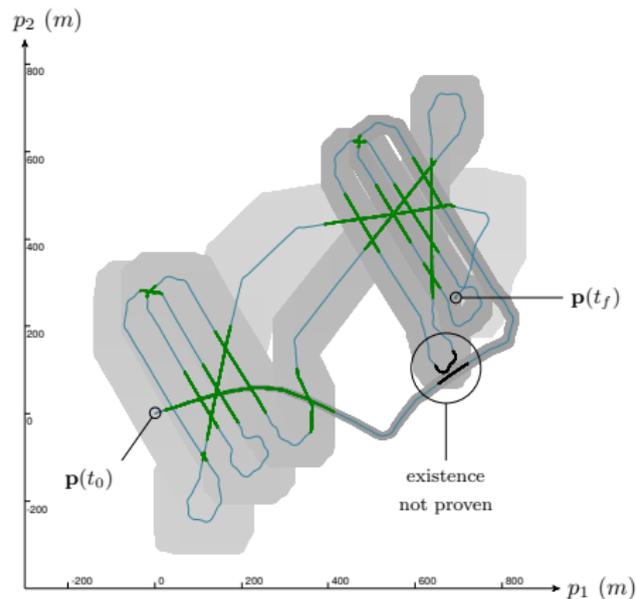# Overview and results

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Topological degree test: $\lambda_{\mathcal{T}} = 24$

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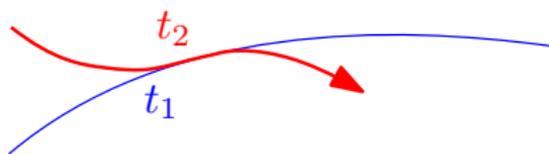
Truth: $\lambda^* = 24$

Conclusion

Loop proof $\Leftrightarrow$ **verified existence of a 0** of an uncertain function:

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▶ have to deal with a reliable approximation of it

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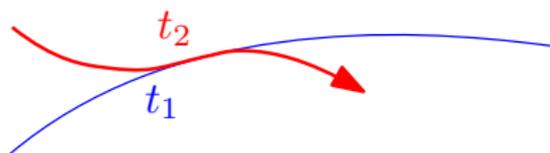
■ Proving the existence of loops in robot trajectories
S. Rohou, P. Franek, C. Aubry, L. Jaulin *The International Journal of Robotics Research*, 2018

Future work

Now that loops are proved with proprioceptive measurements...

...it remains to perform localization by adding environment perceptions.

Tubex library

An open-source C++ (Python) library based on IBEX and providing tools for constraint programming over dynamical systems.

- ▶ Tube, TubeVector, ...
- ▶ contractors (delays, differential eq., time uncertainties...)
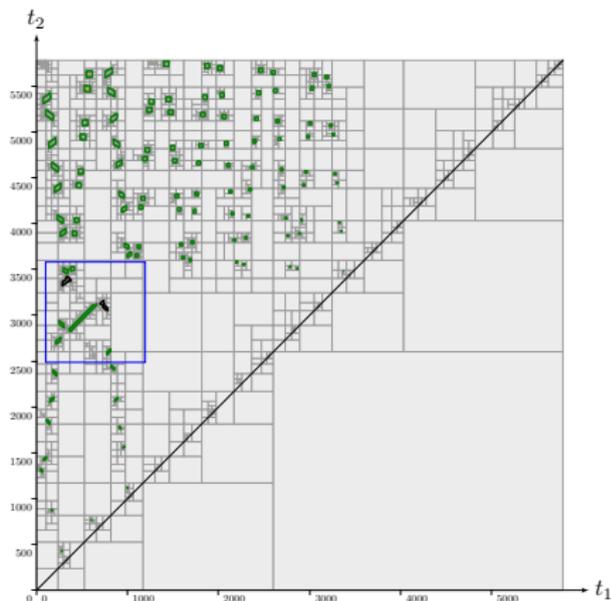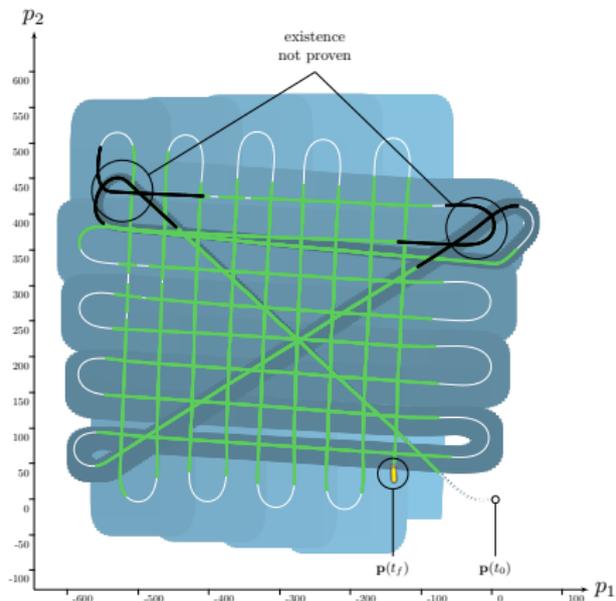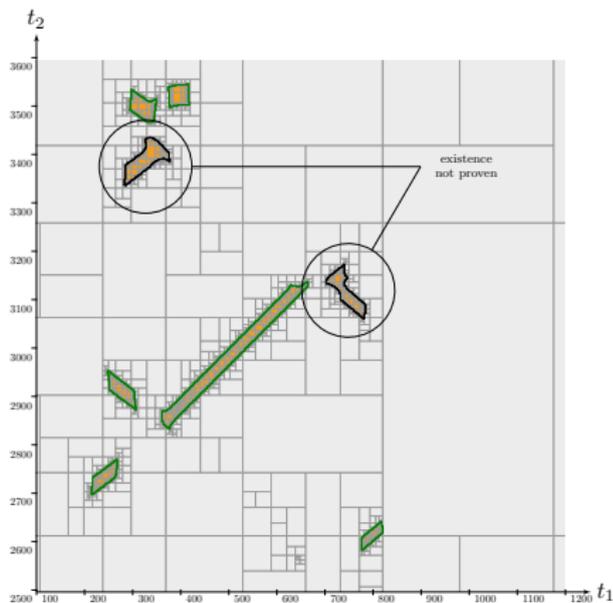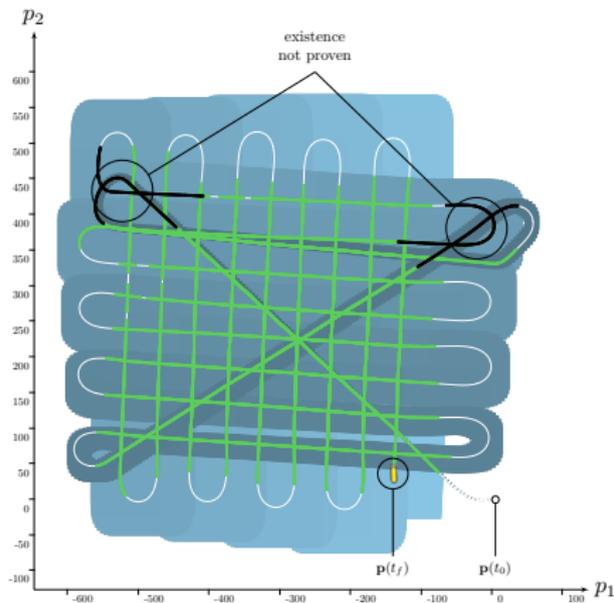- ▶ robotic tools and applications

http://www.simon-rohou.fr/research/tubex-lib/
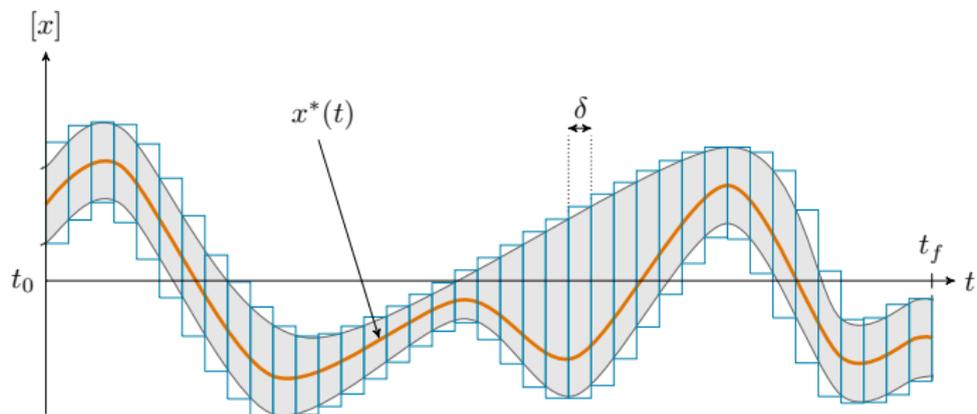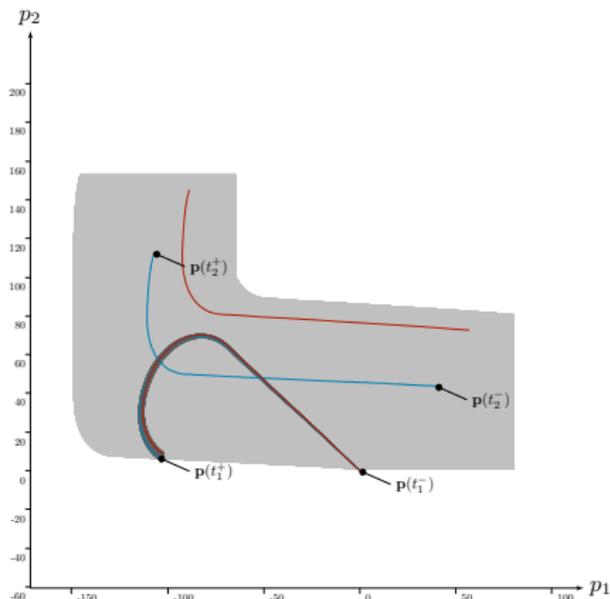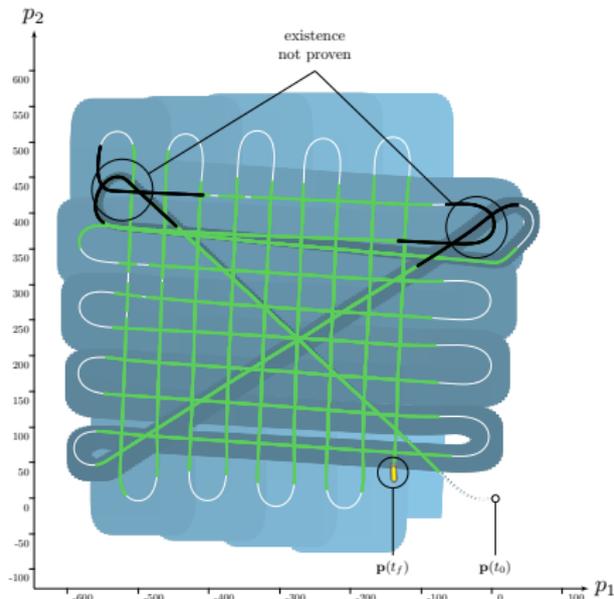
Section 10

Appendix

Appendix
Another experiment (*Daurade* AUV)

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Another experiment (*Daurade* AUV)

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Another experiment (*Daurade* AUV)

Appendix

Reliable approximation of absolute speed $\mathbf{v}^*(\cdot)$

Robot sensors for **absolute speed computation**:

- ▶ velocity sensor (DVL)
- ▶ inertial measurement unit

Appendix

Reliable approximation of absolute speed $\mathbf{v}^*(\cdot)$

Robot sensors for **absolute speed computation**:

► velocity sensor (DVL)

► inertial measurement unit

Uncertainties:

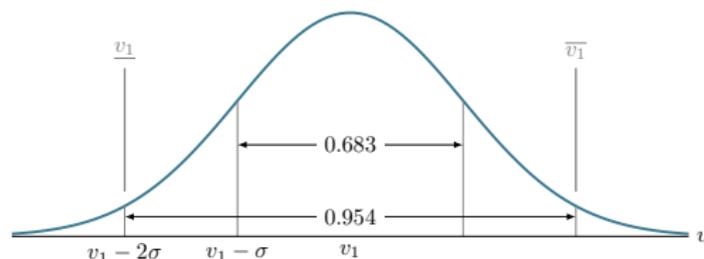► datasheets $\implies$ standard deviation σ for each sensor

Appendix

Reliable approximation of absolute speed $\mathbf{v}^*(\cdot)$

Robot sensors for **absolute speed computation**:

- ▶ velocity sensor (DVL)
- ▶ inertial measurement unit

Uncertainties:

- ▶ datasheets $\implies$ standard deviation σ for each sensor
- ▶ 95% confidence rate: $v_1^* \in [v_1] = [v_1 - 2\sigma, v_1 + 2\sigma]$
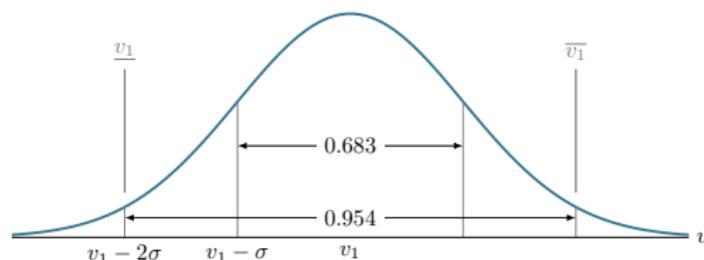
Appendix

Reliable approximation of absolute speed $\mathbf{v}^*(\cdot)$

Robot sensors for **absolute speed computation**:

- ▶ velocity sensor (DVL)
- ▶ inertial measurement unit

Uncertainties:

- ▶ datasheets $\implies$ standard deviation σ for each sensor
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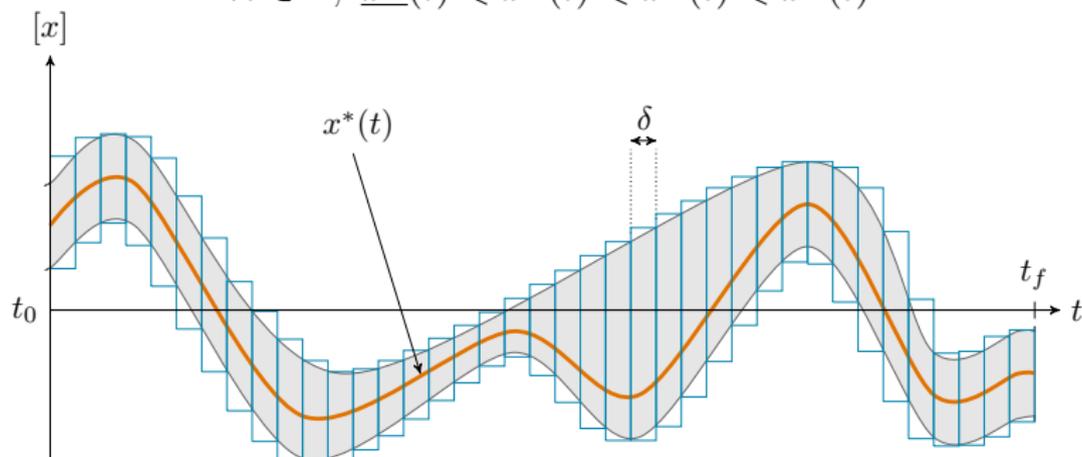

- ▶ uncertainties propagated thanks to interval arithmetic

Appendix

Tubes: computer representation

Implementation **enclosing** $[x^-(\cdot), x^+(\cdot)]$ inside an interval of step functions $[\underline{x^-}(\cdot), \overline{x^+}(\cdot)]$ such that:

$$\forall t \in \mathbb{R}, \ \underline{x^-}(t) \leqslant x^-(t) \leqslant x^+(t) \leqslant \overline{x^+}(t)$$
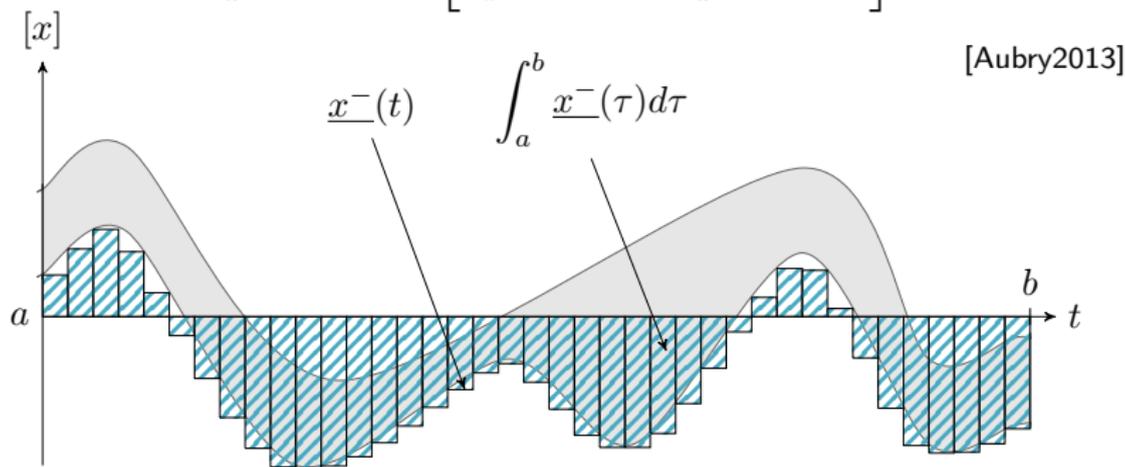

tube implementation with a set of boxes – this outer representation adds pessimism but enables guaranteed and simple computations

Appendix
Tubes integral: implementation

Outer approximation of the integral computed by:

$$\int_a^b [x](\tau)d\tau \subset \left[\int_a^b \underline{x^-}(\tau)d\tau, \int_a^b \overline{x^+}(\tau)d\tau \right]$$

blue area: outer approximation of the lower bound of the tube's integral