Localizing underwater robots with a new bathymetric SLAM approach

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Section 1

Motivations

Motivations

Robot localization: $\mathbf{p}(t) = ?$

Underwater exploration **without surfacing**:

- reasons of discretion and security (military missions)
- case of very deep environments (wrecks search)

Titanic wreck: 3821m deep

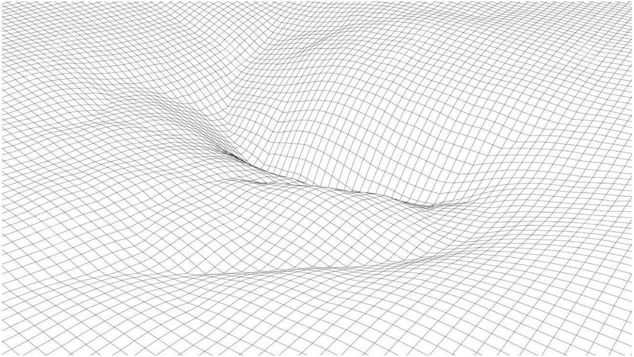
Lost MH370 aircraft: up to 6000m deep

Motivations

Problem: homogeneous environments

Under the surface:

- **no seamarks** or points of interest
- usual SLAM methods do not apply

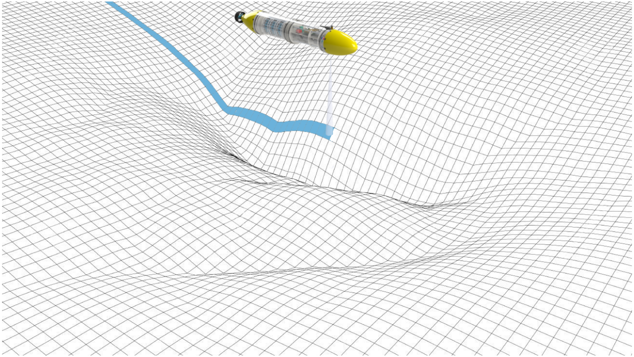


Motivations

Problem: homogeneous environments

Available data:

- **bathymetric** measurements (scalar values)
- using **raw-data SLAM** methods? poor measurements...



Section 2

Formalization

Formalization

Mobile robotics

Robot localization = state estimation problem.
Classically, we have:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) & \text{(navigation)} \\ \end{cases}$$

With:

- $\mathbf{x}$: state vector (position, bearing, ...)
- $\mathbf{u}$: input vector (command)
- $\mathbf{f}$: *evolution* function

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- $\mathbf{z}$: some exteroceptive measurement (camera, sonar...)

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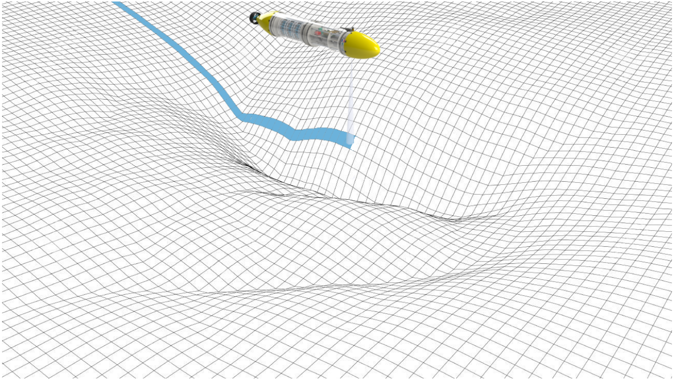
Formalization

Bathymetric localization: observation function g not at hand

Observation equation:

- $z(t) = g(\mathbf{x}(t))$
- expression of g unknown $\implies$ no relation between z and $\mathbf{x}$

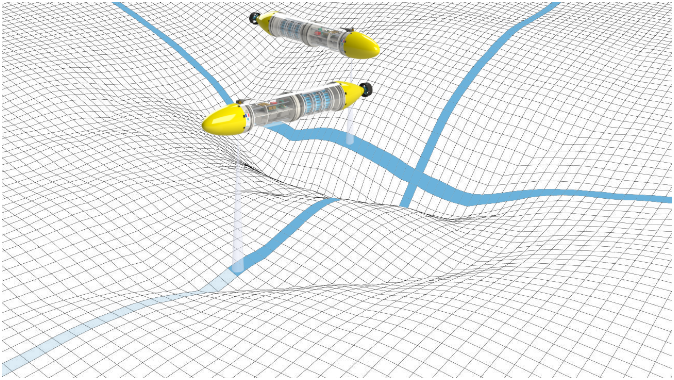


Formalization

Bathymetric localization: observation function g not at hand

Observation equation:

- $z(t) = g(\mathbf{x}(t))$
- expression of g unknown $\implies$ no relation between z and $\mathbf{x}$
- main approach: **inter-temporal measurements**



Formalization

New SLAM formalism: inter-temporal measurements

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \\ \cancel{z(t) = g(\mathbf{x}(t))} \\ \\ \end{cases}$$

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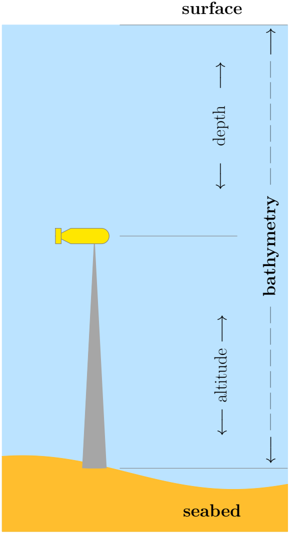

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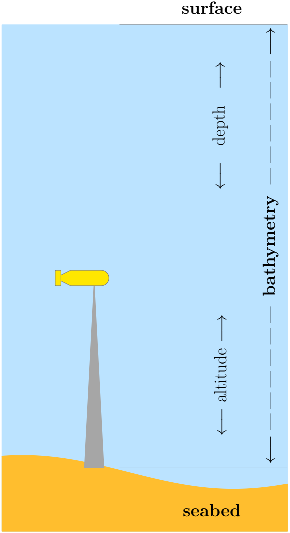

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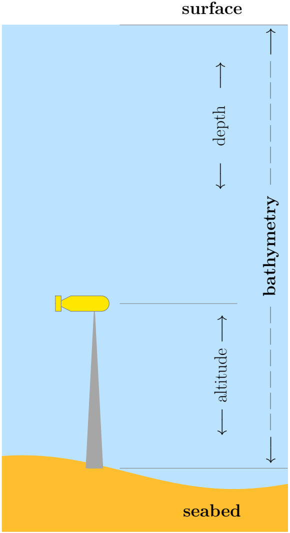

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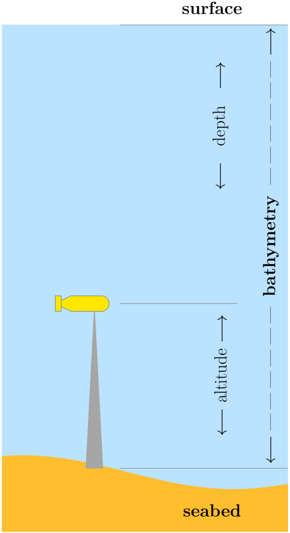
Horizontal position vector $\mathbf{p} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$



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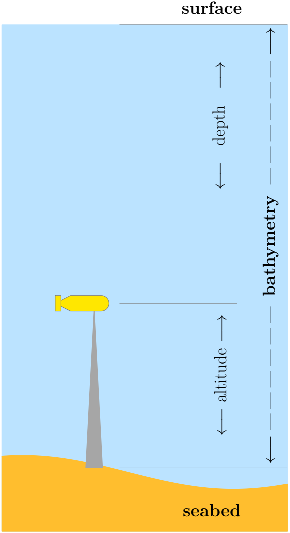
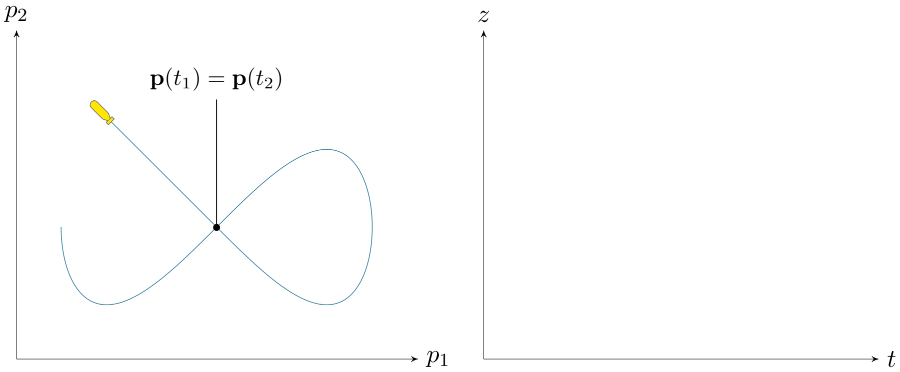
How to deal with these **inter-temporal** equations?



Formalization

Naive resolution

A robot coming back to a previous position $\mathbf{p}$ should sense the same observation z.



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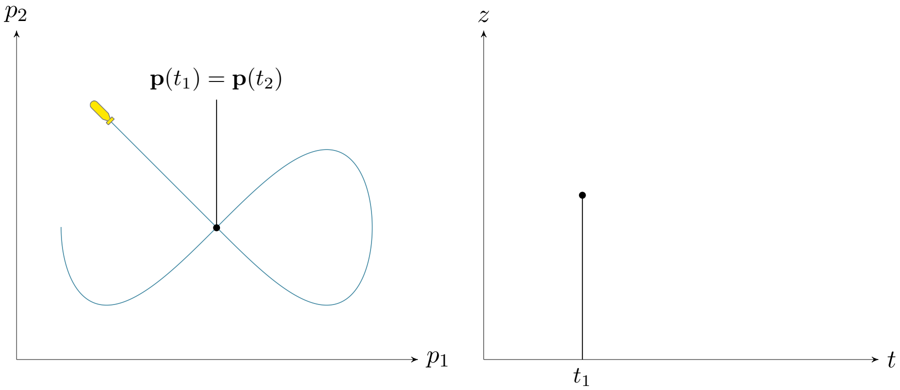

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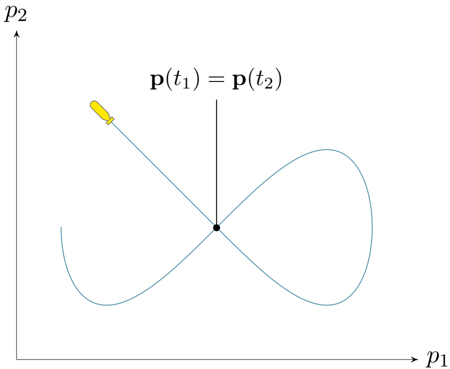

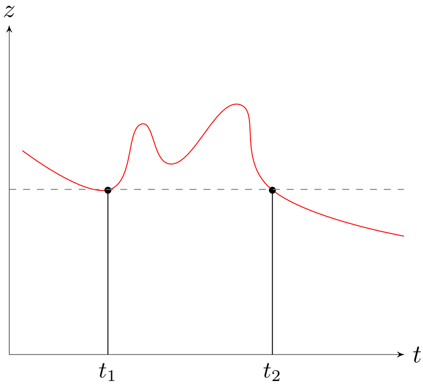

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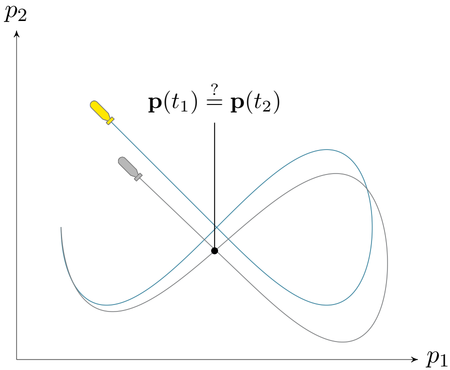

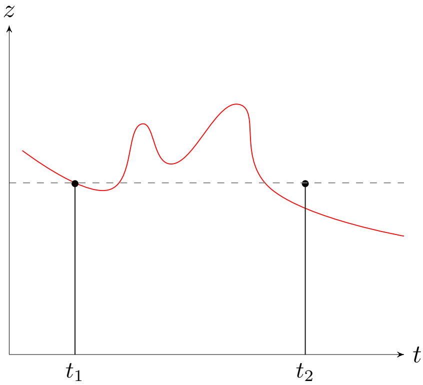

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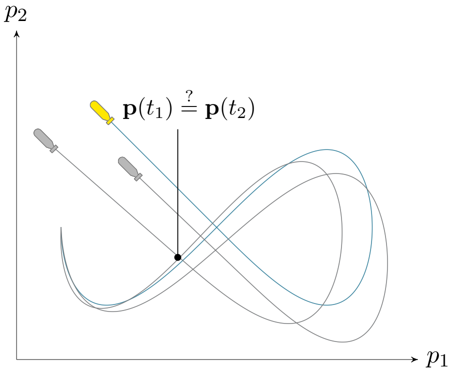

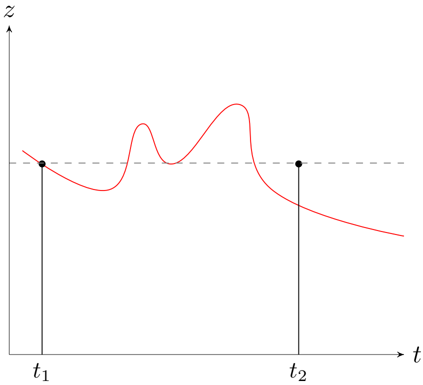

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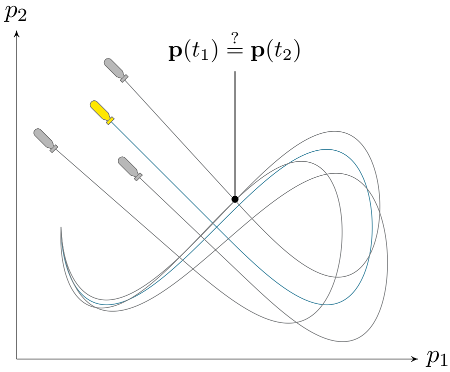

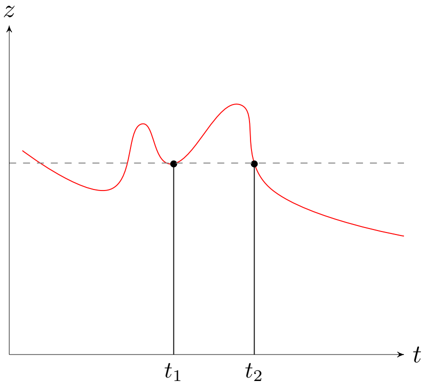

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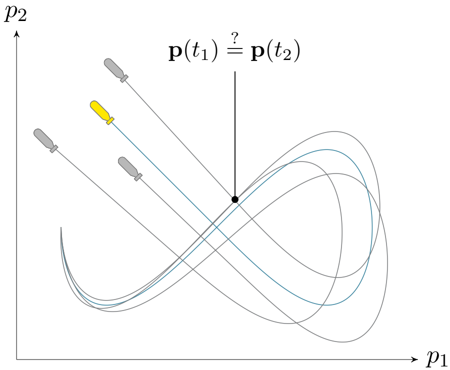

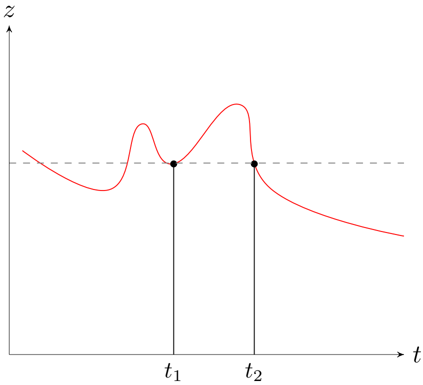

Method: temporal resolution, estimation of feasible pairs (t_1, t_2)

Section 3

Temporal resolution

Temporal resolution

Loops: definitions (Aubry, 2013)

- 2D robot trajectory: $\mathbf{p}(t) : \mathbb{R} \rightarrow \mathbb{R}^2$, $t \in [t_0, t_f]$
- looped trajectory $\Leftrightarrow$ trajectory that crosses itself
  - $\mathbf{p}(t_1) = \mathbf{p}(t_2)$, $t_1 \neq t_2$
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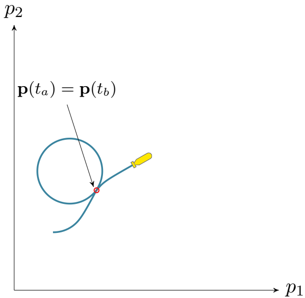

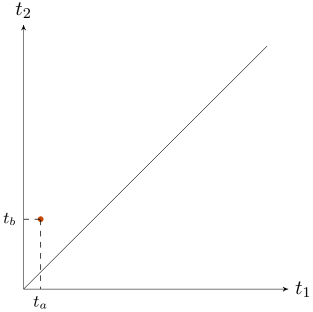

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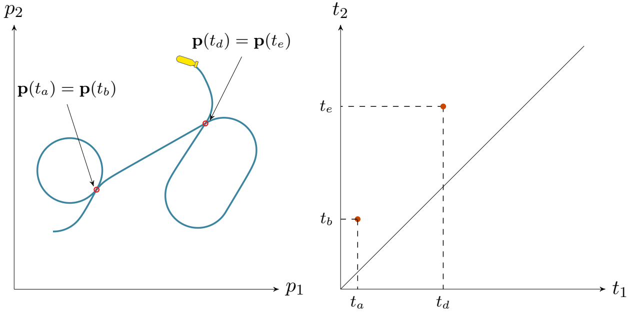

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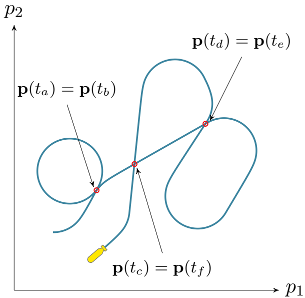

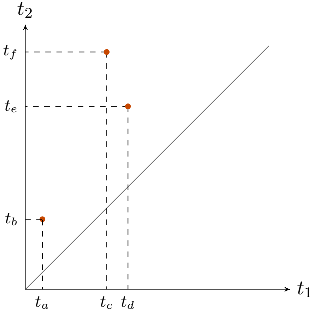

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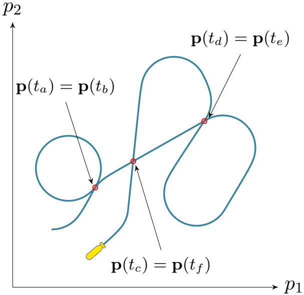
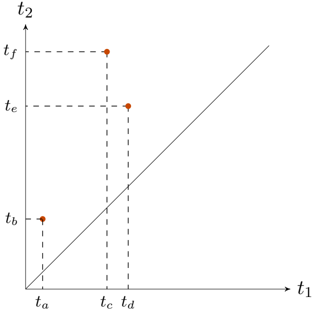
- t-plane $\Leftrightarrow$ all feasible t-pairs $= [t_0, t_f]^2$

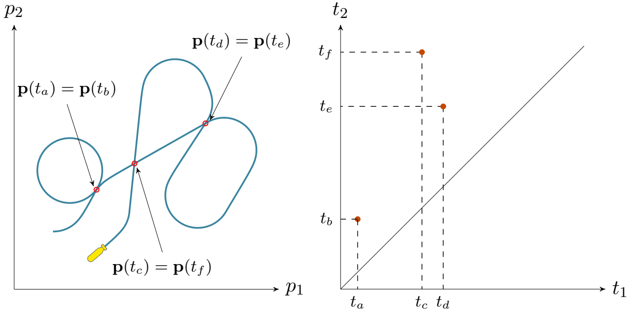




Temporal resolution

Loops: definitions (Aubry, 2013)

- *loop set* $\mathbb{T}^*_{\mathbf{p}}$:
  - $\mathbb{T}^*_{\mathbf{p}} = \left\{(t_1, t_2) \in [t_0, t_f]^2 \mid \mathbf{p}(t_1) = \mathbf{p}(t_2), t_1 < t_2\right\}$
- *loop set* of below example:
  - $\mathbb{T}^*_{\mathbf{p}} = \{(t_a, t_b), (t_c, t_f), (t_d, t_e)\}$



Temporal resolution

Loop set: approximation from sensors

$$\mathbb{T}^*_{\mathbf{p}} = \{(t_1, t_2) \mid \mathbf{p}(t_1) = \mathbf{p}(t_2)\}$$

Trajectory $\mathbf{p}(\cdot)$ unknown, but measurements $\mathbf{v}(\cdot)$, $z(\cdot)$ available:

Temporal resolution

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Proprioceptive sensors
(velocities $\mathbf{v} \in \mathbb{R}^2$)

$$\mathbb{T}^*_{\mathbf{p}} = \left\{(t_1, t_2) \mid \int_{t_0}^{t_1} \mathbf{v}(\tau)d\tau = \int_{t_0}^{t_2} \mathbf{v}(\tau)d\tau\right\}$$

Temporal resolution

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Exteroceptive sensors
(bathymetry $z \in \mathbb{R}$)

$$\mathbb{T}^*_{z} = \left\{(t_1, t_2) \mid z(t_1) = z(t_2)\right\}$$

Temporal resolution

Loop set: approximation from sensors

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Proprioceptive sensors (velocities $\mathbf{v} \in \mathbb{R}^2$)	**Exteroceptive sensors** (bathymetry $z \in \mathbb{R}$)
$\mathbb{T}^*_{\mathbf{p}} = \left\{(t_1, t_2) \mid \int_{t_1}^{t_2} \mathbf{v}(\tau)d\tau = \mathbf{0}\right\}$	$\mathbb{T}^*_{z} = \left\{(t_1, t_2) \mid z(t_1) = z(t_2)\right\}$

Inter-temporal implication:

$$\Big(\mathbf{p}(t_1) = \mathbf{p}(t_2) \implies z(t_1) = z(t_2)\Big) \implies \Big(\mathbb{T}^*_{\mathbf{p}} \subset \mathbb{T}^*_{z}\Big)$$

Temporal resolution

Graphical interpretation

$$\begin{cases} \mathbb{T}^*_{\mathbf{p}} = \left\{(t_1, t_2) \mid \int_{t_1}^{t_2} \mathbf{v}(\tau)d\tau = \mathbf{0}\right\} \\ \mathbb{T}^*_z = \left\{(t_1, t_2) \mid z(t_1) = z(t_2) \right\} \\ \mathbb{T}^*_{\mathbf{p}} \subset \mathbb{T}^*_z \end{cases}$$

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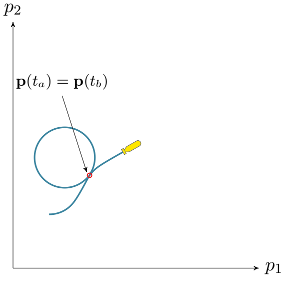

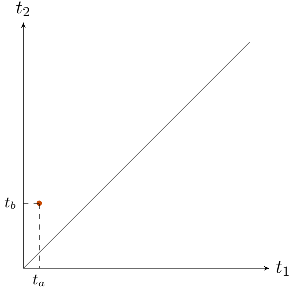

$\mathbb{T}^*_{\mathbf{p}}$

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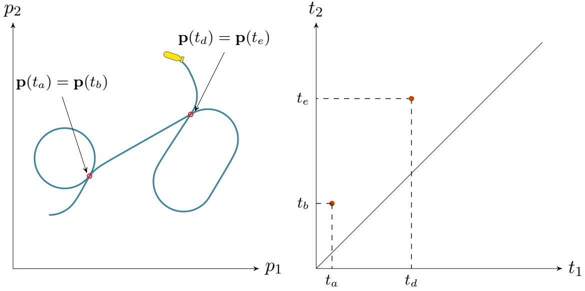

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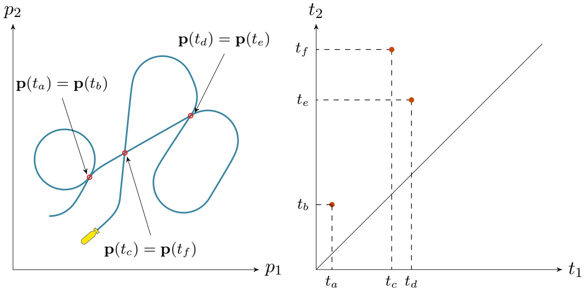

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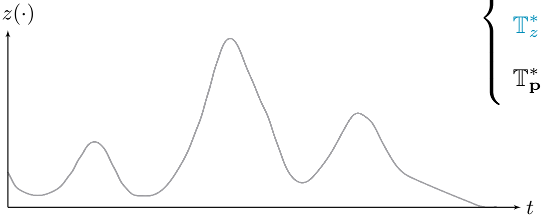

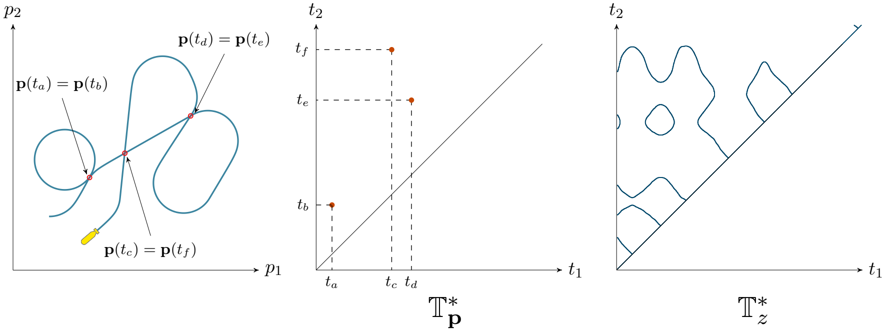

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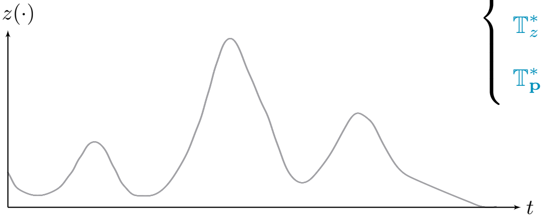
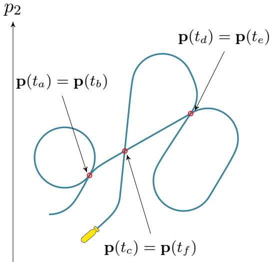
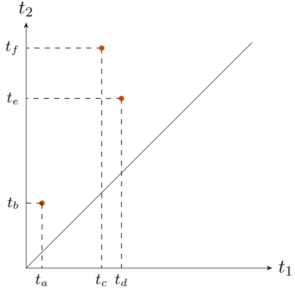
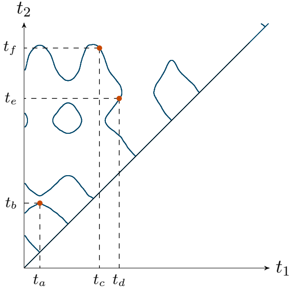






$\mathbb{T}^*_{\mathbf{p}}$

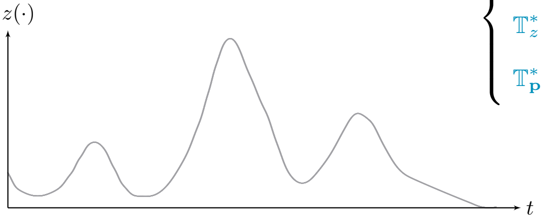


$\mathbb{T}^*_{z}$

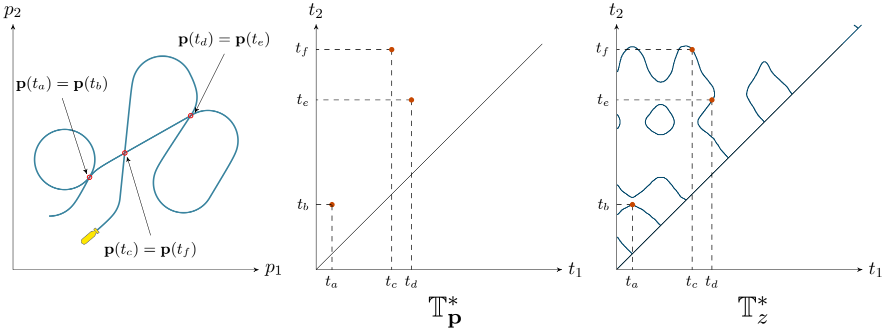
Temporal resolution

Graphical interpretation

$$\begin{cases} \mathbb{T}^*_{\mathbf{p}} = \left\{ (t_1, t_2) \mid \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0} \right\} \\ \mathbb{T}^*_{z} = \left\{ (t_1, t_2) \mid z(t_1) = z(t_2) \right\} \\ \mathbb{T}^*_{\mathbf{p}} \subset \mathbb{T}^*_{z} \end{cases}$$



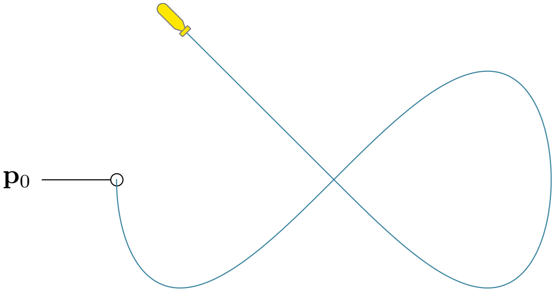
What about uncertainties?



Temporal resolution

Set-membership approach

Actual trajectory: $\mathbf{p}(t) = \int_{t_0}^{t} \mathbf{v}(\tau)d\tau + \mathbf{p}_0$

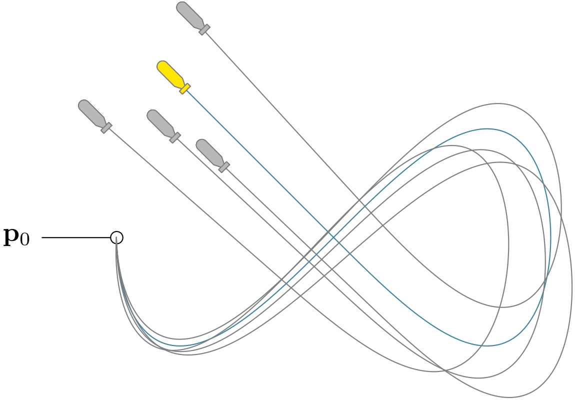


Temporal resolution

Set-membership approach

Actual trajectory: $\mathbf{p}(t) = \int_{t_0}^{t} \mathbf{v}(\tau)d\tau + \mathbf{p}_0$

Drifting trajectory: $\mathbf{p}_e(t) = \int_{t_0}^{t} \big(\mathbf{v}(\tau) + \epsilon(\tau)\big)d\tau + \mathbf{p}_0$

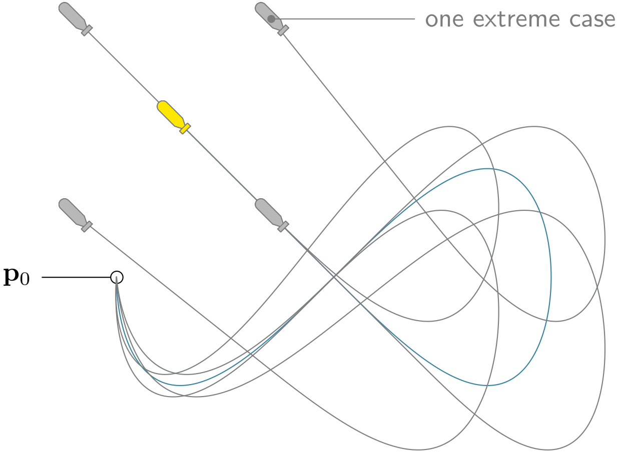


Temporal resolution

Set-membership approach

Actual trajectory: $\mathbf{p}(t)=\int_{t_0}^{t}\mathbf{v}(\tau)d\tau+\mathbf{p}_0$

Approach: consider worst cases by defining bounded solutions



Temporal resolution

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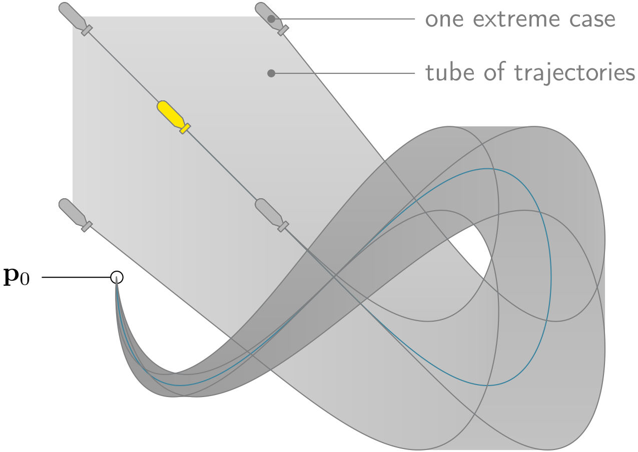
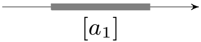


Temporal resolution

Interval Analysis

An interval $[x]$:

- a closed and connected subset of $\mathbb{R}$ delimited by two bounds
- $[x] = [x^-, x^+] = \{x \in \mathbb{R} \mid x^- \leqslant x \leqslant x^+\}$
- $[x] \in \mathbb{IR}$



Temporal resolution

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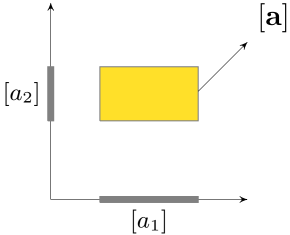
A box $[\mathbf{x}]$:

- a cartesian product of n intervals
- $[\mathbf{x}] \in \mathbb{IR}^n$



Temporal resolution

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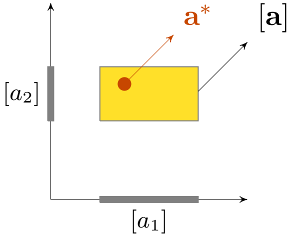


Notation: actual value denoted x^*, $\mathbf{x}^*$, ...

Temporal resolution

Interval Analysis

Based on the extension of all classical **real arithmetic operators**:

- $+, -, \times, \div$
- *ex:* $[x] + [y] = [x^- + y^-, x^+ + y^+]$
- *ex:* $[x] - [y] = [x^- - y^+, x^+ - y^-]$

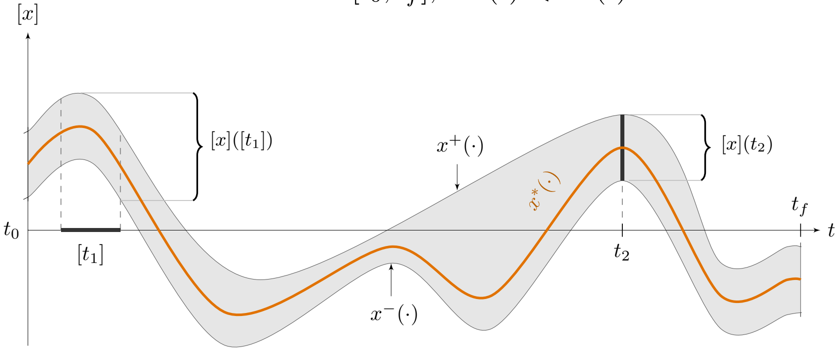
Adaptation of **elementary functions** such as:

- cos, exp, tan, etc.
- output is the smallest interval containing all the images of all defined inputs through the function

Temporal resolution

Tubes enclosing uncertain trajectories

Tube $[x](\cdot)$: interval of trajectories $[x^-(\cdot), x^+(\cdot)]$
such that $\forall t \in [t_0, t_f],\ x^-(t) \leqslant x^+(t)$



Tube $[x](\cdot)$ enclosing an uncertain trajectory $x^*(\cdot)$

Temporal resolution

Tubes enclosing uncertain trajectories

Tube $[x](\cdot)$**:** interval of trajectories $[x^-(\cdot), x^+(\cdot)]$
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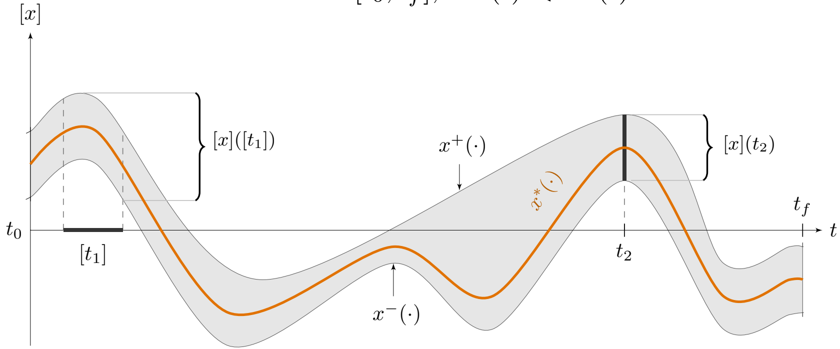


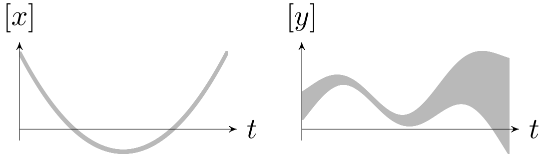
Tube $[x](\cdot)$ enclosing an uncertain trajectory $x^*(\cdot)$

Set-membership approach:

$x^*(\cdot) \in [x](\cdot)$, computations on bounds $\Rightarrow$ **guaranteed outputs**

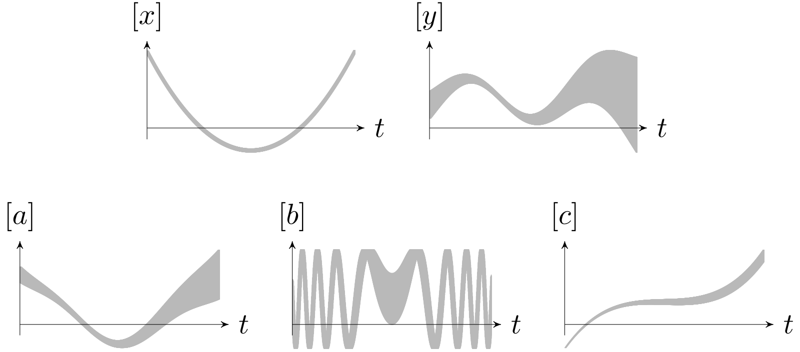
Temporal resolution

Tubes arithmetic



Temporal resolution

Tubes arithmetic



$[a](\cdot) = [x](\cdot) + [y](\cdot)$

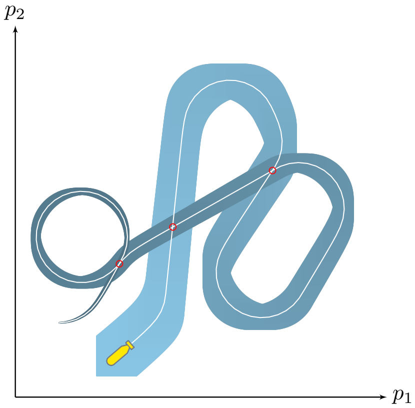
$[b](\cdot) = \sin\big([x](\cdot)\big)$

$[c](\cdot) = \int_0 [x](\tau) d\tau$

Temporal resolution

Bounded-error context

Uncertain trajectories enclosed in tubes.

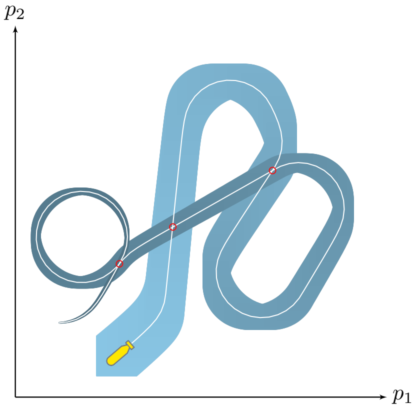


(a) Bounded trajectories

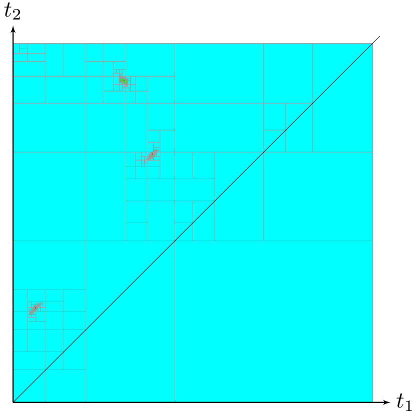
Temporal resolution

Bounded-error context

Uncertain trajectories enclosed in tubes.
Approximation of the enclosure of t-sets with SIVIA algorithms:



(c) Bounded trajectories

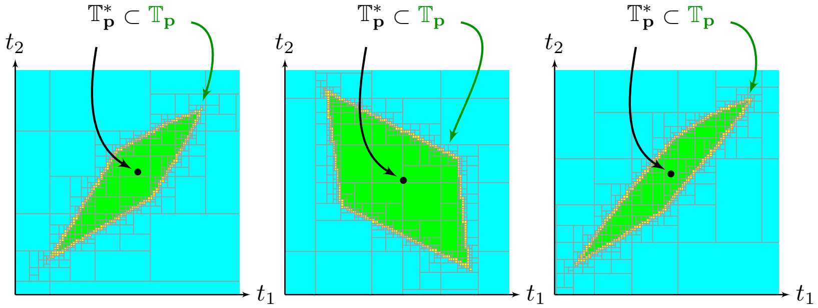


(d) Approximation of $\mathbb{T}_{\mathbf{p}}$

Temporal resolution

Bounded-error context

Approximation of the enclosure of t-sets with SIVIA algorithms:

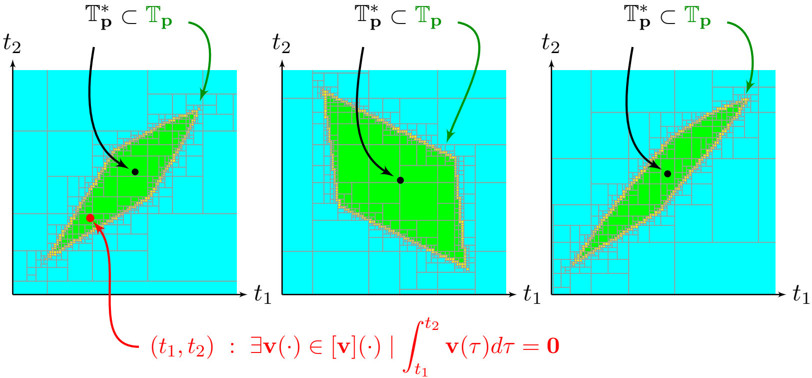


Zoom on the components of $\mathbb{T}_{\mathbf{p}}$

Temporal resolution

Bounded-error context

Approximation of the enclosure of t-sets with SIVIA algorithms:



Zoom on the components of $\mathbb{T}_{\mathbf{p}}$

Temporal resolution

Bounded-error context

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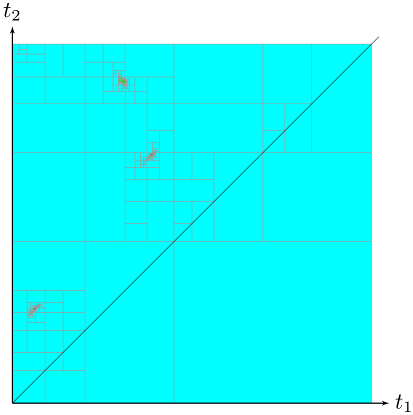
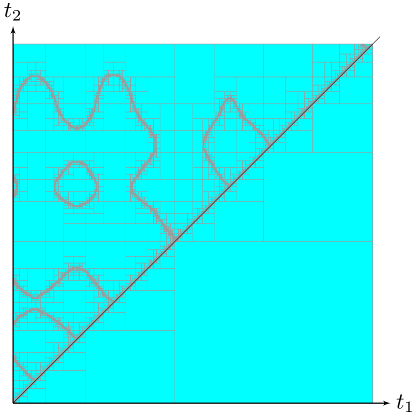


(a) Approximation of $\mathbb{T}_{\mathbf{p}}$



(b) Approximation of $\mathbb{T}_{z}$

Temporal resolution

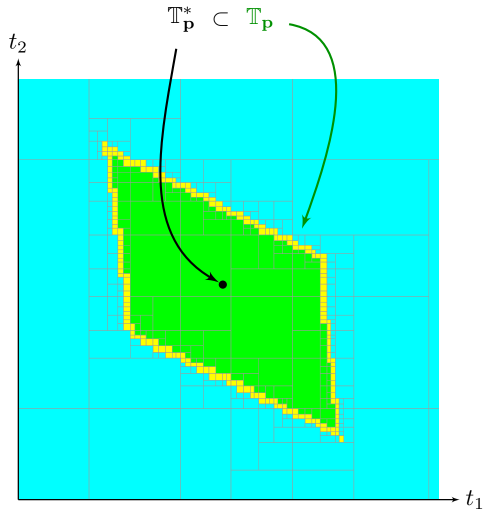
Intersection of the t-sets: fusion

Constraint:

- $\mathbb{T}^*_{\mathbf{p}} \subset \mathbb{T}^*_z$

Domains $\mathbb{T}_{\mathbf{p}}$, $\mathbb{T}_z$:

- $\mathbb{T}^*_{\mathbf{p}} \subset \mathbb{T}_{\mathbf{p}}$
- $\mathbb{T}^*_z \subset \mathbb{T}_z$



Approximation of $\mathbb{T}_{\mathbf{p}}$

Temporal resolution

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Domains $\mathbb{T}_{\mathbf{p}}$, $\mathbb{T}_z$:

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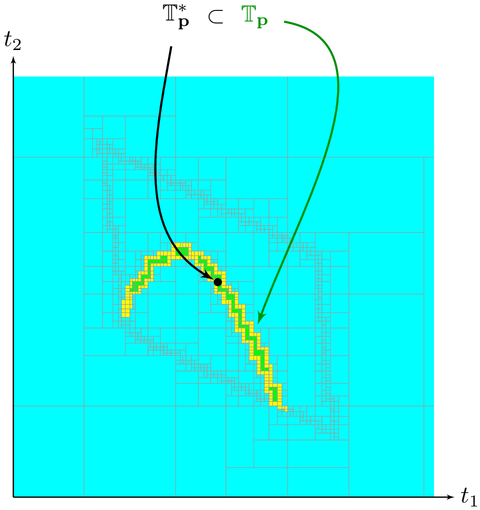
Contraction:

- $\mathbb{T}_{\mathbf{p}} := \mathbb{T}_{\mathbf{p}} \cap \mathbb{T}_z$



Approximation of $\mathbb{T}_{\mathbf{p}}$

Temporal resolution

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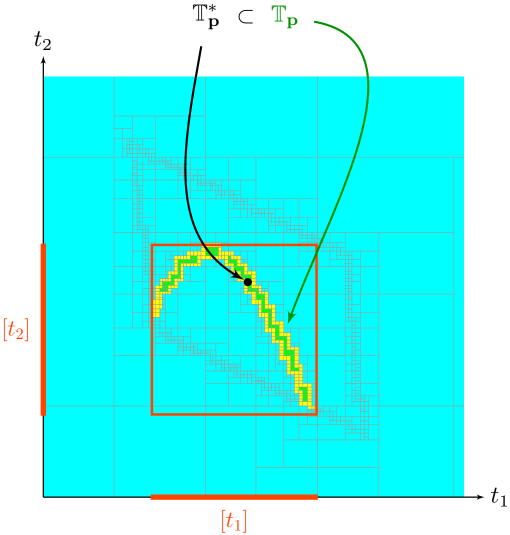


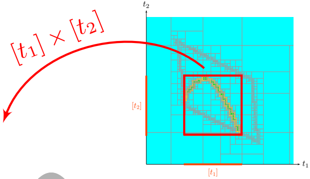
Approximation of $\mathbb{T}_{\mathbf{p}}$

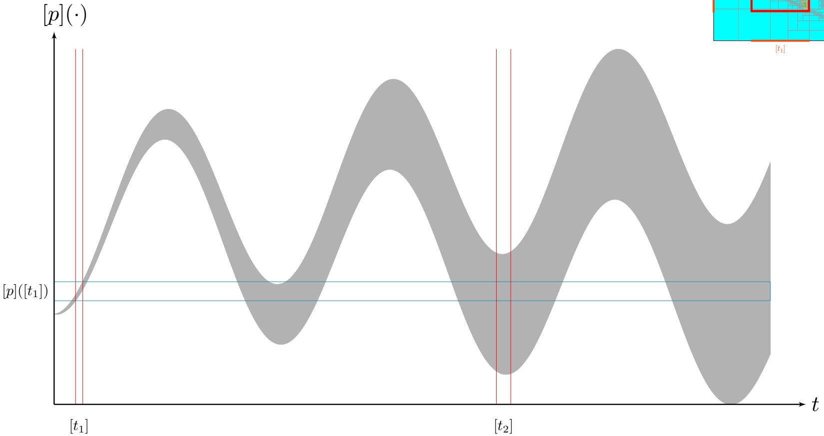
Temporal resolution

From times to positions

Robot localization = contraction of the tube $[\mathbf{p}](\cdot)$

- $\mathbf{t}^* \in [t_1] \times [t_2], \quad \mathbf{p}^*(\cdot) \in [\mathbf{p}](\cdot)$
- constraint: $\mathbf{p}(t_1) = \mathbf{p}(t_2)$





Temporal resolution

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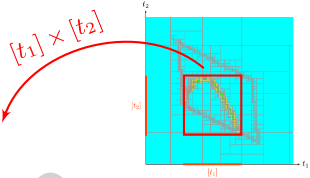

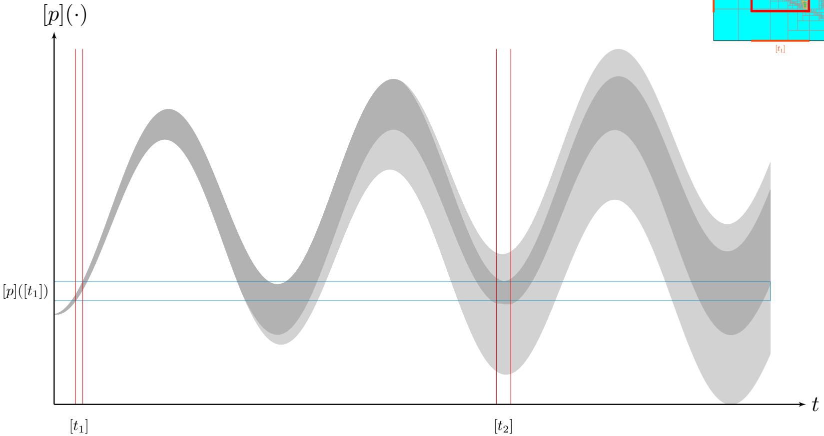

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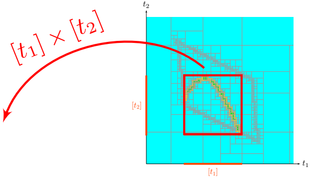

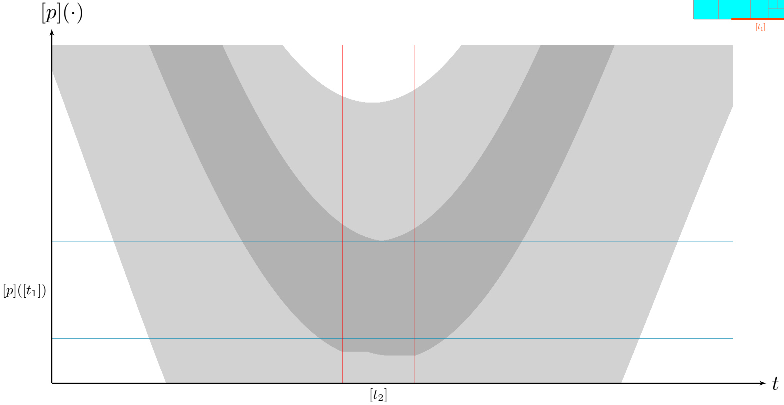

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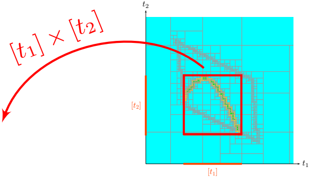

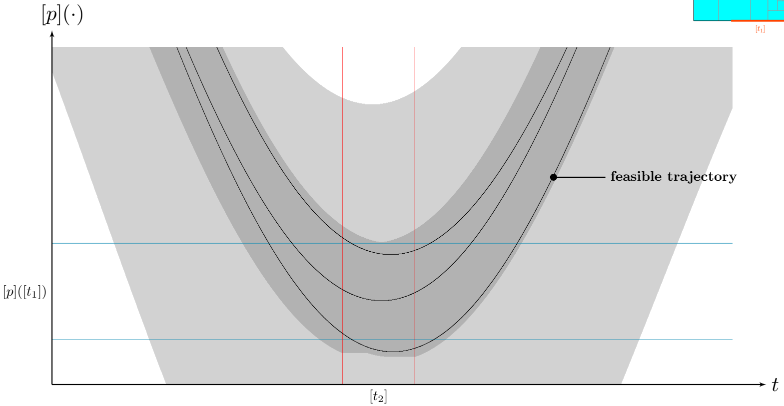

Section 4

Sea trials

Sea trials

Experimental mission with the Daurade AUV

- Daurade: Autonomous Underwater Vehicle
- weight: 1010kg – length: 5m – max depth: 300m

Special thanks to DGA-TN Brest (formerly GESMA) and SHOM, France

Sea trials

Experimental mission with the Daurade AUV

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Sea trials

Experimental mission with the Daurade AUV

- 2 hours experimental mission
- in the *Rade de Brest*, Brittany, France

Sea trials

Experimental mission with the Daurade AUV

- 2 hours experimental mission
- in the *Rade de Brest*, Brittany, France

Sea trials

Experimental mission with the Daurade AUV

- 2 hours experimental mission
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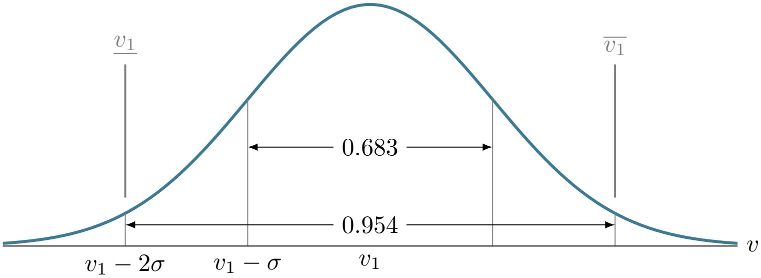
Sea trials

Bounded measurements

Uncertainties:

- datasheets $\Longrightarrow$ standard deviation σ for each sensor
- 95% confidence rate: $v_1^* \in [v_1] = [v_1 - 2\sigma, v_1 + 2\sigma]$

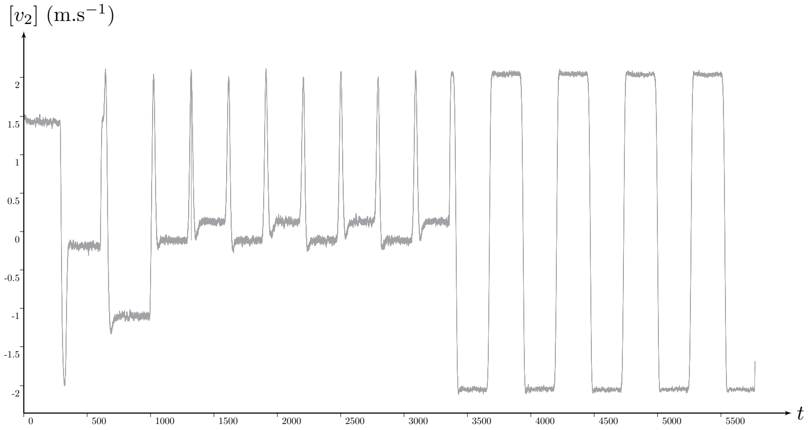


- uncertainties propagated thanks to interval arithmetic

Sea trials

Evolution measurements

- velocity measurements obtained with a DVL
- considering uncertainties, building a tube $[\mathbf{v}](\cdot)$

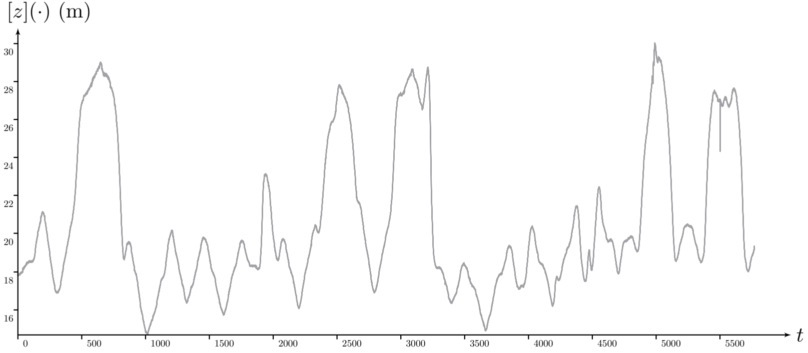


North speed velocity tube $[v_2](\cdot)$

Sea trials

Observations measurements: bathymetric values

- DVL, same sensor, can provide **altitude measurements** z_{alt}
- pressure sensor: depth values z_{depth}
- time-dependent measurements, use of **tide models**
- $z = z_{\mathrm{alt}} + z_{\mathrm{depth}} + z_{\mathrm{tide}}$



Observation tube $[z](\cdot)$

Sea trials

Dead-reckoning

Actual trajectory:

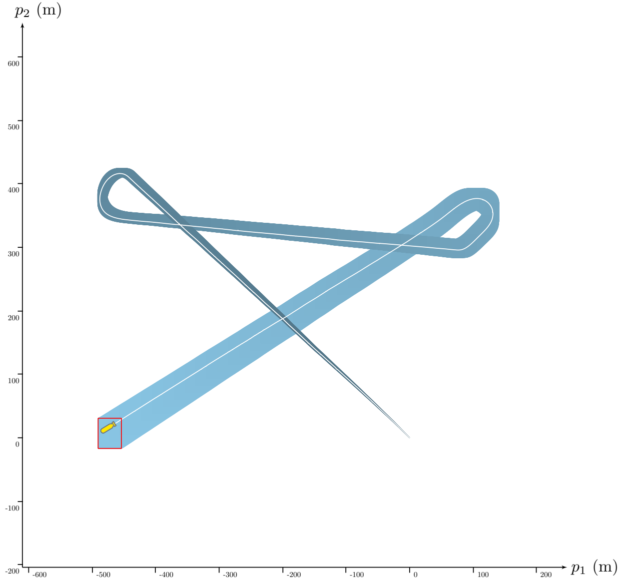
- white

Tube of positions:

- blue

Last position box:

- red



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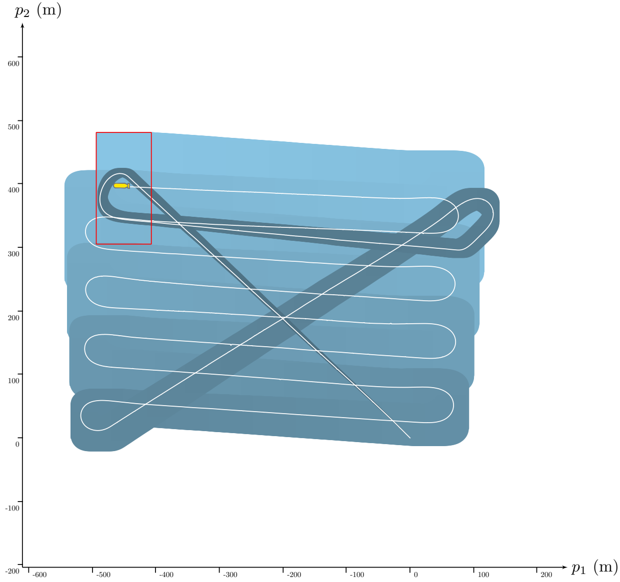

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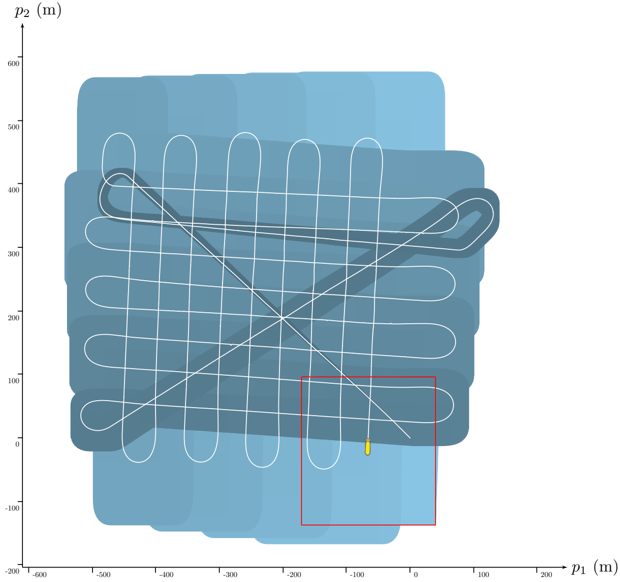

Sea trials

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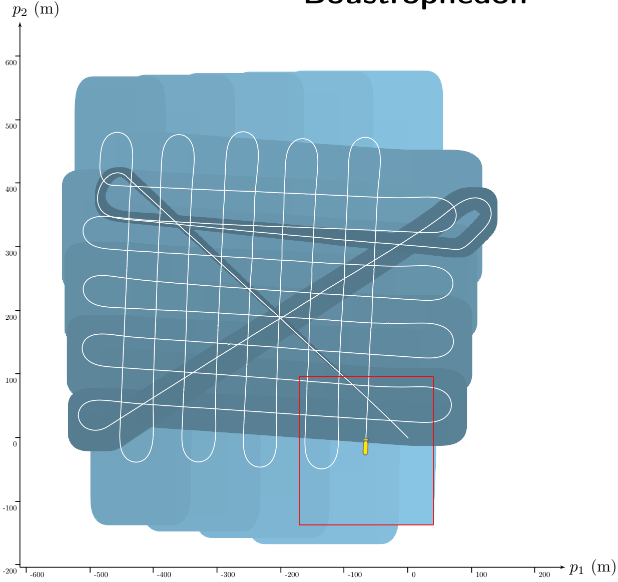
Boustrophedon



Sea trials

USBL

Actual trajectory:

- obtained thanks to an external USBL system
- used only for algorithm validation

Ultra-short baseLine (USBL), from the mothership

Sea trials

USBL

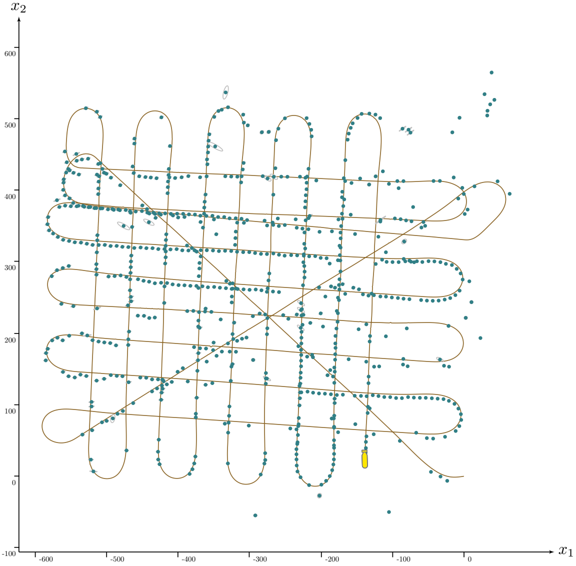
Actual trajectory:

- obtained thanks to an external USBL system
- used only for algorithm validation

Ultra-short baseLine (USBL), from the robot

Sea trials

USBL



Measurements and filtered trajectory

Sea trials

SLAM results

Actual trajectory:

- white

Tube of positions:

- blue

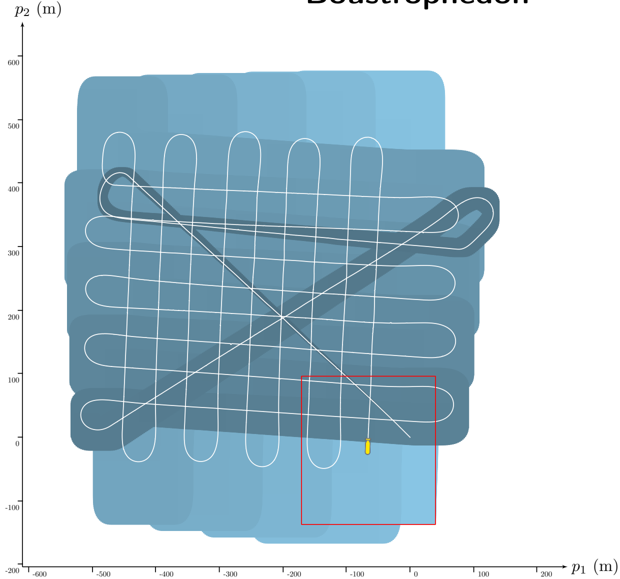
Last position box:

- red

Contracted parts:

- gray

Boustrophedon



Sea trials

SLAM results

Actual trajectory:

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Tube of positions:

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Contracted parts:

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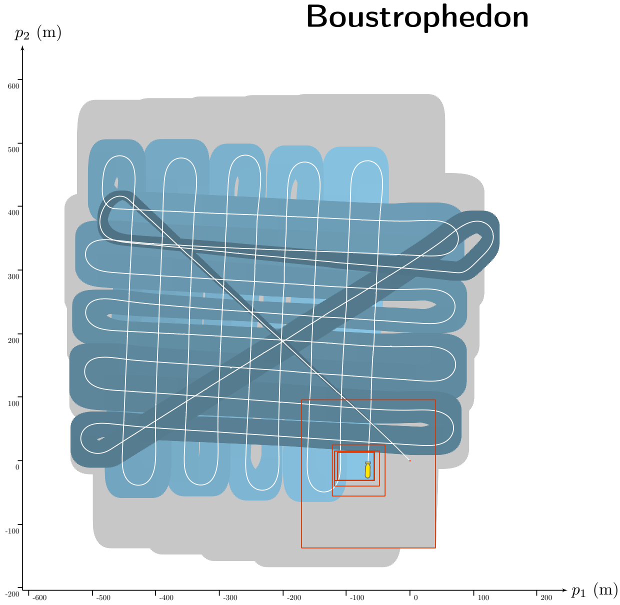
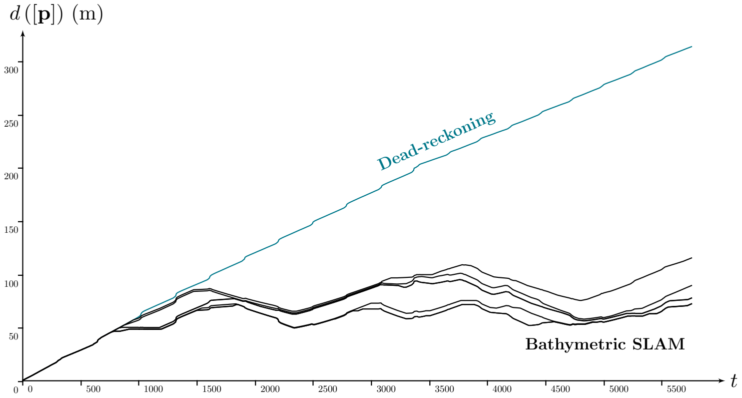


Sea trials

SLAM results



Thicknesses of the tube $[\mathbf{p}](\cdot)$

Localization:

- dead-reckoning: linear drift
- SLAM: no cumulated drift

Constraint method:

- iterative resolution
- reliable outputs, pessimism

Section 5

Conclusions

Conclusions

On this application

Bathymetric localization method:

- scalar measurements, **poor dataset**

Conclusions

On this application

Bathymetric localization method:

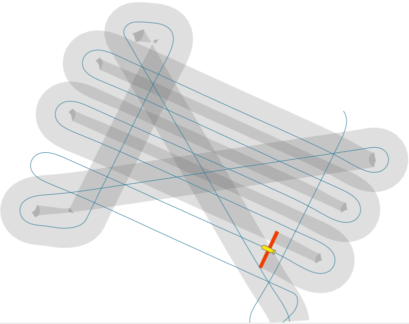
- scalar measurements, **poor dataset**
- **basic sensors**:
  IMU (attitude), DVL (velocities, altitude), pressure sensor (depth)

Conclusions

On this application

Bathymetric localization method:

- scalar measurements, **poor dataset**
- **basic sensors**:
  IMU (attitude), DVL (velocities, altitude), pressure sensor (depth)
- localization approach suitable for **survey missions**:
  boustrophedons involve several loops



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Originality of this work

- localization even in case of **unknown observation function** $\mathbf{g}$
  inter-temporal measurements

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Tubex library: open-source library providing tools for constraint programming over dynamical systems

http://www.simon-rohou.fr/research/tubex-lib

A temporal approach for the SLAM problem

— thank you for your attention —

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