Dealing with evaluation constraints in uncertain dynamical systems

Simon Rohou[1], Luc Jaulin[2], Lyudmila Mihaylova[3], Fabrice Le Bars[2], and Sandor M. Veres[3]

[1] IMT Atlantique, LS2N, Nantes, France
[2] ENSTA Bretagne, Lab-STICC, France
[3] The University of Sheffield, United Kingdom
simon.rohou@imt-atlantique.fr

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Problem: we consider the guaranteed non-linear state estimation of a robot described by the following state equations:

$$\begin{cases} \dot{\mathbf{x}}(\cdot) = \mathbf{f}\big(\mathbf{x}(\cdot), \mathbf{u}(\cdot)\big), & \text{(1a)} \\ \mathbf{z}_i = \mathbf{g}\big(\mathbf{x}(t_i)\big), & \text{(1b)} \end{cases}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector representing the system at time t and $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ a non-linear function depicting the evolution of the system based on input vectors $\mathbf{u}(t) \in \mathbb{R}^m$. The observation function $\mathbf{g} : \mathbb{R}^n \to \mathbb{R}^p$ may also be non-linear. The $t_i \in \mathbb{R}$, $i \in \mathbb{N}$, are measurement times and the $\mathbf{z}_i \in \mathbb{R}^p$ are the related outputs.

Approach: we will deal with this dynamical system by using a constraint programming approach. In a nutshell, the method consists in breaking the Equations (1a),(1b) into a set of elementary constraints that must be satisfied by the variables of the problem. In our case, the constraints may be non-linear or differential equations and the variables are vectors (*e.g.* $\mathbf{z}_i \in \mathbb{R}^p$) or trajectories (*e.g.* $\mathbf{x}(\cdot) \in \mathbb{R} \to \mathbb{R}^n$).

The variables are known to belong to some domains. For vectors of $\mathbb{R}^n$, we will use boxes in $\mathbb{IR}^n$. For trajectories, we will use *tubes* denoted by $[\mathbf{x}](\cdot) : \mathbb{R} \to \mathbb{IR}^n$. Constraints will be applied on these domains by means of operators called *contractors* $\mathcal{C}$ [2].

Contribution: the problem involves algebraic and differential constraints on trajectories such as $a(\cdot) = \sin\big(b(\cdot)\big)$ or $\dot{x}(\cdot) = v(\cdot)$. The related contractors have been the subject of some work. It remains to deal with the following elementary constraint denoted by $\mathcal{L}_{\text{eval}}$:

$$\mathcal{L}_{\text{eval}}\big(t, z, y(\cdot), w(\cdot)\big) : \big\{z = y(t)\ ,\ \dot{y}(\cdot) = w(\cdot)\big\} \tag{2}$$

with $t \in [t], z \in [z], y(\cdot) \in [y](\cdot), w(\cdot) \in [w](\cdot)$. Here, $w(\cdot)$ is the derivative of the signal to be evaluated. The problem is complex as the uncertainties related to t are difficult to propagate through the differential equation.

We propose the related contractor $\mathcal{C}_{\text{eval}}$, see [1], that will reliably reduce the sets of feasible solutions by contracting the bounds of the tube $[y](\cdot)$ and the intervals $[t]$ and $[z]$. Figure 1 provides an illustration of the evaluation of a trajectory in a bounded-error context.

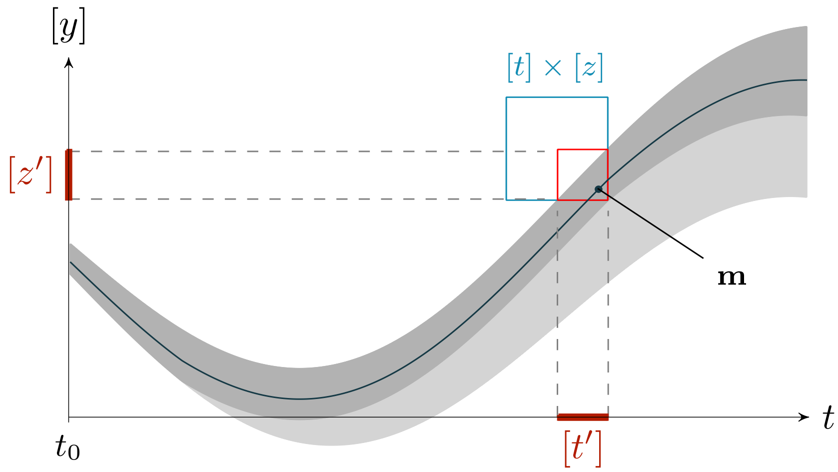

Figure 1: Evaluation on a tube $[y](\cdot)$. A given measurement $\mathbf{m} \in \mathbb{R}^2$, pictured by a black dot, is known to belong to the blue box $[t] \times [z]$. The tube is contracted by means of $\mathcal{C}_{\text{eval}}$; the contracted part is depicted in light gray. Meanwhile, the bounded observation itself is contracted to $[t'] \times [z']$ with $[t'] \subseteq [t]$ and $[z'] \subseteq [z]$. This is illustrated by the red box. The dark line is an example of a compliant trajectory.

References

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