

Robot localization in an unknown but symmetric environment

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Section 1

Context



Motivations: underwater localization

Context

Localization problem.

Considering a known speed of sound and linear acoustic rays:

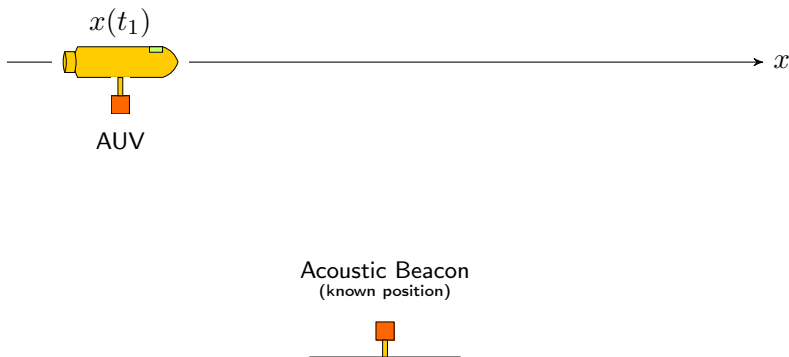


Figure: AUV's localization with an acoustic beacon on the seabed

Motivations: underwater localization

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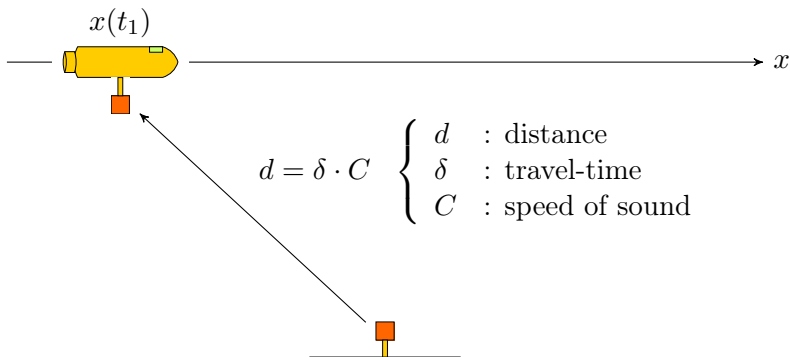


Figure: AUV's localization with an acoustic beacon on the seabed

Motivations: underwater localization

Context

Localization problem.

Considering refractions (Snell-Descartes) and no knowledge on C :

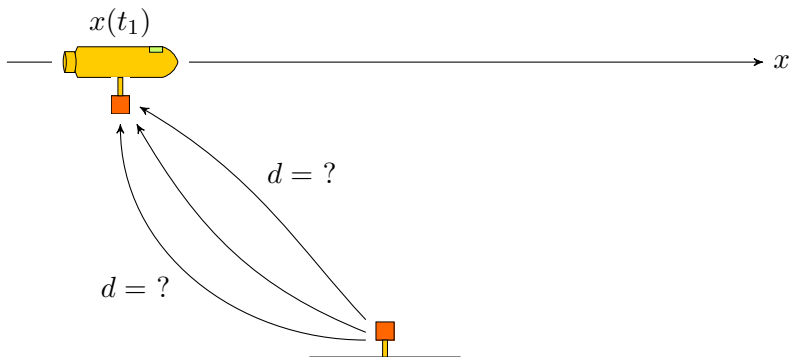


Figure: AUV's localization with an acoustic beacon on the seabed



Motivations: underwater localization

Context

Compensation of uncertainties with inter-temporal measurements.

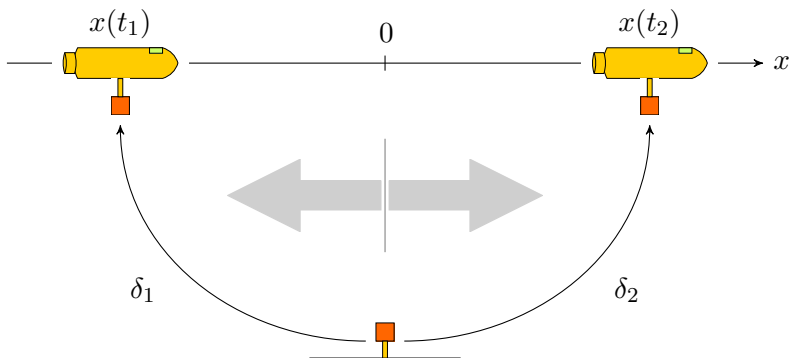


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Motivations: underwater localization

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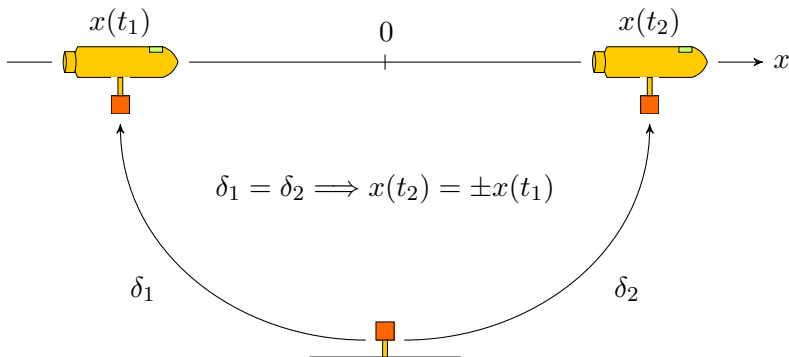


Figure: AUV's localization with an acoustic beacon on the seabed

Section 2

Formalization



System

Formalization

State estimation problem. Classically, we have:

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) &= g(\mathbf{x}(t)) \end{cases}$$

Where:

- ▶ $\mathbf{x} \in \mathbb{R}^n$ is the state vector
- ▶ $\mathbf{u} \in \mathbb{R}^m$ is the input vector
- ▶ $y \in \mathbb{R}$ is a measurement (assumed to be scalar)
- ▶ $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is the *evolution* function
- ▶ $g : \mathbb{R}^n \rightarrow \mathbb{R}$ is the *observation* function



Distorsion function

Formalization

We introduce $h : \mathbb{R} \rightarrow \mathbb{R}$ the **distorsion function** describing the uncertainties on the environment.

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) &= h \circ g(\mathbf{x}(t)) \end{cases}$$

- ▶ the analytic expression of h is considered unknown
- ▶ we admit h is strictly increasing (\Leftrightarrow *environmental gradient*)



Inter-temporality

Formalization

An inter-temporal state relation can be established by considering two identical measurements at different times:

$$\underbrace{h \circ g(\mathbf{x}(t_1))}_{y(t_1)} = \underbrace{h \circ g(\mathbf{x}(t_2))}_{y(t_2)}$$



Inter-temporality

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Since h is an injective function:

$$y(t_1) = y(t_2) \implies g(\mathbf{x}(t_1)) = g(\mathbf{x}(t_2))$$



Inter-temporality

Formalization

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Since h is an injective function:

$$y(t_1) = y(t_2) \implies g(\mathbf{x}(t_1)) = g(\mathbf{x}(t_2))$$

For resolution purposes, we define:

$$\begin{aligned}\tilde{y}(t_1, t_2) &= y(t_2) - y(t_1) \\ \tilde{g}(t_1, t_2) &= g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1))\end{aligned}$$



The symmetric set

Formalization

We define the **symmetric set** $\mathbb{S} \in \mathbb{T}$ such that:

$$\mathbb{S} = \{(t_1, t_2) \in [0, t_f]^2 \mid \tilde{y}(t_1, t_2) = 0, t_1 < t_2\}$$

[Aub13]

- ▶ (t_1, t_2) is called a t -pair
- ▶ the set \mathbb{T} of all t -pairs is called a t -plane



Understanding the symmetric set

Formalization

$$\mathbb{S} = \{(t_1, t_2) \in [0, t_f]^2 \mid \tilde{y}(t_1, t_2) = 0, t_1 < t_2\}$$

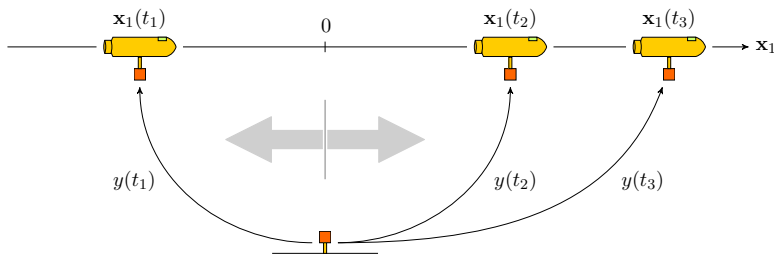


Figure: inter-temporal measurements

$$(t_1, t_2) \in \mathbb{S}$$

$$(t_1, t_3) \notin \mathbb{S}$$

$$(t_2, t_3) \notin \mathbb{S}$$



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Section 3

Resolution with an interval method



Set-membership estimation

Resolution with an interval method

The state estimation becomes:

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ y(t) &= h \circ g(\mathbf{x}(t)) \end{cases}$$

With:

- ▶ $\mathbf{x}(t) \in [\mathbf{x}](t)$
- ▶ $\mathbf{u}(t) \in [\mathbf{u}](t)$
- ▶ $y(t) \in [y](t)$

$[\mathbf{x}](0)$ is supposed to be known.

Values evolving with time are pictured with **tubes**.



Tubes

Resolution with an interval method

Tube $[f](t)$: interval of functions $[f^-, f^+]$ such that: $\forall t \in \mathbb{R}, f^-(t) \leq f^+(t)$

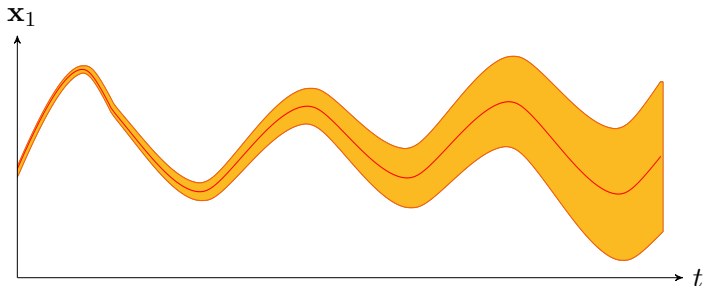


Figure: Tube $[x_1](t)$ (orange) enclosing true values (red)

Proposition

Resolution with an interval method

The symmetric set \mathbb{S} is:

$$\mathbb{S} = \{(t_1, t_2) \mid \tilde{y}(t_1, t_2) = 0\}$$



Proposition

Resolution with an interval method

The symmetric set \mathbb{S} is:

$$\mathbb{S} = \{(t_1, t_2) \mid \tilde{y}(t_1, t_2) = 0\}$$

Considering: $\tilde{y} \in [\tilde{y}]$,
we can estimate \mathbb{S}^+ enclosing \mathbb{S} with:

$$\mathbb{S}^+ = \{(t_1, t_2) \mid 0 \in [\tilde{y}](t_1, t_2)\}$$



Estimation of \mathbb{S}^+

Resolution with an interval method

$$\mathbb{S} = \{(t_1, t_2) \mid \tilde{y}(t_1, t_2) = 0\}$$

$$\mathbb{S}^+ = \{(t_1, t_2) \mid 0 \in [\tilde{y}](t_1, t_2)\}$$

A state $\mathbf{x}(t_1)$ can be associated to $\mathbf{x}(t_2)$ only if $(t_1, t_2) \in \mathbb{S}$.

For now, let us fix the value for t_2 :

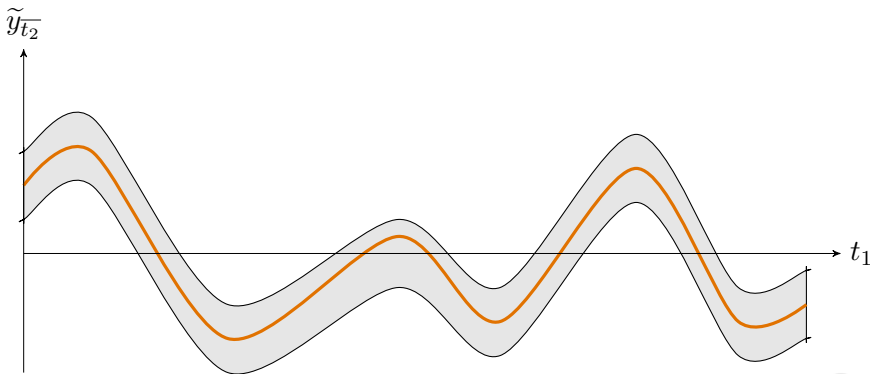


Figure: the tube $[\tilde{y}]_{t_2}(t_1)$



Estimation of \mathbb{S}^+

Resolution with an interval method

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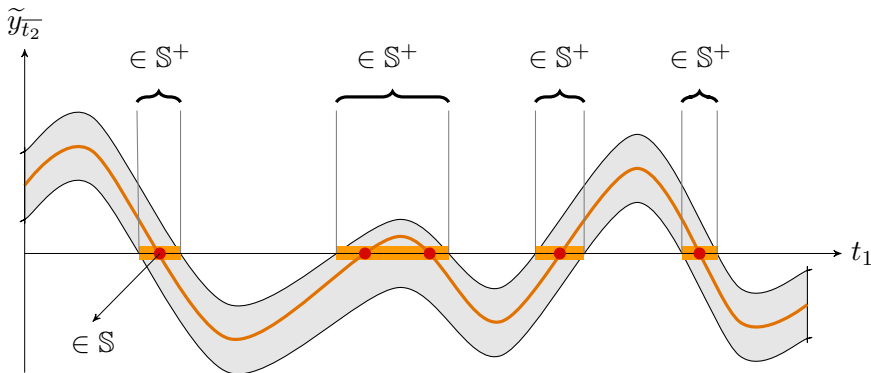


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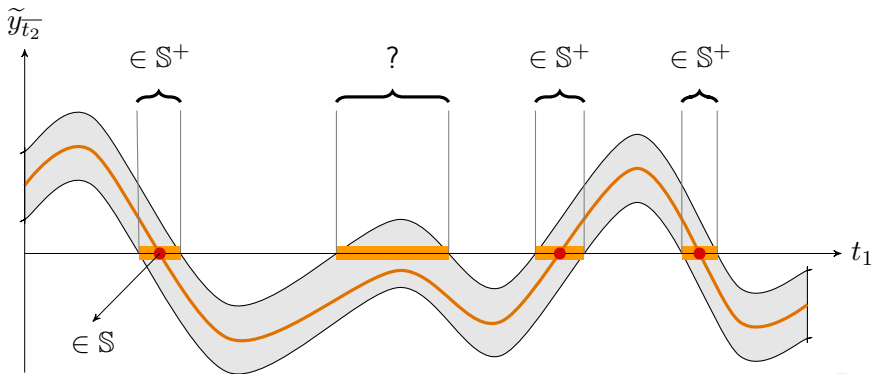


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Estimation of \mathbb{S}^+

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$$\mathbb{S}^+ = \{(t_1, t_2) \mid 0 \in [\tilde{y}](t_1, t_2)\}$$

We apply the intermediate value theorem.

We search solutions for $\tilde{y}(t_1, t_2) \leq 0$: the **pre-symmetric set** \mathbb{P} .

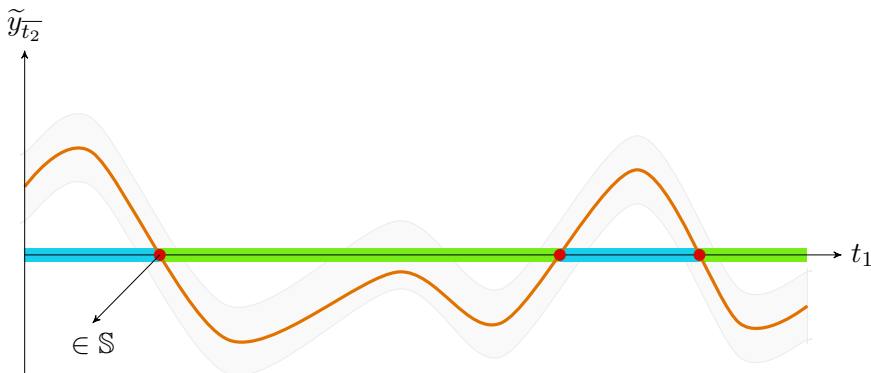


Figure: the pre-symmetric set \mathbb{P} :



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Estimation of \mathbb{S}^+

Resolution with an interval method

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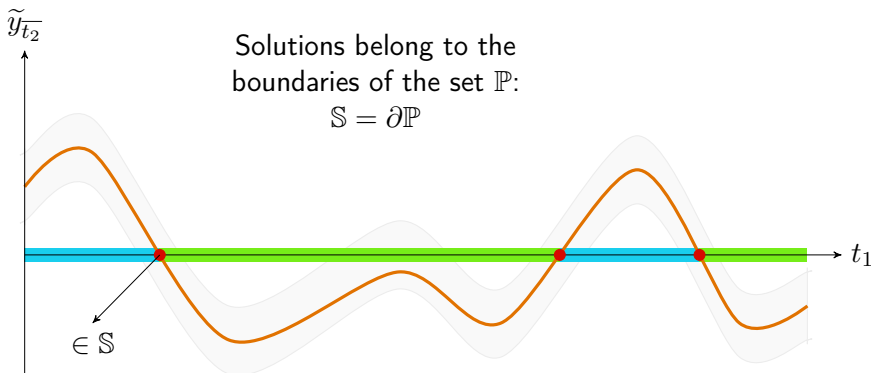


Figure: the pre-symmetric set \mathbb{P} :



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Estimation of the pre-symmetric set \mathbb{P}

Resolution with an interval method

$$\mathbb{P}^- = \text{green box}$$

$$\mathbb{P}^+ = \text{green box} + \text{yellow box}$$

The symmetric set is a boundary of the pre-symmetric set: $\mathbb{S} = \partial\mathbb{P}$

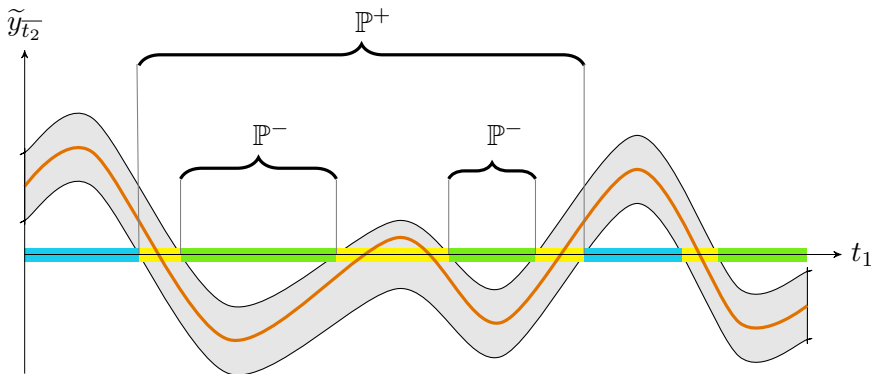


Figure: representation of the pre-symmetric set



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Estimation of the pre-symmetric set \mathbb{P}

Resolution with an interval method

$$\mathbb{P}^- = \text{green box}$$

$$\mathbb{P}^+ = \text{green box} + \text{yellow box} + \text{orange box}$$

The symmetric set is a boundary of the pre-symmetric set: $\mathbb{S} = \partial\mathbb{P}$

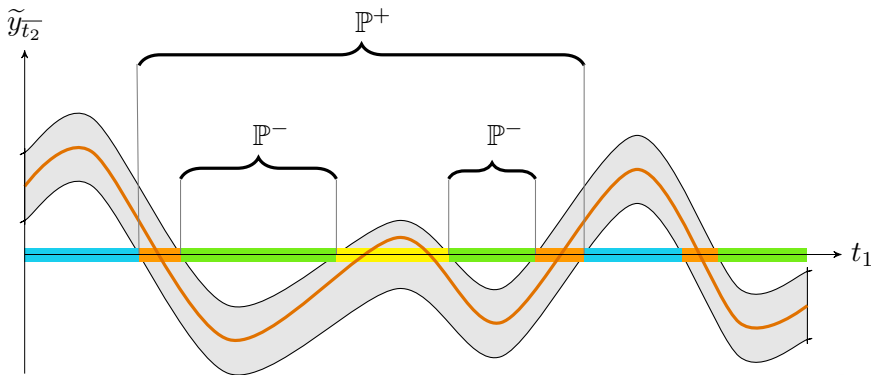


Figure: the set = sure boundaries of \mathbb{P} = true solutions for \mathbb{S}



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Estimation of the pre-symmetric set \mathbb{P}

Resolution with an interval method

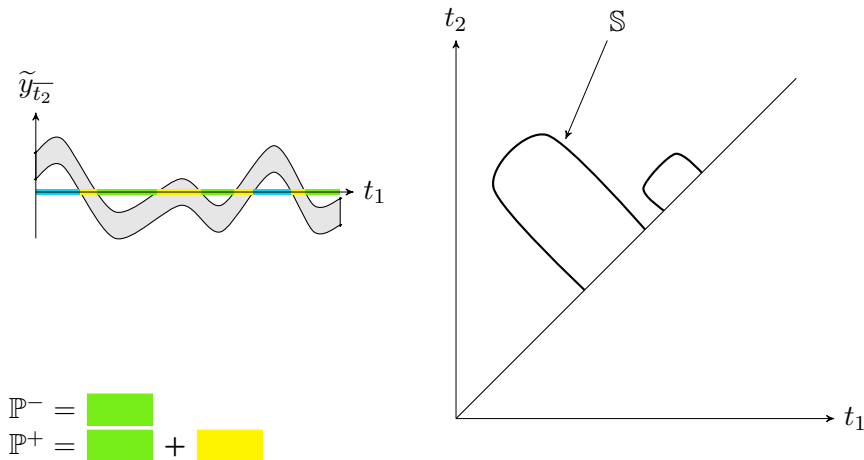


Figure: representation of \mathbb{S} and \mathbb{P} in a t -plane



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Estimation of the pre-symmetric set \mathbb{P}

Resolution with an interval method

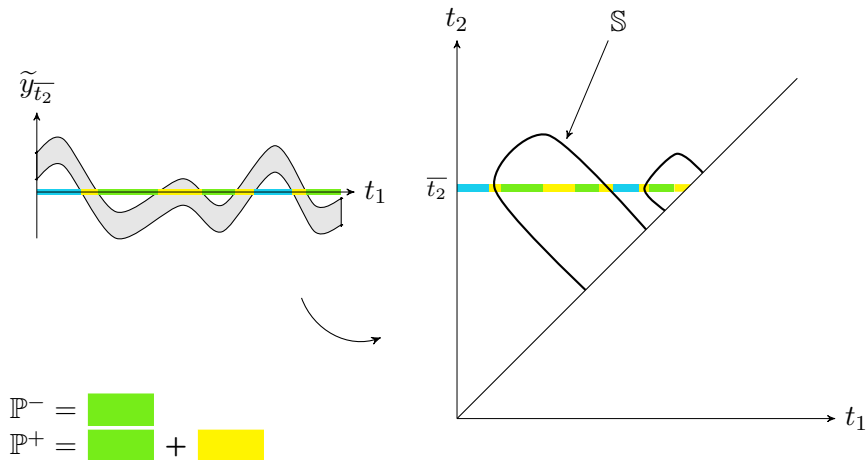


Figure: representation of \mathbb{S} and \mathbb{P} in a t -plane



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Constraint Network

Resolution with an interval method

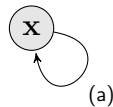
Variables: \mathbf{x} , \mathbb{P}



Constraint Network

Resolution with an interval method

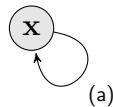
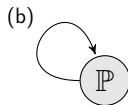
$$\left\{ \begin{array}{l} \text{Variables: } \mathbf{x}, \mathbb{P} \\ \text{Constraints:} \\ \quad \text{(a) } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \end{array} \right.$$



Constraint Network

Resolution with an interval method

$$\left\{ \begin{array}{l} \text{Variables: } \mathbf{x}, \mathbb{P} \\ \text{Constraints:} \\ \quad \text{(a) } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \quad \text{(b) } \mathbb{P} = \{(t_1, t_2) \mid \tilde{y}(t_1, t_2) \leq 0\} \end{array} \right.$$



Constraint Network

Resolution with an interval method

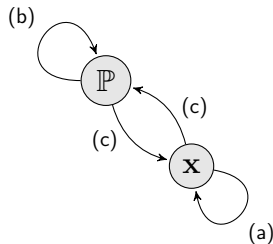
Variables: \mathbf{x} , \mathbb{P}

Constraints:

(a) $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$

(b) $\mathbb{P} = \{(t_1, t_2) \mid \tilde{y}(t_1, t_2) \leq 0\}$

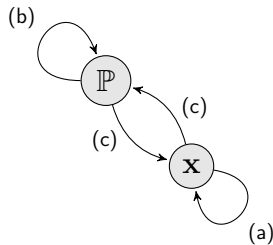
(c) $\mathbb{P} = \{(t_1, t_2) \mid \tilde{g}(t_1, t_2) \leq 0\}$



Constraint Network

Resolution with an interval method

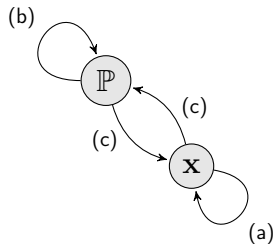
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 \quad \text{(c) } \mathbb{P} = \{(t_1, t_2) \mid \tilde{g}(t_1, t_2) \leq 0\} \\
 \text{Domains: } [\mathbf{x}], [\mathbb{P}]
 \end{array} \right.$$



Constraint Network

Resolution with an interval method

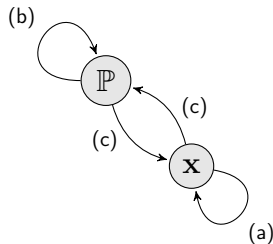
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 \quad \text{(c) } \mathbb{P} = \{(t_1, t_2) \mid \tilde{g}(t_1, t_2) \leq 0\} \\
 \text{Domains: } [\mathbf{x}], [\mathbb{P}] \\
 \text{Initialization: } \mathbf{x} \in [-\infty, +\infty]^n, [\mathbb{P}] = [\emptyset, [0, t_f]^2]
 \end{array} \right.$$



Constraint Network

Resolution with an interval method

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 \text{Variables: } \mathbf{x}, \mathbb{P} \\
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 \end{array} \right.$$



h does not appear anymore.

Section 4

Example



Simulation: an Autonomous Underwater Vehicle

Example

Estimation of \mathbb{S}

Example

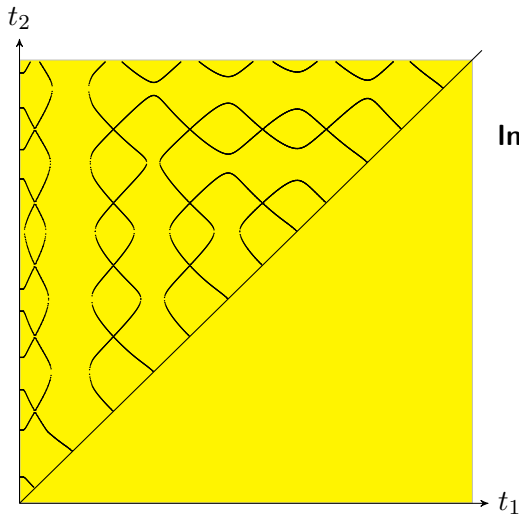
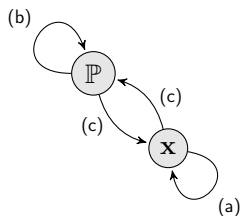


Figure: t -plane (\mathbb{S} pictured in black)



Initialization:

$$\begin{aligned} [\mathbb{P}] &= [\emptyset, [0, t_f]^2] \\ &= \text{yellow square} \end{aligned}$$

Estimation of \mathbb{P}

Example

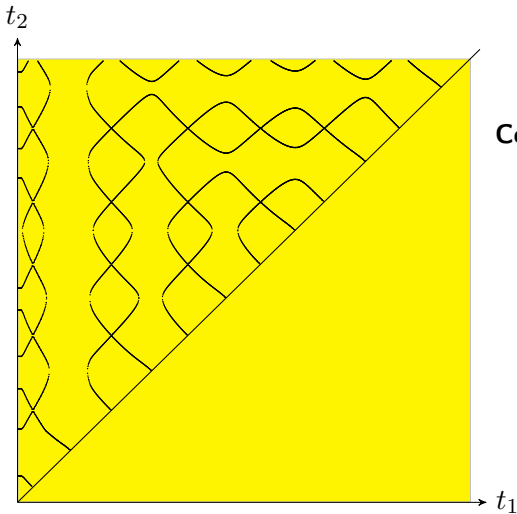
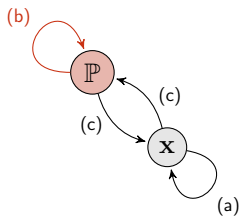


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (b):

$$\mathbb{P} = \{(t_1, t_2) \mid \tilde{y}(t_1, t_2) \leq 0\}$$

$$\underbrace{(\tilde{y}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\tilde{y}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

$$\mathbb{P}^- = \text{green box}$$

$$\mathbb{P}^+ = \text{green box} + \text{yellow box}$$



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Estimation of \mathbb{P}

Example

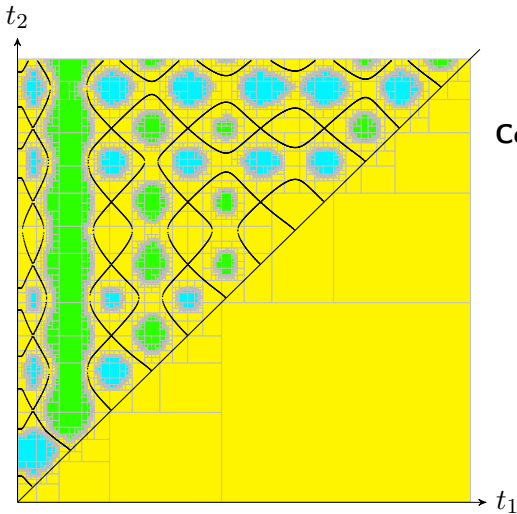
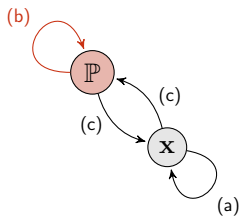


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Estimation of \mathbb{P}

Example

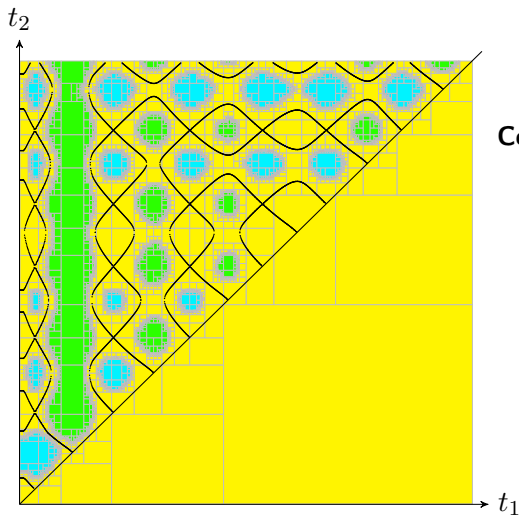
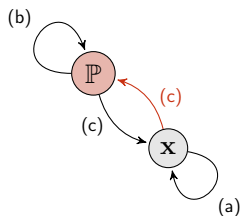


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (c):

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$$\underbrace{(\tilde{g}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\tilde{g}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

$$\mathbb{P}^- = \text{[Green Box]}$$

$$\mathbb{P}^+ = \text{[Green Box]} + \text{[Yellow Box]}$$



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Estimation of \mathbb{P}

Example

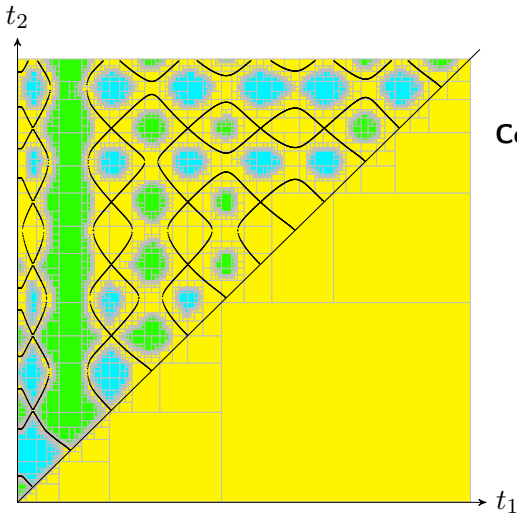
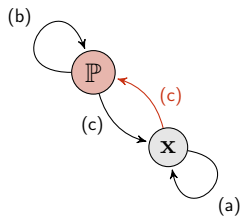


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (c):

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Contraction of $[\mathbf{x}]$

Example

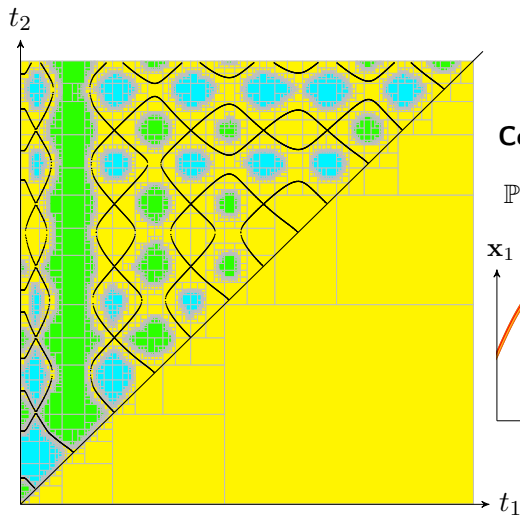
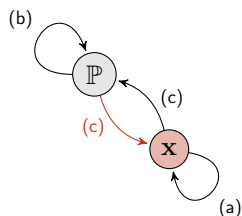


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1)) \leq 0\}$$

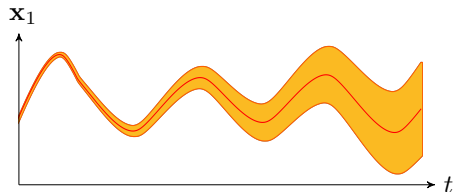


Figure: Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example

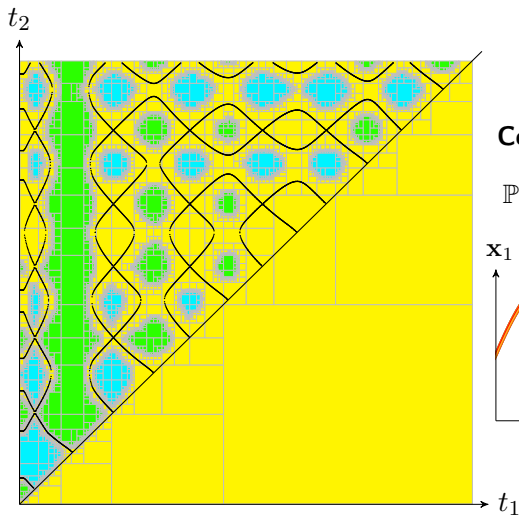
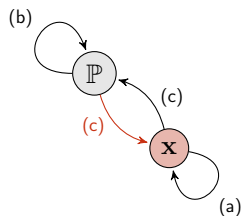


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid g(\mathbf{x}(t_2)) - g(\mathbf{x}(t_1)) \leq 0\}$$

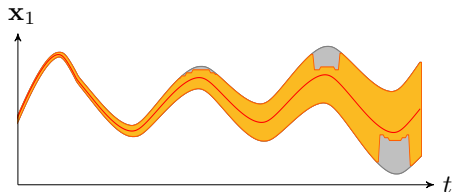


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Contraction of $[\mathbf{x}]$

Example

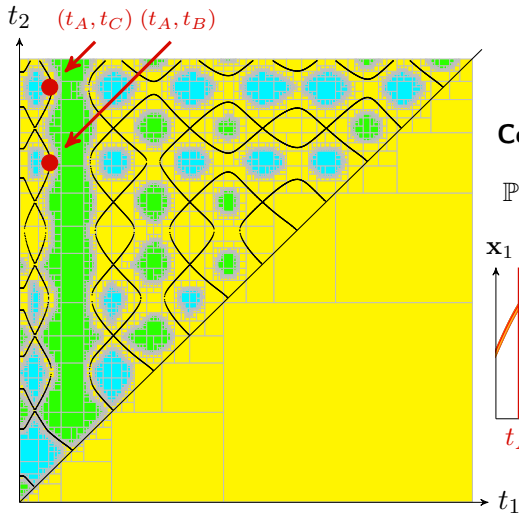
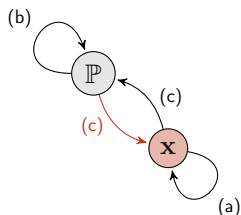


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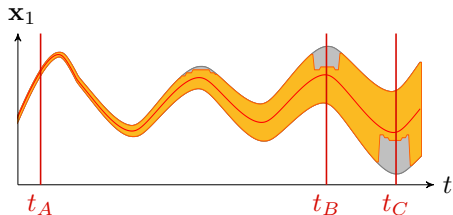


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Contraction of $[\mathbf{x}]$

Example

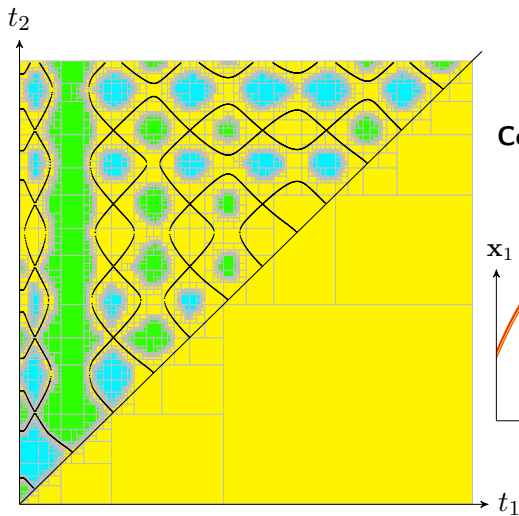
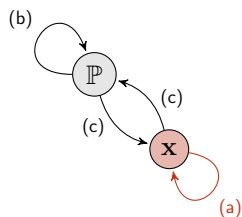


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (a):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

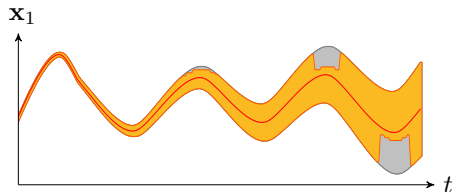


Figure: Contraction of tube $[\mathbf{x}_1](t)$



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Contraction of $[\mathbf{x}]$

Example

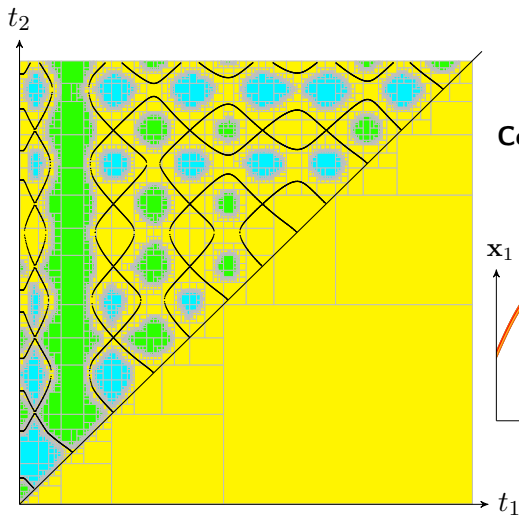
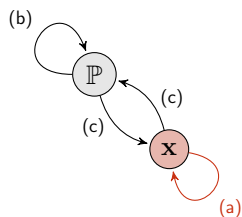


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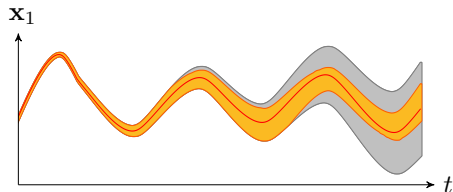


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Estimation of \mathbb{P}

Example

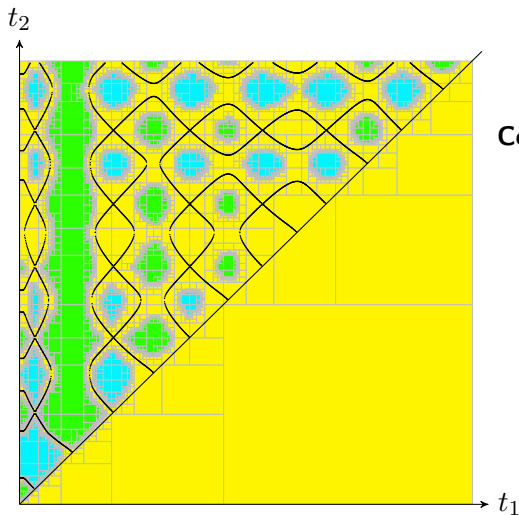
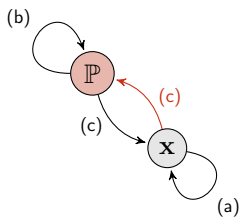


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (c):

$$\mathbb{P} = \{(t_1, t_2) \mid \tilde{g}(t_2, t_1) \leq 0\}$$

$$\underbrace{(\tilde{g}^+)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^-} \subset \mathbb{P} \subset \underbrace{(\tilde{g}^-)^{-1}(\mathbb{R}^-)}_{\mathbb{P}^+}$$

$$\mathbb{P}^- = \text{green box}$$

$$\mathbb{P}^+ = \text{green box} + \text{yellow box}$$



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Estimation of \mathbb{P}

Example

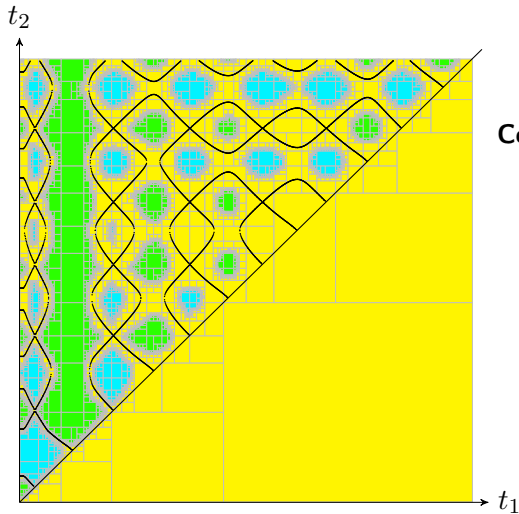
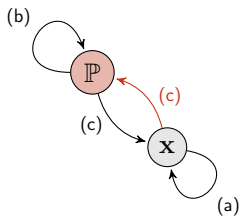


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



Constraint (c):

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Contraction of $[\mathbf{x}]$

Example

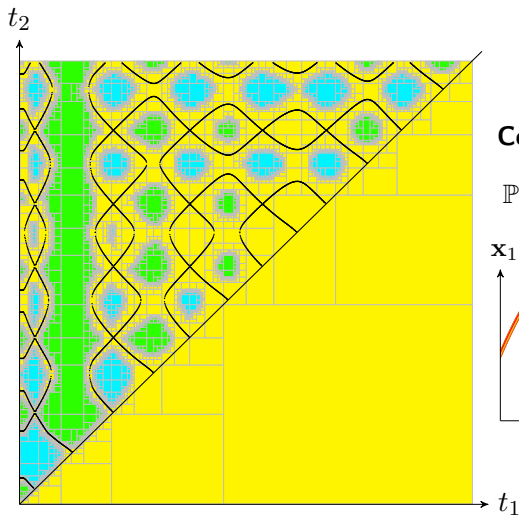
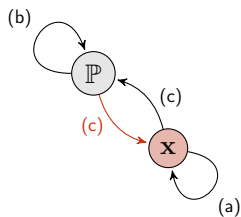


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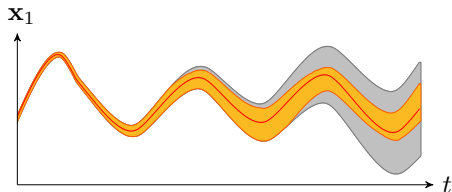


Figure: Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example

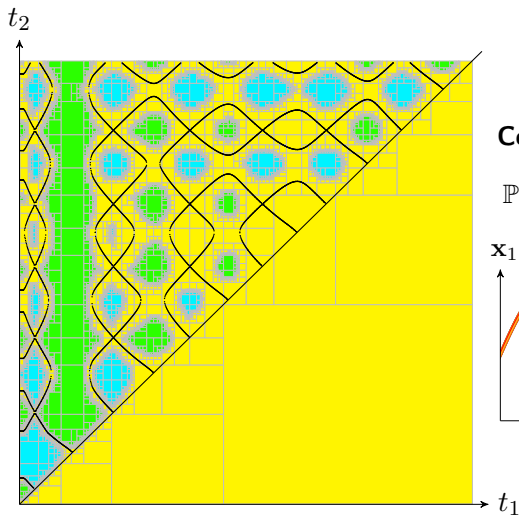
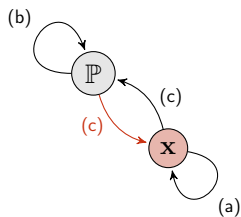


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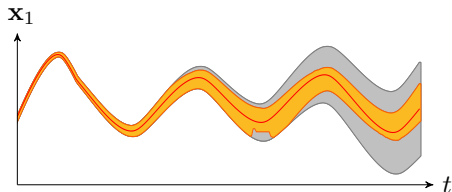


Figure: Contraction of tube $[\mathbf{x}_1](t)$

Contraction of $[\mathbf{x}]$

Example

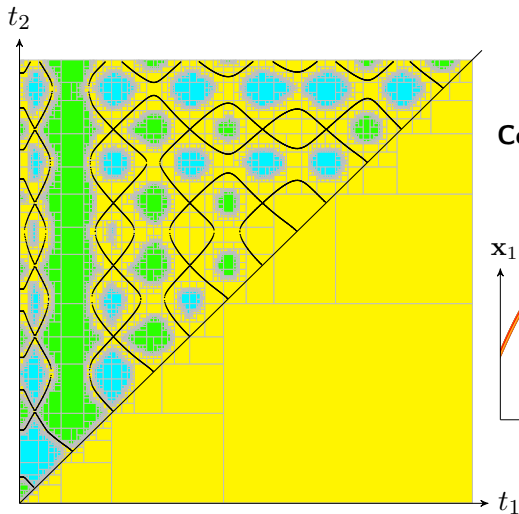
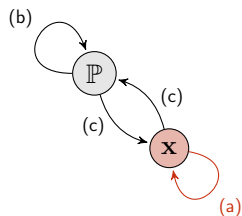


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



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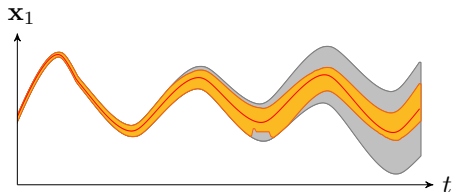


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Contraction of $[\mathbf{x}]$

Example

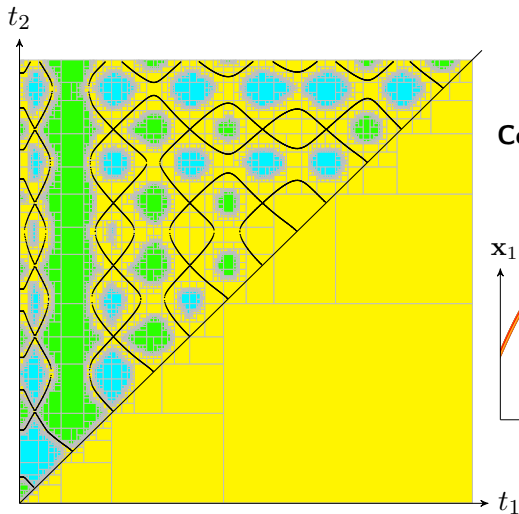
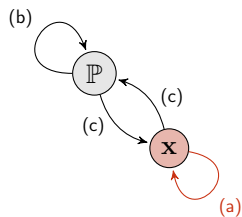


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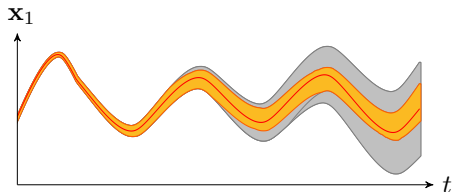


Figure: Contraction of tube $[\mathbf{x}_1](t)$



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Estimation of \mathbb{P}

Example

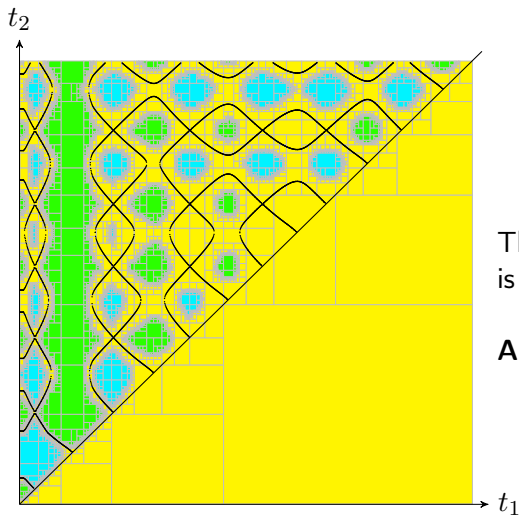
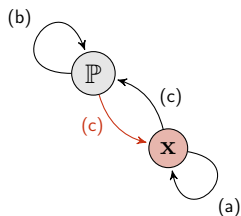


Figure: t -plane of \mathbb{P} with a SIVIA algorithm



The simulation is run until there is no more contraction for x .

A fixpoint has been reached.

Conclusion

Robot localization in an unknown but symmetric environment:

- ▶ use of inter-temporal measurements $y(t_1, t_2)$, $y(t_3, t_4)$, ...
- ▶ uncertainties compensated
- ▶ state estimation

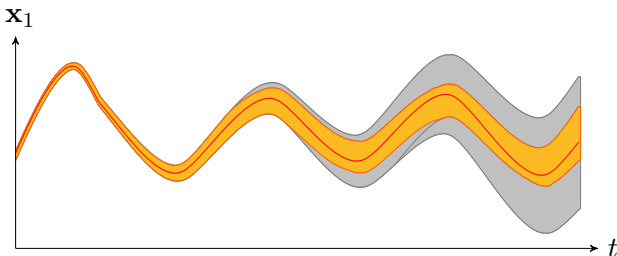


Figure: final contraction of robot's position



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Support: *Direction Générale de l'Armement* (DGA - FR)

References:

[Aub13] C. Aubry, R. Desmare and L. Jaulin,
Loop detection of mobile robots using interval analysis,
Automatica, 2013.

[Bar12] F. LeBars, J. Sliwka, O. Reynet and L. Jaulin,
State estimation with fleeting data,
Automatica, 2012.

Tools:



IBEX library
used for interval arithmetic



VIBES
used for rendering



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ENSTA
Bretagne