Robust Polygon-Based Localization

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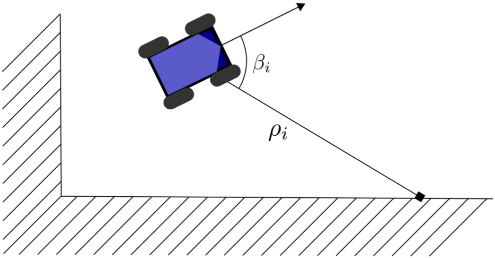
Outline

1. Motivation
2. Static Localization
   - Map Representation
   - Polygon-Based Localization
   - Experiments
3. Dynamic Localization
   - Constraints Network
   - Experiments
4. Conclusion

Motivation

Localization problem.
Consider a pose estimation problem based on rangefinder readings.



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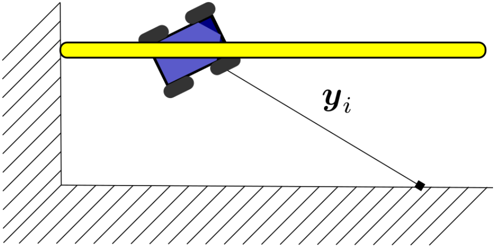

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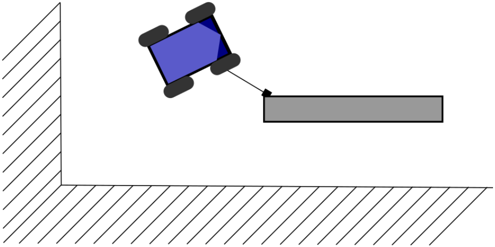

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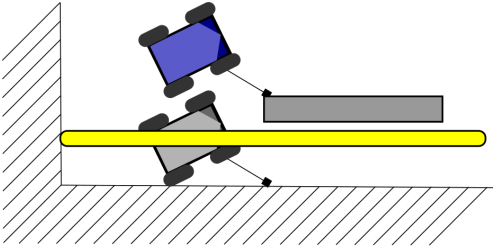

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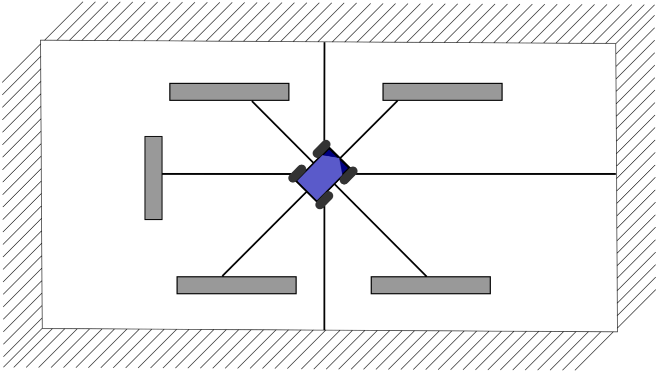

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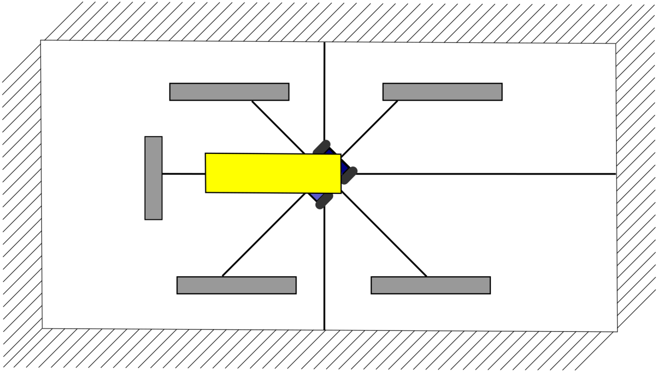

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Static Localization

$$\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}_i) \in \mathbb{M}$$

where:

- $\boldsymbol{x} = (x, y, \theta)$ is the robot's pose to be estimated
- $\boldsymbol{y}_i = (\rho_i, \beta_i)$, given by a distance ρ_i and bearing β_i
- $\mathbb{M}$ is the known map.

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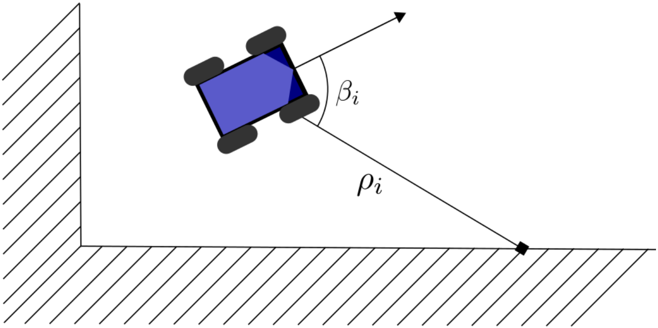
$$\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}_i) = \boldsymbol{h}_i(\boldsymbol{x})$$

Static Localization

$$\boldsymbol{h}_i(\boldsymbol{x}) = \begin{bmatrix} x + \rho_i \cdot cos(\beta_i + \theta) \\ y + \rho_i \cdot sin(\beta_i + \theta) \end{bmatrix}$$



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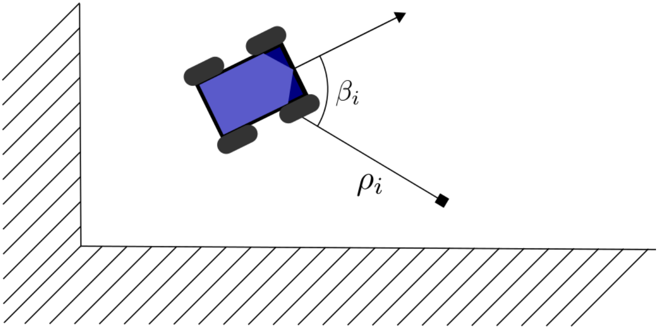
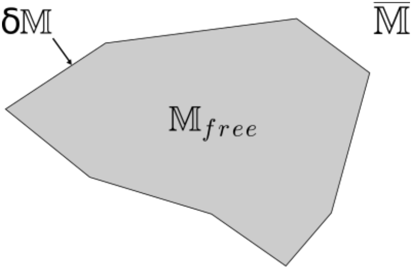
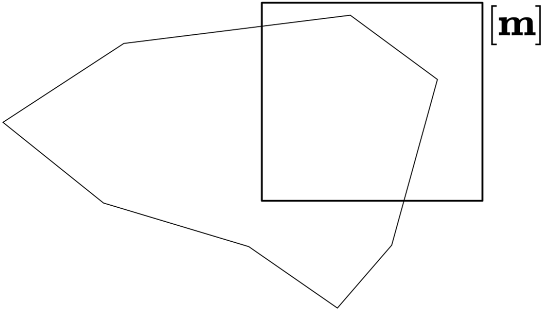


Map Representation - Static Localization



$$\mathbb{M} = \delta\mathbb{M} \cup \mathbb{M}_{free}$$

Map Representation - Segment Contractor



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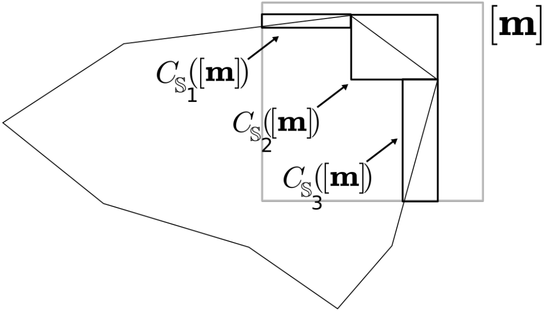

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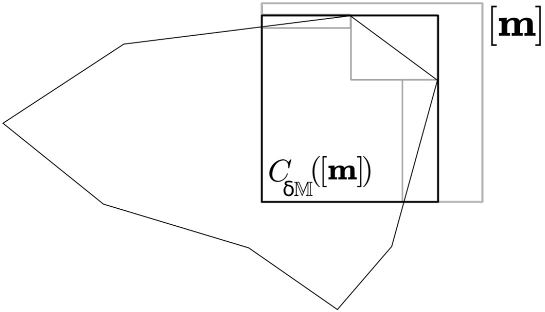


$$C_{\delta\mathbb{M}} = \bigcup_{j=1}^{card(\mathbb{S})} C_{\mathbb{S}_j}$$

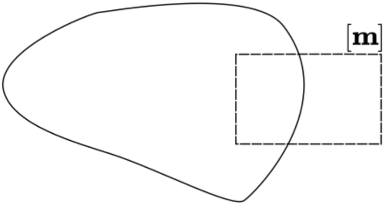
Map Representation - Segment Constraints

$$c_1 : det(\boldsymbol{b}_j - \boldsymbol{a}_j, \boldsymbol{a}_j - \boldsymbol{m}) = \begin{vmatrix} b_{j,0} - a_{j,0}, & a_{j,0} - m_0 \\ b_{j,1} - a_{j,1}, & a_{j,1} - m_1 \end{vmatrix} = 0,$$
$$c_2 : min(\boldsymbol{a}_j, \boldsymbol{b}_j) \leq \boldsymbol{m} \leq max(\boldsymbol{a}_j, \boldsymbol{b}_j)$$

where:

- $(\boldsymbol{a}_j, \boldsymbol{b}_j)$ are the extremities of a segment $\mathbb{S}_j$
- $\boldsymbol{m} = (m_0, m_1) \in \delta\mathbb{M}$.

Map Representation



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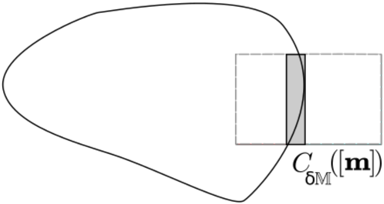

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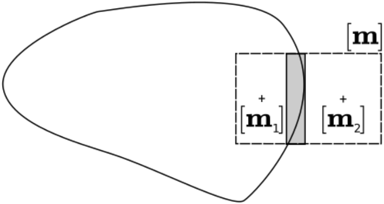
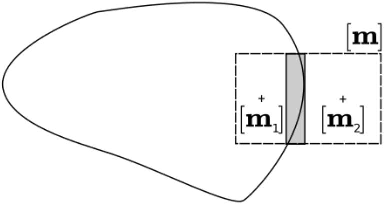


$$\begin{cases} [\boldsymbol{m}_k] \subset \mathbb{M}_{free}, & \mathcal{T}(\boldsymbol{m}_k) = true \\ [\boldsymbol{m}_k] \subset \overline{\mathbb{M}}, & Otherwise \end{cases}$$

Map Representation



$$\begin{cases} [\boldsymbol{m}_1] \subset \mathbb{M}_{free}, \\ [\boldsymbol{m}_2] \subset \overline{\mathbb{M}}, \end{cases}$$

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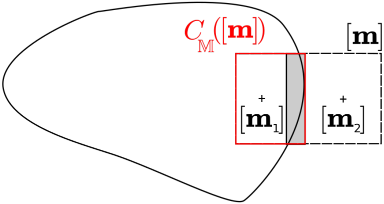

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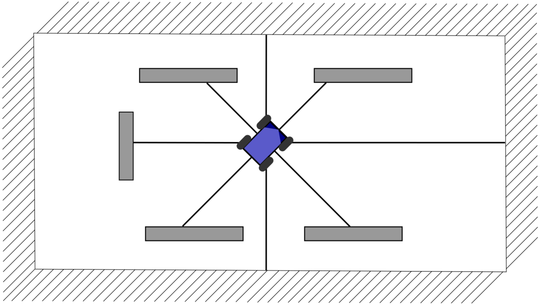
Polygon-Based Localization

$$\mathbb{X}_i = \boldsymbol{h}_i^{-1}(\mathbb{M}) = \{\boldsymbol{x} \in \mathbb{R}^3 : \boldsymbol{h}_i(\boldsymbol{x}) \in \mathbb{M}\}$$

where:

- $\mathbb{X}_i$ is a set of possible robot's poses

Polygon-Based Localization



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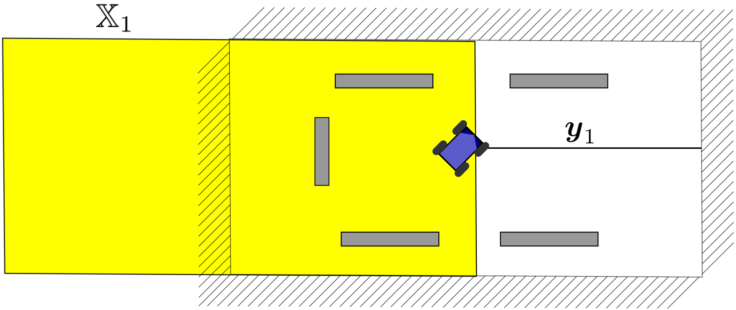

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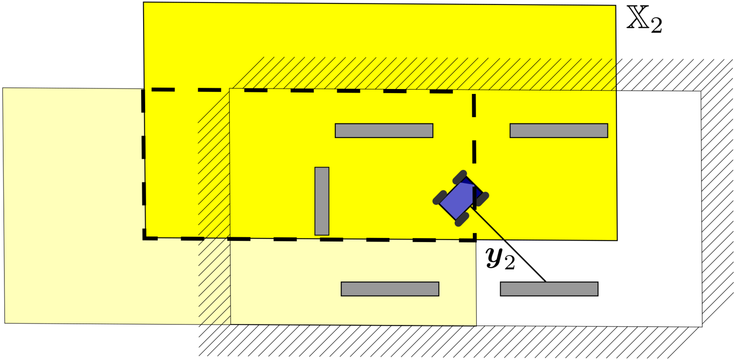

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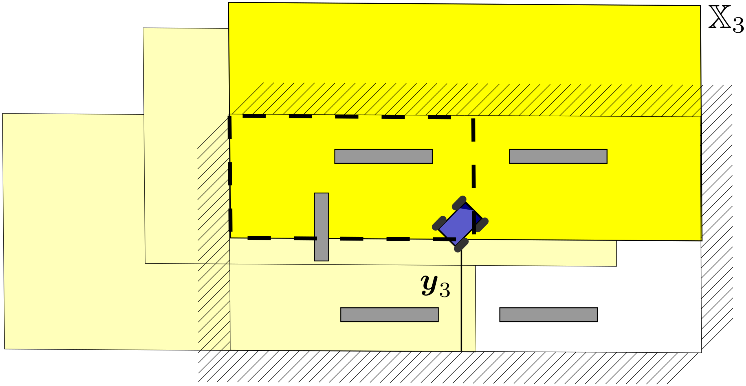

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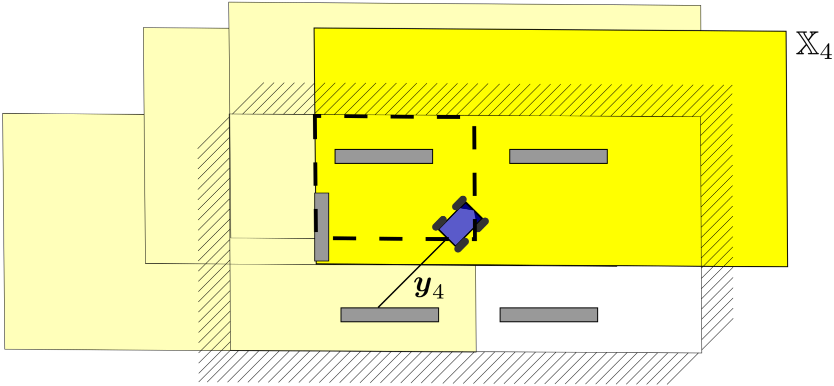

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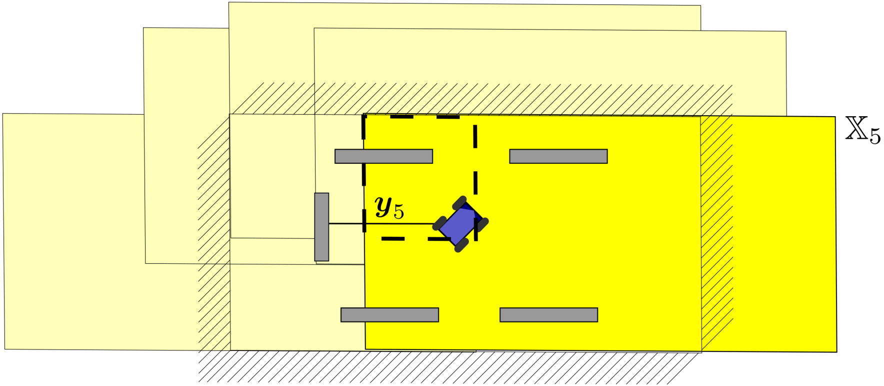

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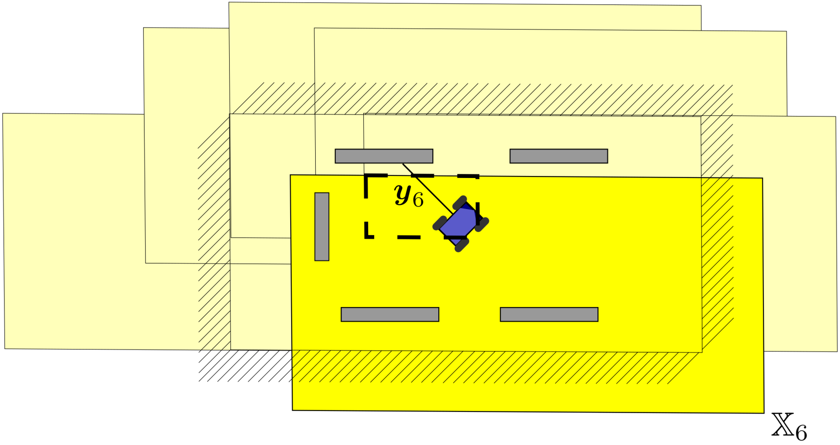

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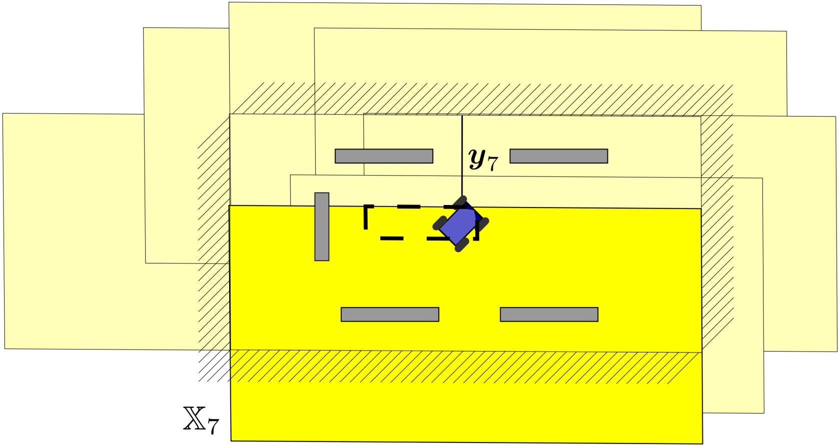

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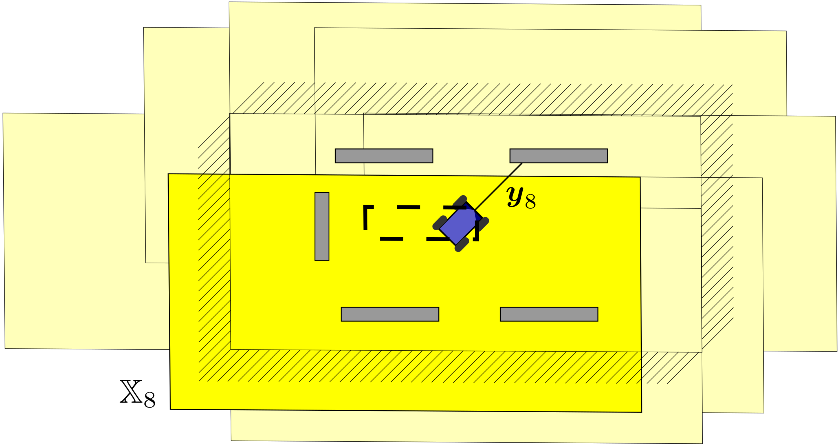

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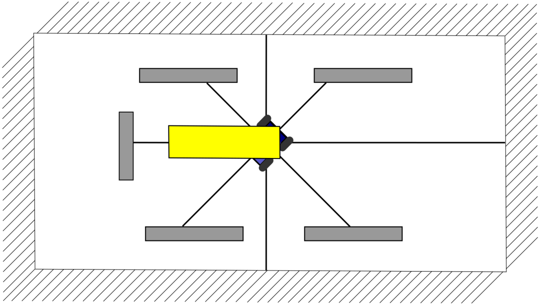
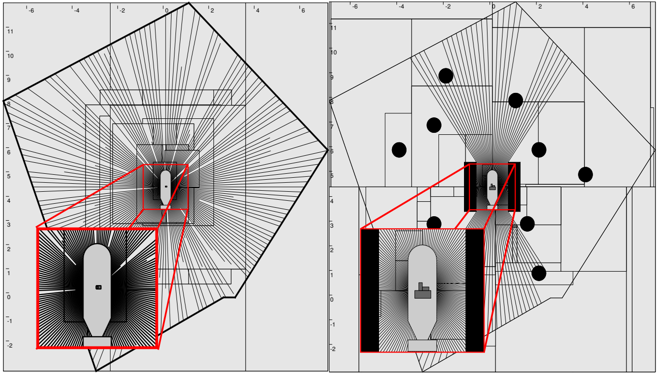


$$\mathbb{X} = \bigcap_{i=1}^{r} \mathbb{X}_i$$

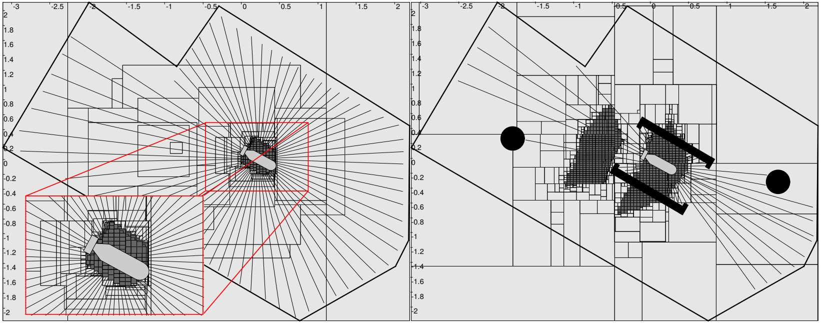
Experiments - Simulated Scenario



(a) (b)

Figure: Static localization performed in the simulated scenarios, without (a) and with (b) unknown obstacles. The pavings were computed with $\epsilon_\theta = 10^\circ$ for the θ space and $\epsilon = 50$cm for coordinates x and y.

Experiments - Real Scenario



(a) (b)

Figure: Static localization performed in the real scenarios, without (a) and with (b) unknown obstacles. The pavings were computed with $\epsilon_\theta = 10^\circ$ for the θ space and $\epsilon = 5$cm for coordinates x and y.

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Dynamic Localization

$$\begin{cases} \boldsymbol{x}_{k+1} & = & \boldsymbol{f}\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right) \\ \boldsymbol{y}_k & = & \boldsymbol{g}\left(\boldsymbol{x}_k\right) \end{cases}$$

where:

- $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector
- $\boldsymbol{u} \in \mathbb{R}^m$ is the input vector
- $\boldsymbol{y} \in \mathbb{R}^p$ are the measurements
- $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ is the *evolution* function
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$$\boldsymbol{x} = (x, y, \theta) \text{ and } \boldsymbol{u} = (v, \omega)$$

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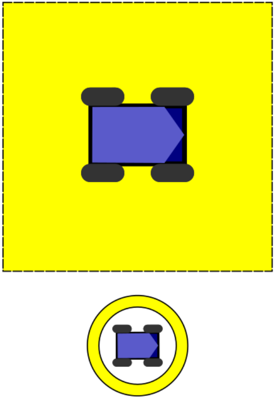

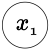

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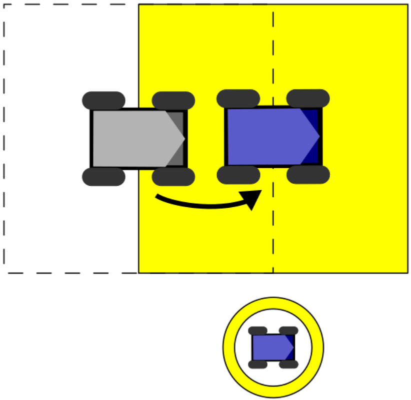

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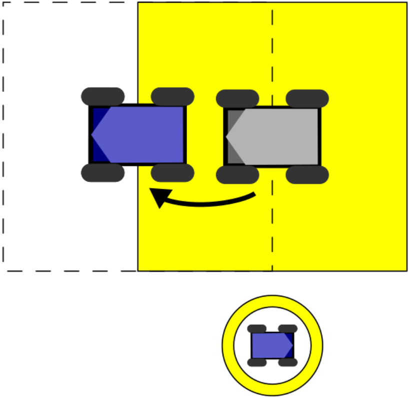

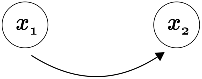

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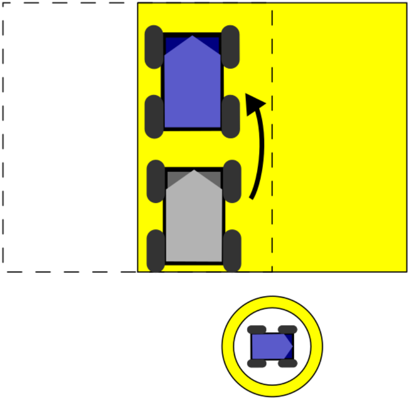

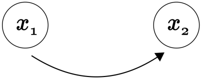

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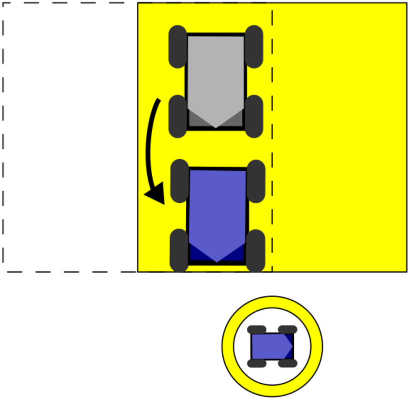

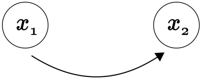

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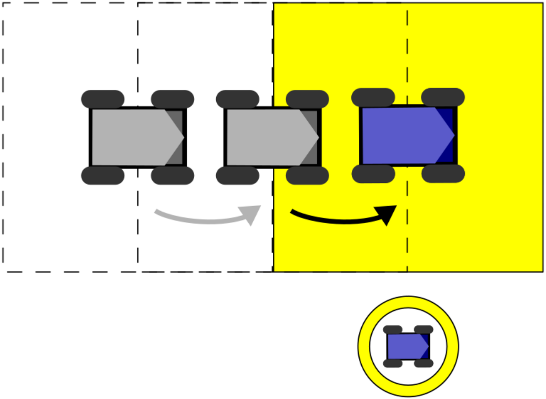

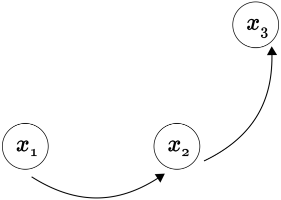

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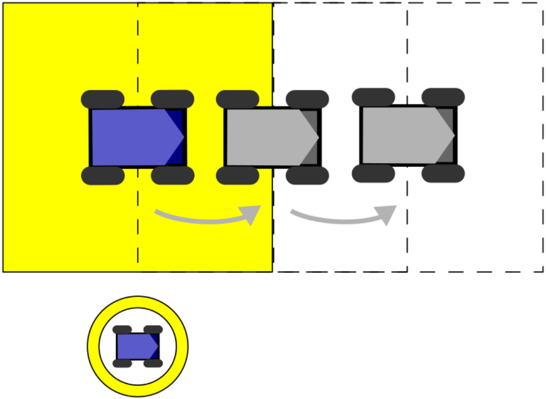

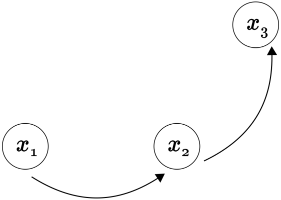

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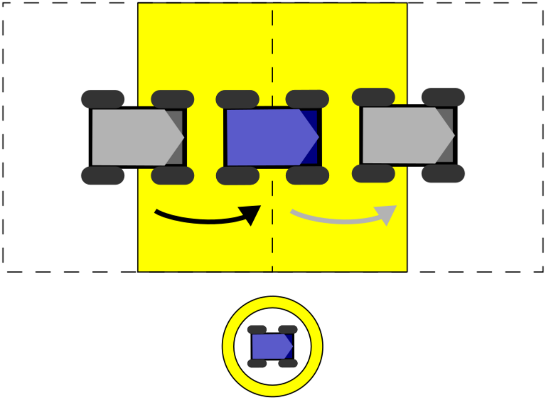

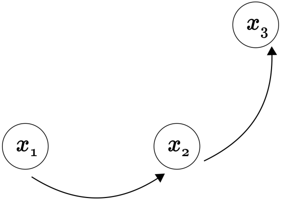

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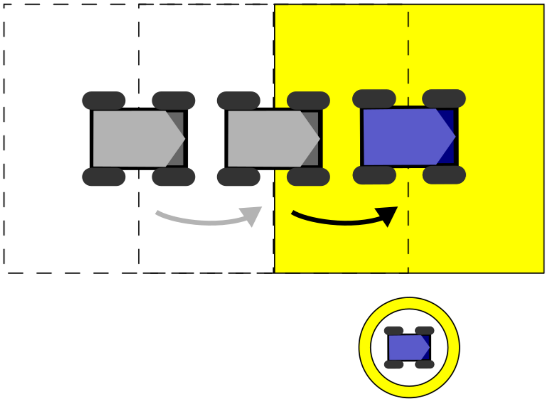

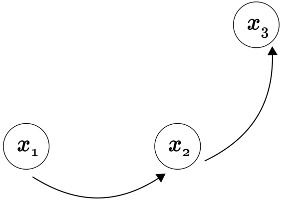

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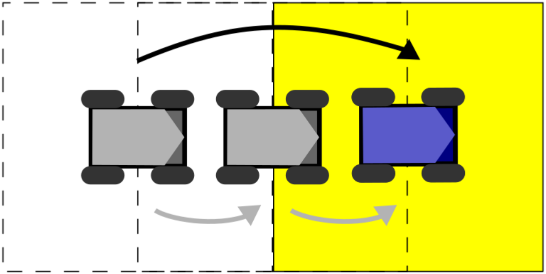

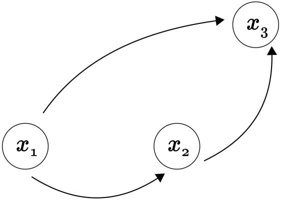

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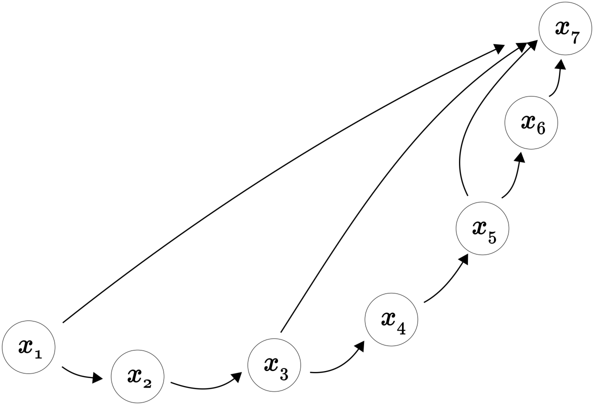
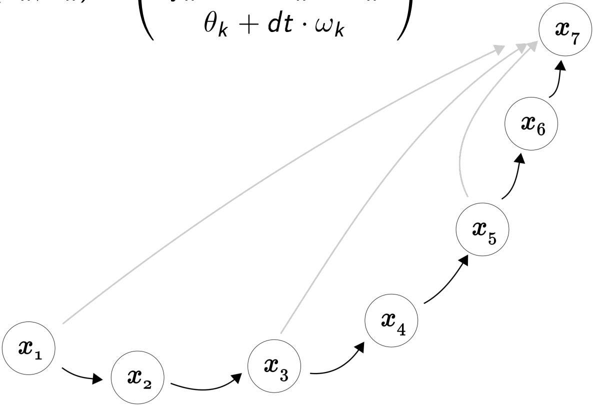


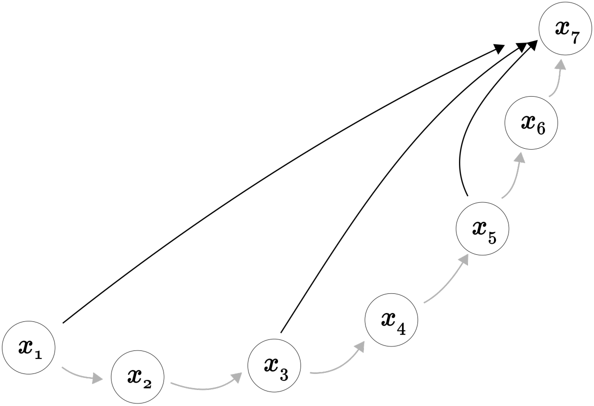
Constraints Network

$$\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_1}(\boldsymbol{x}_k, \boldsymbol{u}_k) = \begin{pmatrix} x_k + dt \cdot v_k \cos\theta_k \\ y_k + dt \cdot v_k \sin\theta_k \\ \theta_k + dt \cdot \omega_k \end{pmatrix}$$



Constraints Network

$$\mathbf{x}_{k+l} = \mathbf{f}_{G_2}(\mathbf{x}_k, \mathbf{u}_{k:k+l})$$



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$$\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_2}\left(\boldsymbol{x}_k, \boldsymbol{u}_{k:k+l}\right) =$$

$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ \omega_i \end{pmatrix}$$

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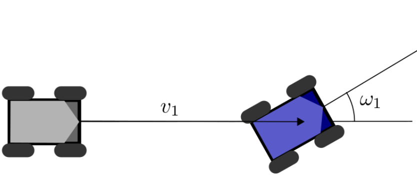

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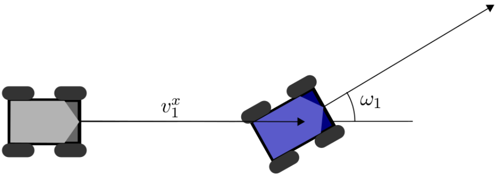
$$\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_2}(\boldsymbol{x}_k, \boldsymbol{u}_{k:k+l}) =$$

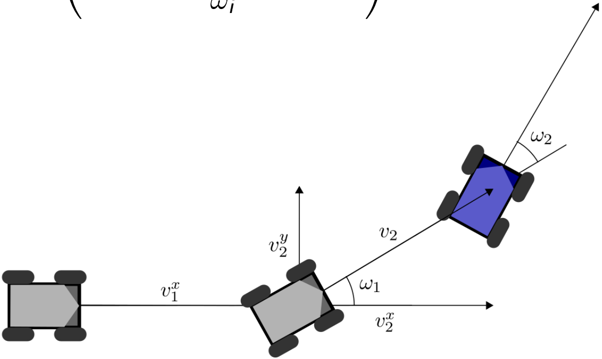
$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ \omega_i \end{pmatrix}$$



Constraints Network

$$\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_2}(\boldsymbol{x}_k, \boldsymbol{u}_{k:k+l}) =$$

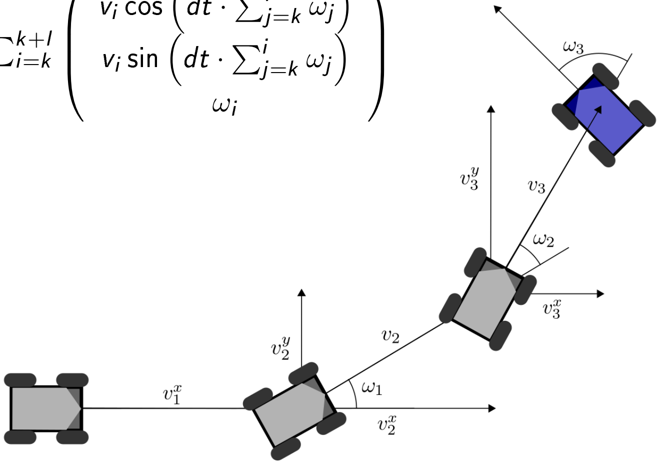
$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ \omega_i \end{pmatrix}$$



Constraints Network

$$\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_2}\left(\boldsymbol{x}_k, \boldsymbol{u}_{k:k+l}\right) =$$

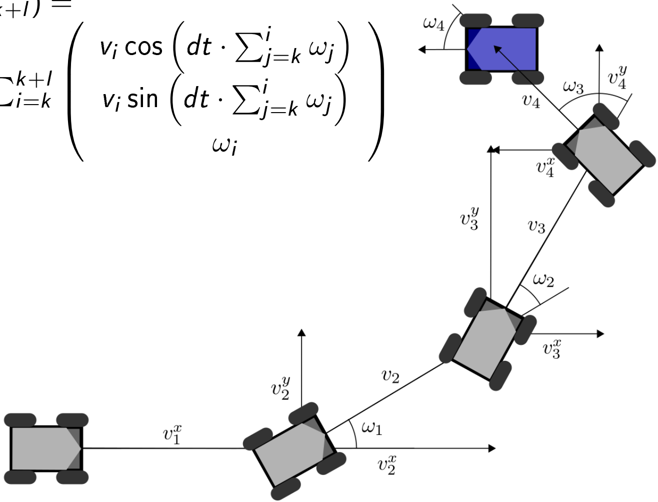
$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ \omega_i \end{pmatrix}$$



Constraints Network

$$\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_2}\left(\boldsymbol{x}_k, \boldsymbol{u}_{k:k+l}\right) =$$

$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^{i} \omega_j\right) \\ \omega_i \end{pmatrix}$$



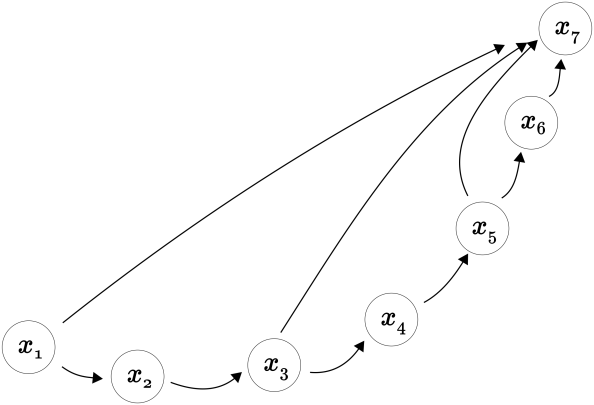
Constraints Network - Global Movement Model

$$A = \begin{pmatrix} \cos\theta_k & -\sin\theta_k & 0 \\ \sin\theta_k & \cos\theta_k & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

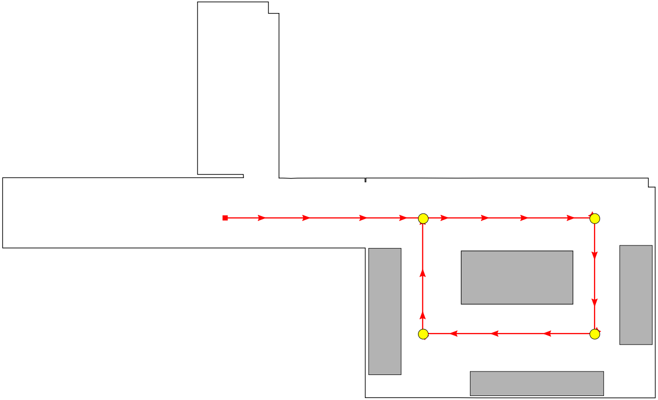
$$\mathbf{x}_{k+l} =$$

$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot \begin{pmatrix} \cos\theta_k & -\sin\theta_k & 0 \\ \sin\theta_k & \cos\theta_k & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \sum_{j=k}^{i} \omega_j\right) \\ v_i \sin\left(dt \sum_{j=k}^{i} \omega_j\right) \\ \omega_i \end{pmatrix}$$

Constraints Network



Experiments

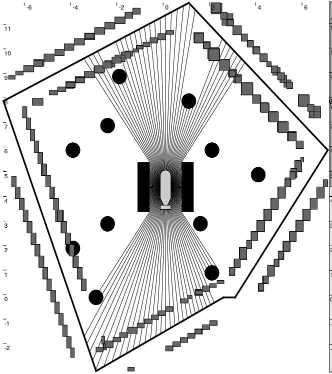


Experiments

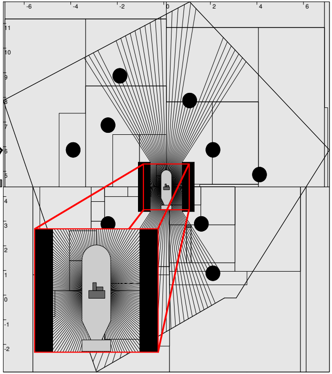
- Video

Conclusion

Experiments - Simulated Scenario



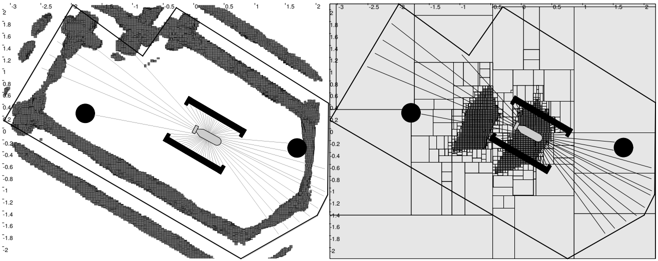
(a)



(b)

Figure: Static localization performed in the simulated scenario. (a) Segment approach and (b) Polygon-based approach. The pavings were computed with $\epsilon_\theta = 10°$ for the θ space and $\epsilon = 50$cm for coordinates x and y.

Experiments - Real Scenario



(a) (b)

Figure: Static localization performed in a real scenario. (a) Segment approach and (b) Polygon-based approach. The pavings were computed with $\epsilon_\theta = 10^\circ$ for the θ space and $\epsilon = 50$cm for coordinates x and y.