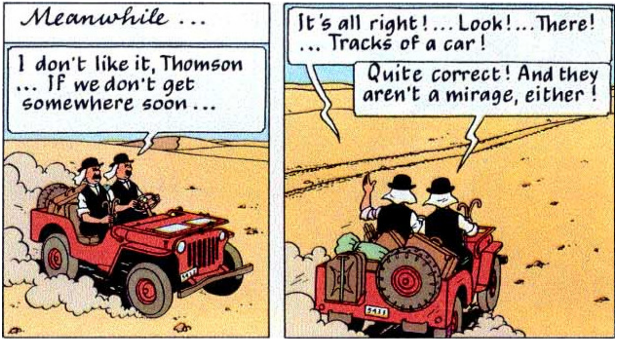
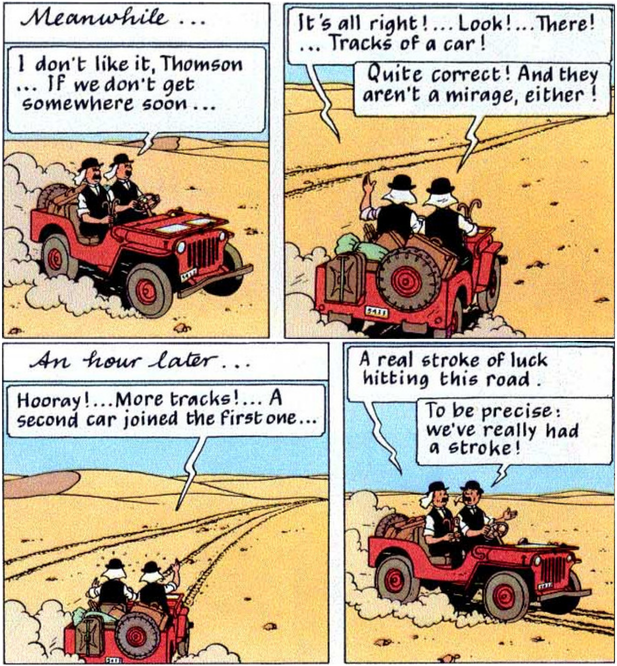


From *Tintin, Land of Black Gold.*



From *Tintin, Land of Black Gold.*

Reliable underwater SLAM using seabed roughness

Simon Rohou, Michel Legris

15^{th} December 2023, Paris

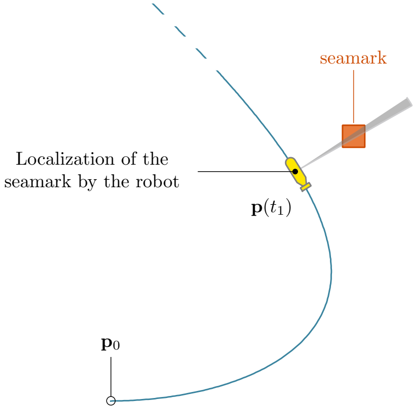
Section 2

Motivations

Motivations

Simultaneous Localization and Mapping

- **come back** to a previous pose and **recognize** the environment
- problem: loop closure detection



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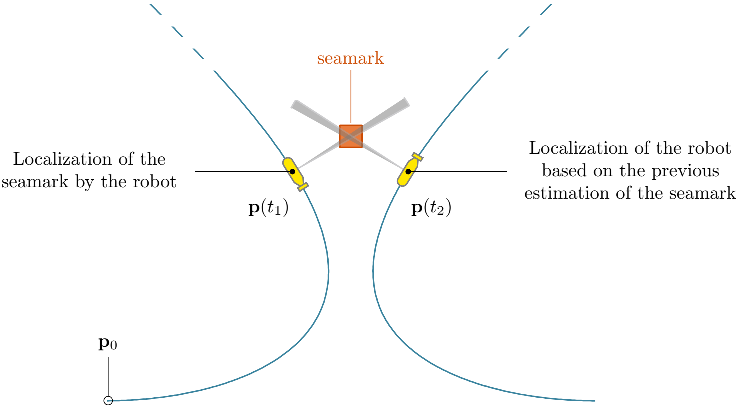


Motivations

Simultaneous Localization and Mapping

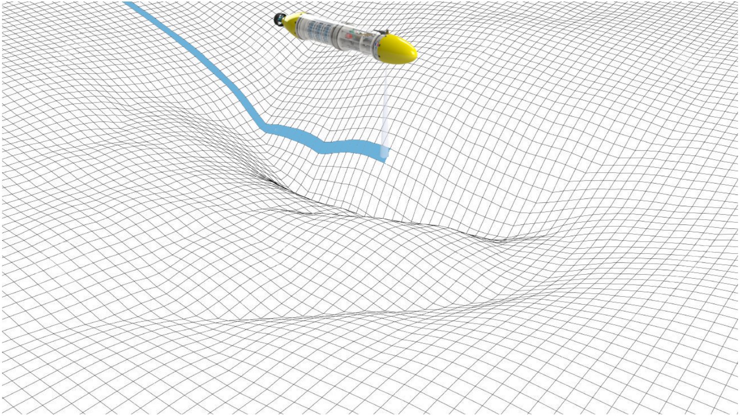
The problem of **false loop detections** in similar environments.

In Versailles' gardens: similar places. *Did we really come back to a previous place?*

Motivations

Underwater robot localization

An underwater robot performing a loop during an exploration:



Robot's trajectory is projected in blue on the seabed.

Motivations

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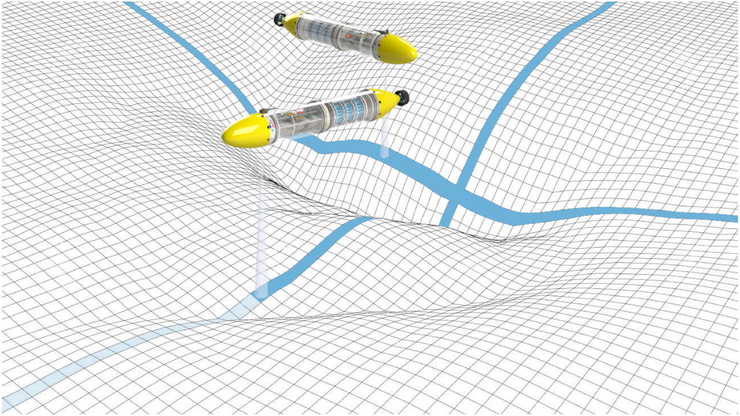
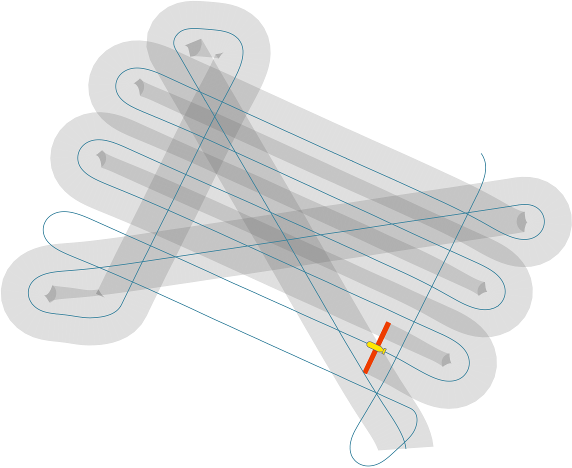


Robot's trajectory is projected in blue on the seabed.

Motivations

Underwater robot localization

Typical path involving numerous loops:

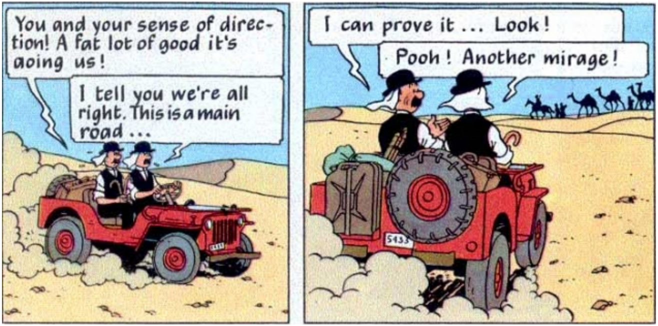


Underwater robot performing a survey with a multibeam sonar.

Motivations

Reliability of loops

Can we prove that we revisited the same place only from observations?

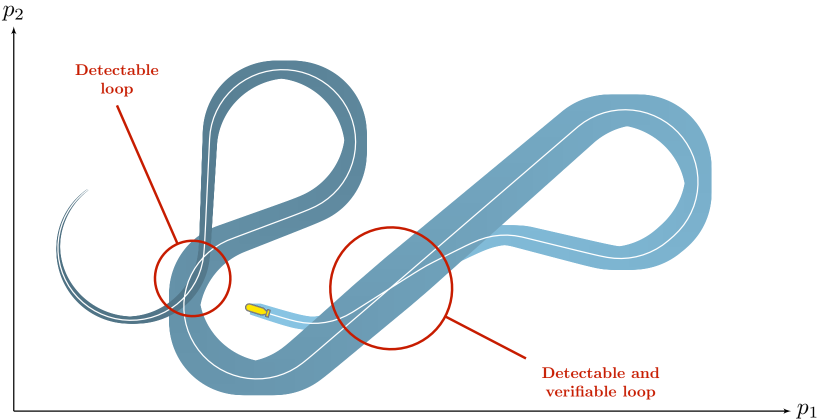


..they are wrong!

Motivations

Uncertainties: detection vs verification

Uncertain trajectories enclosed by **tubes**.



Only one loop can be verified – at least two feasible loops are detected

Section 3

Looped trajectories

Looped trajectories

Definitions (Aubry, 2013)

- robot position: $\mathbf{p} = (x, y)^\intercal \in \mathbb{R}^2$
- 2D robot trajectory: $\mathbf{p}(t) : \mathbb{R} \to \mathbb{R}^2$, $t \in [t_0, t_f]$
- looped trajectory $\Leftrightarrow$ trajectory that crosses itself
  - $\mathbf{p}(t_1) = \mathbf{p}(t_2)$, $t_1 \neq t_2$
  - 1 loop $\Leftrightarrow$ 1 t-pair (t_1, t_2)

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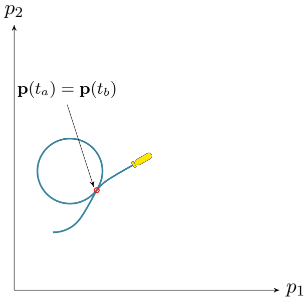

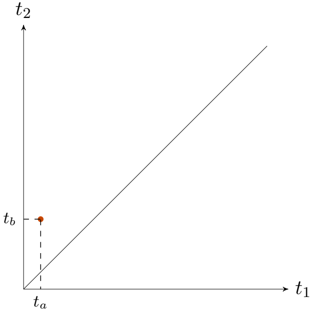

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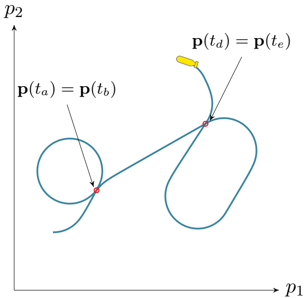

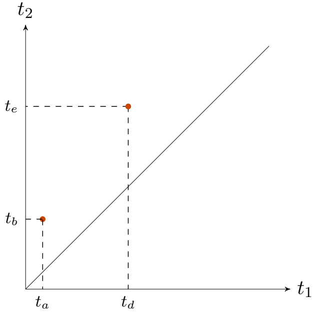

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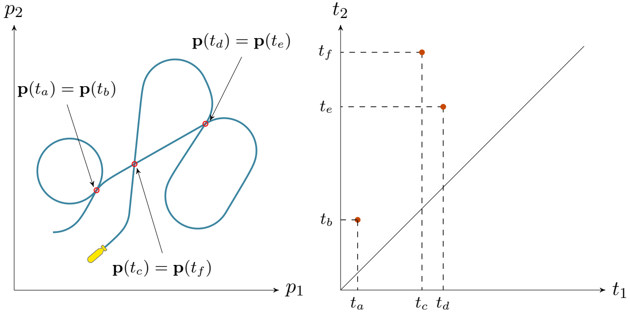
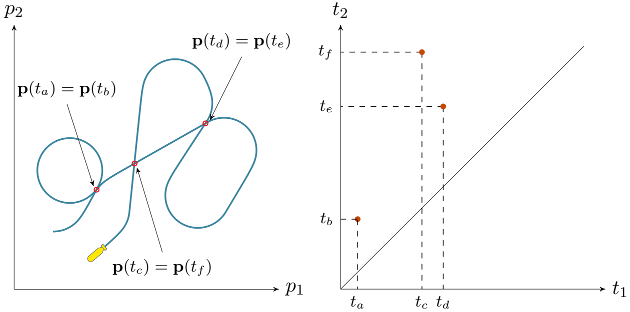


Looped trajectories

Definitions (Aubry, 2013)

- t-plane $\Leftrightarrow$ all feasible t-pairs $= [t_0, t_f]^2$
- *loop set* $\mathbb{T}^*$:
  - $\mathbb{T}^* = \left\{(t_1, t_2) \in [t_0, t_f]^2 \mid \mathbf{p}(t_1) = \mathbf{p}(t_2), t_1 < t_2\right\}$
- *loop set* of below example:
  - $\mathbb{T}^* = \{(t_a, t_b), (t_c, t_f), (t_d, t_e)\}$



Looped trajectories

Computing loops from robot sensors

Context: robot trajectory $\mathbf{p}(t)$ cannot be directly sensed.
Computation from speed measurements:

$$\mathbf{p}(t) = \int_{t_0}^{t} \mathbf{v}(\tau)d\tau + \mathbf{p}_0, \quad (1)$$

with $\mathbf{v}(t) \in \mathbb{R}^2$: robot velocity vector at time $t \in [t_0, t_f]$.

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Loop-set from velocity:

$$\mathbb{T}^* = \left\{(t_1, t_2) \in [t_0, t_f]^2 \mid \mathbf{p}(t_1) = \mathbf{p}(t_2), t_1 < t_2\right\} \quad (2)$$

$$= \left\{(t_1, t_2) \in [t_0, t_f]^2 \mid \int_{t_1}^{t_2} \mathbf{v}(\tau)d\tau = \mathbf{0}, t_1 < t_2\right\} \quad (3)$$

Section 4

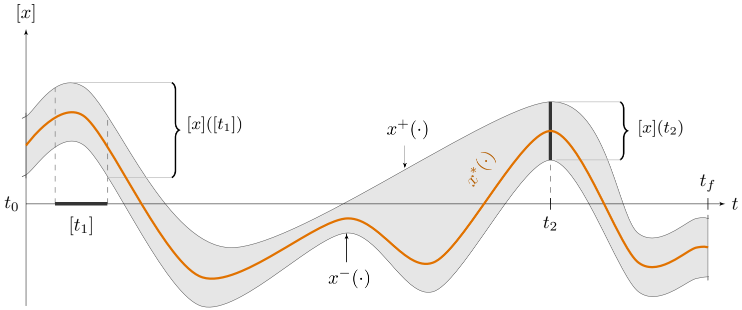
Loop detection

Loop detection

Bounded-error context

Tubes: sets of trajectories

$[x](\cdot)$, interval of trajectories $[x^-(\cdot), x^+(\cdot)]$
such that $\forall t \in \mathbb{R},\ x^-(t) \leqslant x^+(t)$



Tube $[x](\cdot)$ enclosing an uncertain trajectory $x^*(\cdot)$

Loop detection

Bounded-error context

Actual loop-set $\mathbb{T}^*$ (error free):

$$\mathbb{T}^* = \left\{ (t_1, t_2) \mid \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau = \mathbf{0} \right\} \tag{4}$$

Loop detection

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Bounded-error context, assuming $\mathbf{v}^*(\cdot) \in [\mathbf{v}](\cdot)$:

$$\mathbb{T} = \left\{ (t_1, t_2) \mid \exists \mathbf{v}(\cdot) \in [\mathbf{v}](\cdot) \ , \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0} \right\} \tag{5}$$

Loop detection

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Set-membership approach:

$$\mathbb{T}^* \subset \mathbb{T} \subset [t_0, t_f]^2 \tag{6}$$

Loop detection

Inclusion function

Simplification:
defining the actual but unknown function $\mathbf{f}^* : \mathbb{R}^2 \to \mathbb{R}^2$

$$\mathbf{f}^*(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau \qquad (7)$$

Loop detection

Inclusion function

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$$\mathbf{f}^*(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau \tag{7}$$

Assessed knowledge:
$[\mathbf{f}] : \mathbb{R}^2 \to \mathbb{IR}^2$ is an *interval function* of $\mathbf{f}^*$:

$$\mathbf{f}^*(t_1, t_2) \in [\mathbf{f}](t_1, t_2) = \int_{t_1}^{t_2} [\mathbf{v}](\tau) d\tau \tag{8}$$

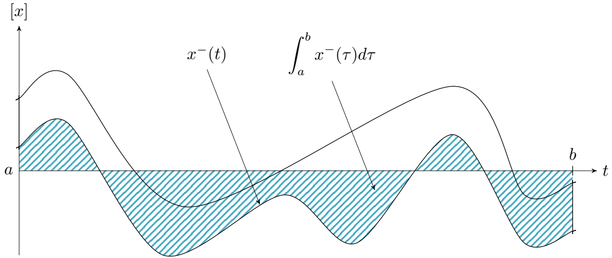
Loop detection

Integral of tubes

Definition: the integral of a tube $[x](\cdot) = [x^-, x^+]$ is an interval:

$$\int_a^b [x](\tau)d\tau = \left\{ \int_a^b x(\tau)d\tau \mid x(\cdot) \in [x](\cdot) \right\} = \left[\int_a^b x^-(\tau)d\tau, \int_a^b x^+(\tau)d\tau \right]$$

[Aubry2013]



blue area: lower bound of the tube's integral

Loop detection

Integral of tubes

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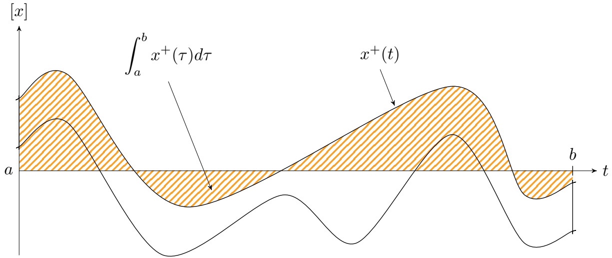
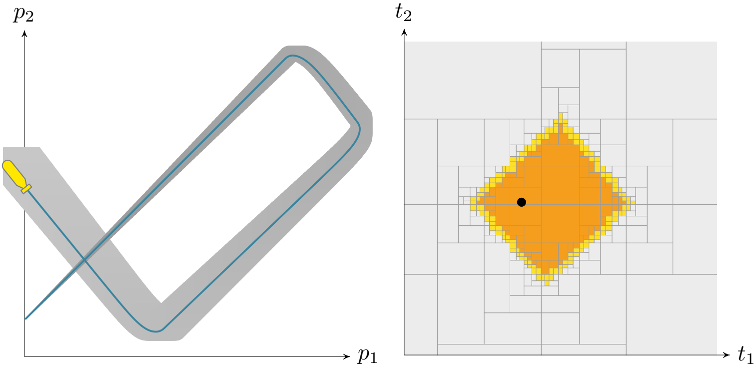
[Aubry2013]



orange area: upper bound of the tube's integral

Loop detection

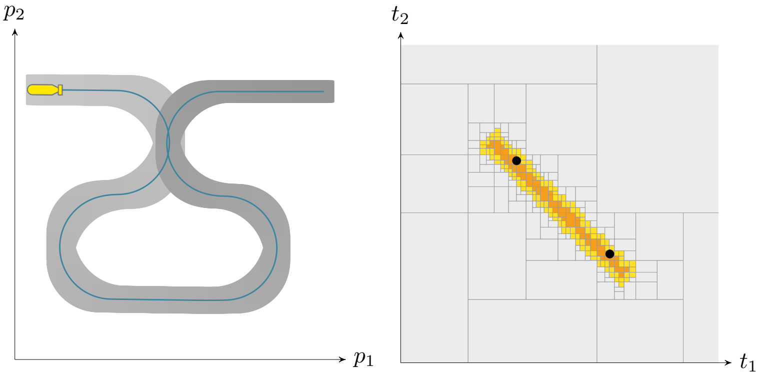
Reliable approximation of a loop set



Undeniable looped trajectory

Loop detection

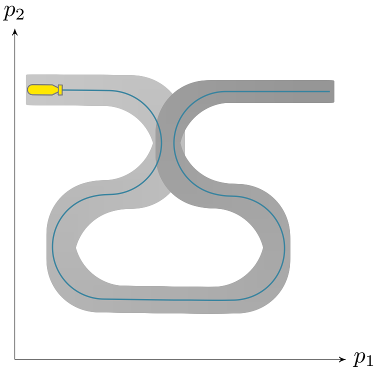
Reliable approximation of a loop set

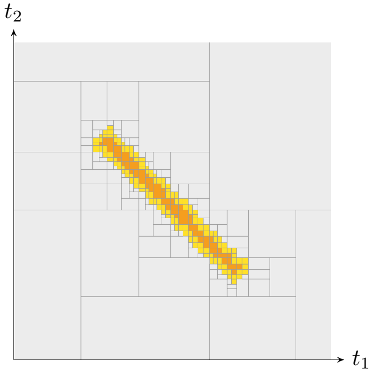


Doubtful looped trajectory

Loop detection

Reliable approximation of a loop set

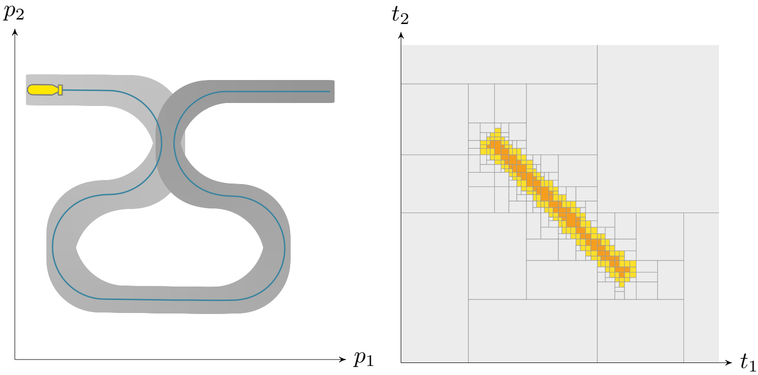




Doubtful looped trajectory

Loop detection

Reliable approximation of a loop set

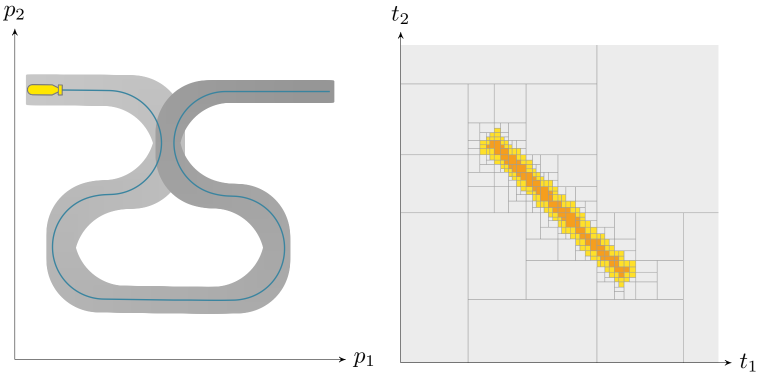


Doubtful looped trajectory

$$\forall \mathbf{f} \in [\mathbf{f}], \exists \mathbf{t} \in \mathbb{T}_i \mid \mathbf{f}(\mathbf{t}) = \mathbf{0} \implies \underbrace{\exists \mathbf{t} \in \mathbb{T}_i \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0}}_{\text{loop existence proof}} \quad (9)$$

Loop detection

Reliable approximation of a loop set



Doubtful looped trajectory

Proving the existence of loops in robot trajectories
Simon Rohou, Peter Franek, Clément Aubry, Luc Jaulin
The International Journal of Robotics Research, 2018

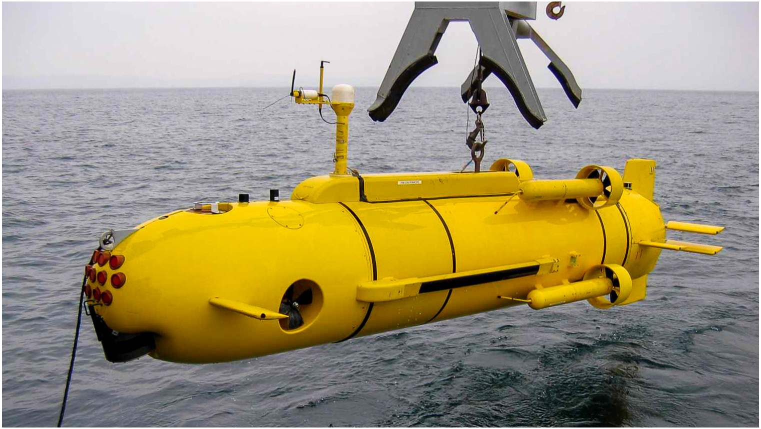
Section 5

Application (loops detections/proofs)

Application (loops detections/proofs)

Redermor mission

2 hours experimental mission in Brittany (France)

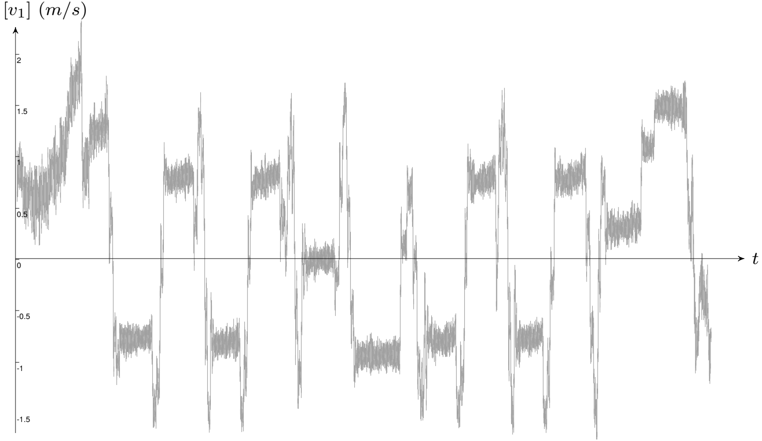


The *Redermor* Autonomous Underwater Vehicle (AUV)

Application (loops detections/proofs)

Redermor mission

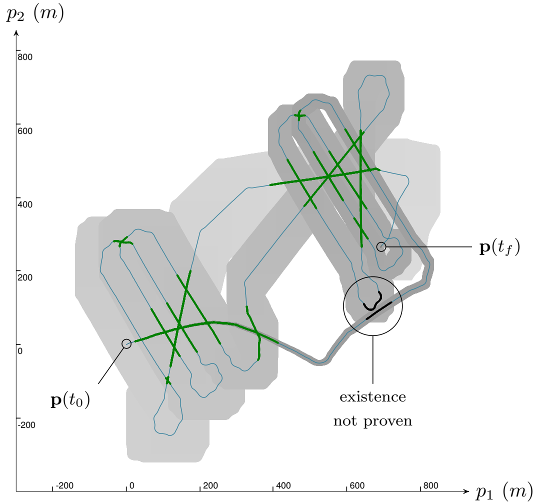
Tube of proprioceptive measurements $[\mathbf{v}](\cdot)$:



East speed velocity tube $[v_1](\cdot)$

Application (loops detections/proofs)

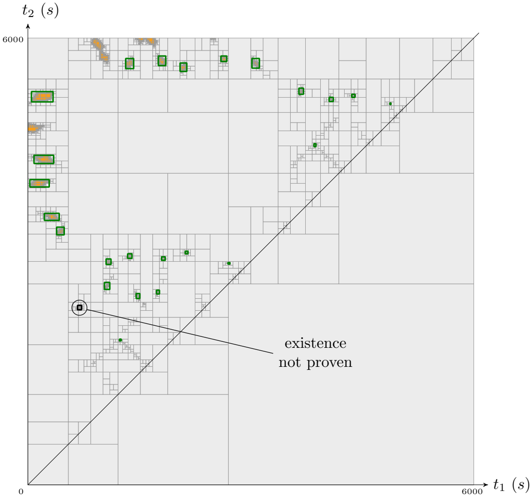
Guaranteed computation of the trajectory



2d trace of *Redermor* AUV

Application (loops detections/proofs)

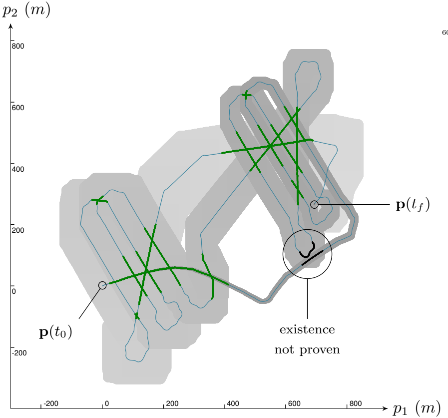
t-plane of the mission: $\mathbb{T} = \{(t_1, t_2) \mid \mathbf{0} \in [\mathbf{f}](t_1, t_2), t_1 < t_2\}$

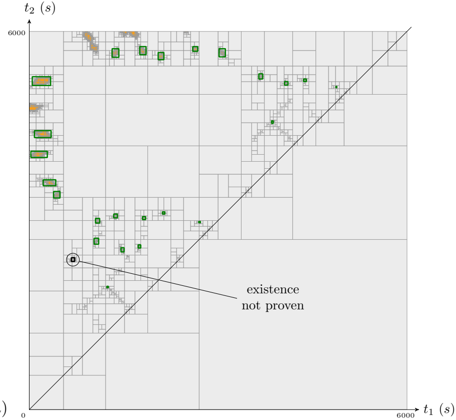


t-plane corresponding to *Redermor*'s mission

Application (loops detections/proofs)

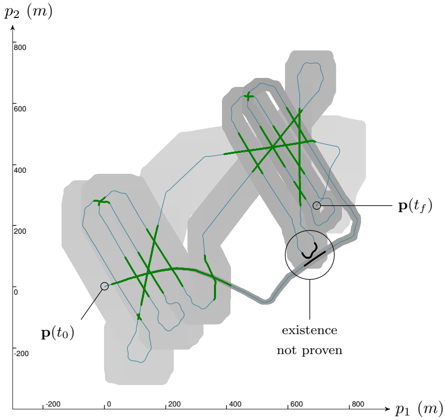
Overview and results

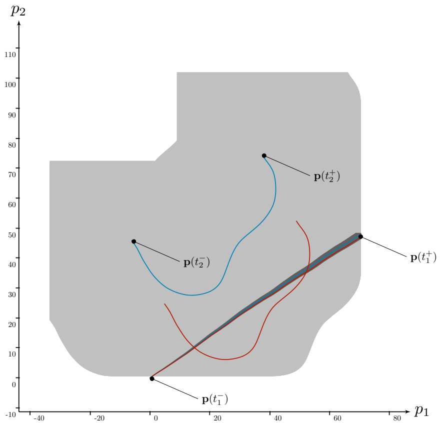




Application (loops detections/proofs)

Overview and results





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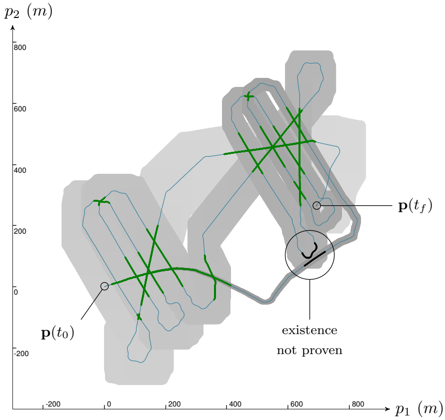


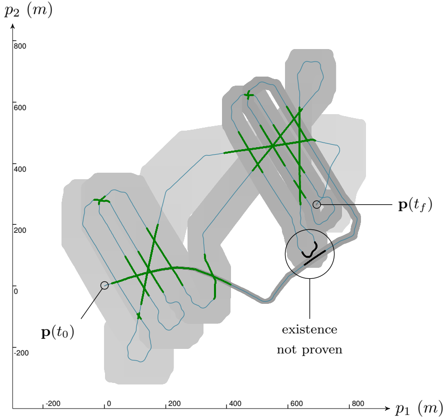
Loop proof number

Without uncertainties:

$$\lambda^* = \#\left\{\mathbf{t} \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0}, t_1 < t_2\right\}$$

Application (loops detections/proofs)

Overview and results



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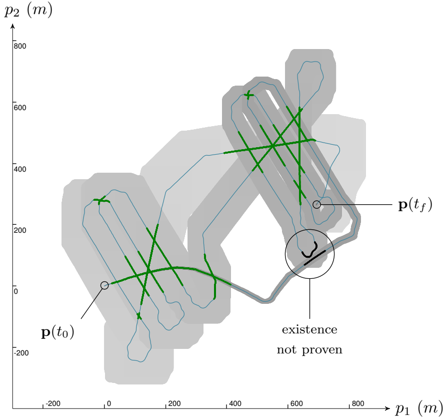
Results:

Newton operator test: $\lambda_{\mathcal{N}} = 14$

Application (loops detections/proofs)

Overview and results



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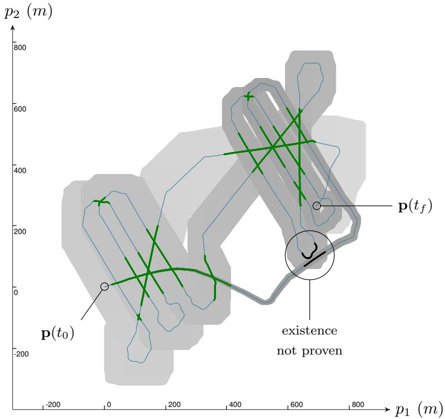
Results:

Newton operator test: $\lambda_{\mathcal{N}} = 14$

Topological degree test: $\lambda_{\mathcal{T}} = 24$

Application (loops detections/proofs)

Overview and results



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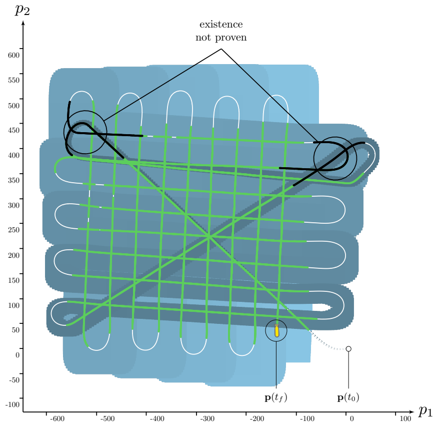
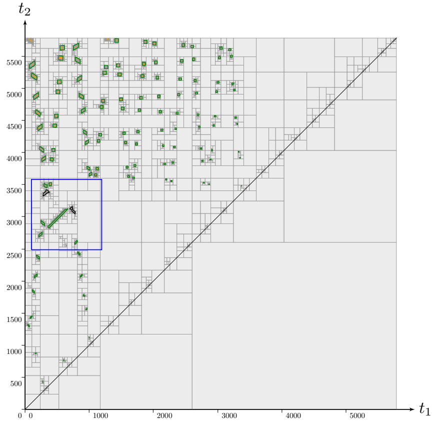
Topological degree test: $\lambda_{\mathcal{T}} = 24$

Truth: $\lambda^* = 24$

Application (loops detections/proofs)

Another experiment (*Daurade* AUV)





Application (loops detections/proofs)

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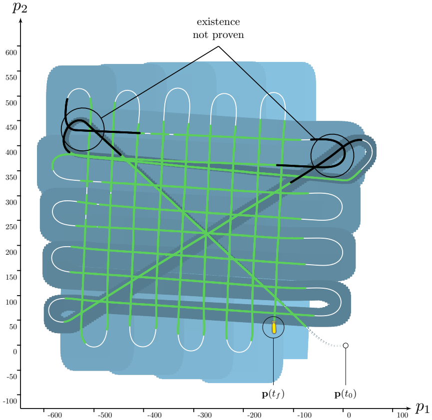
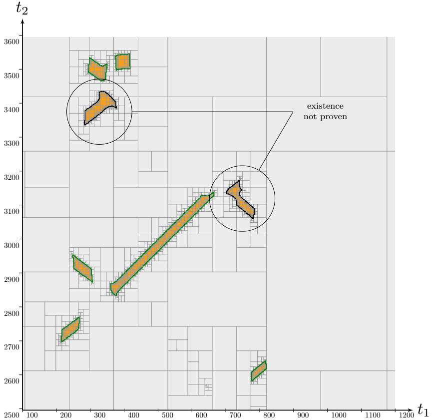
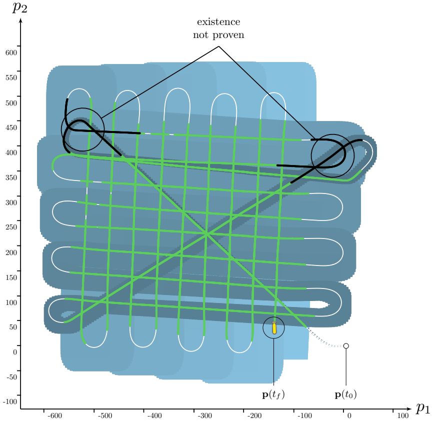
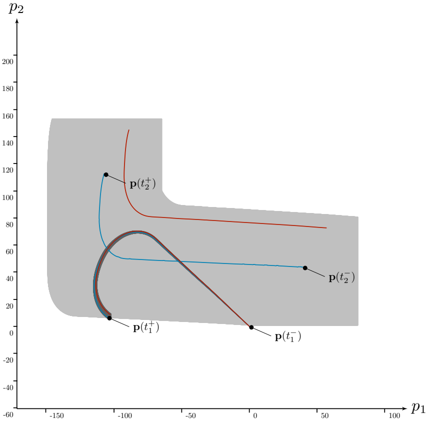




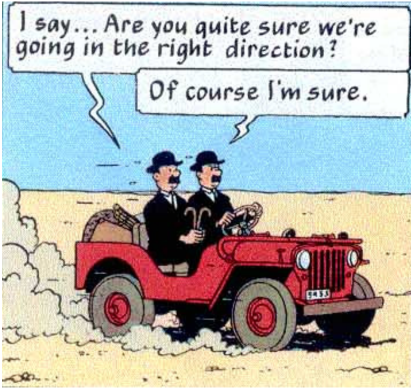
Application (loops detections/proofs)

Another experiment (*Daurade* AUV)





Now that loops are proved with proprioceptive measurements...



...it remains to perform localization by adding environment perceptions.

Section 8

Towards SLAM

Towards SLAM

Experiment (3rd December, 2014)

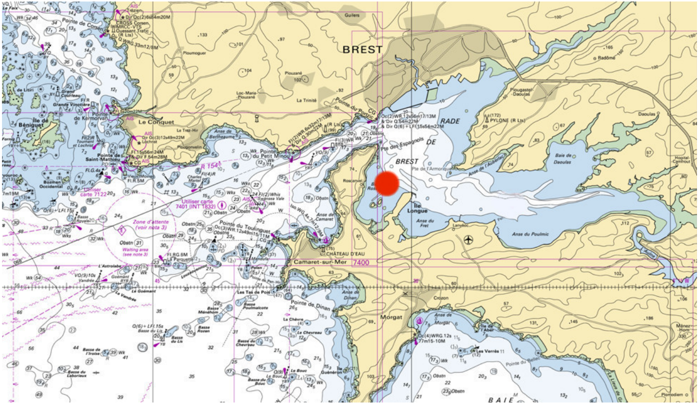
- Daurade: Autonomous Underwater Vehicle (AUV)
- weight: 1010kg – length: 5m – max depth: 300m

Special thanks to DGA-TN Brest (formerly GESMA)

Towards SLAM

Experiment (3$^{\text{rd}}$ December, 2014)

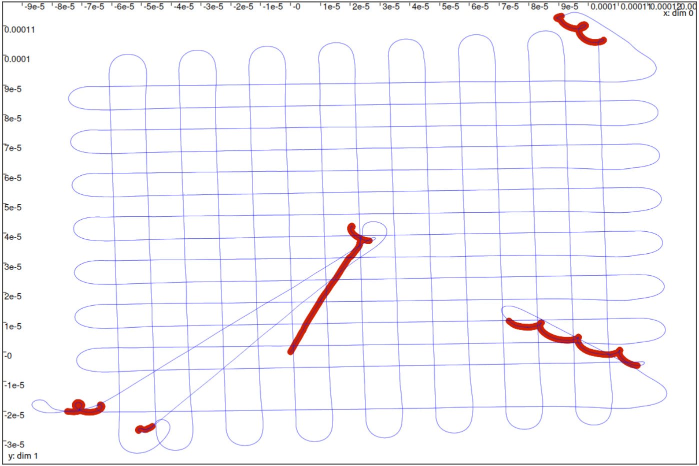
- 3.5 hours experimental mission
- *Rade de Brest*, Brittany



Location: *Baie de Roscanvel* – Credits: Shom

Towards SLAM

Experiment (Boustrophédon)

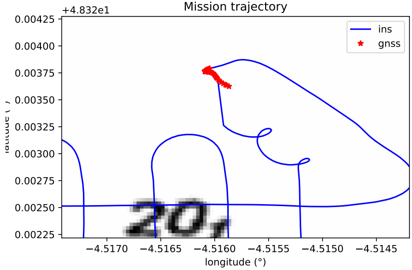


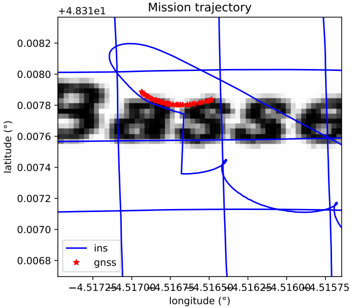
Multi-boustrophedon pattern
(actual trajectory followed by Daurade)

Towards SLAM

Actual positioning drift (40m)

Current Kalman algorithms, from the constructor, are able to filter the trajectory with GNSS fixes (when surfacing).
→ up to 40m of positioning drift after ∼1h of deadreckoning





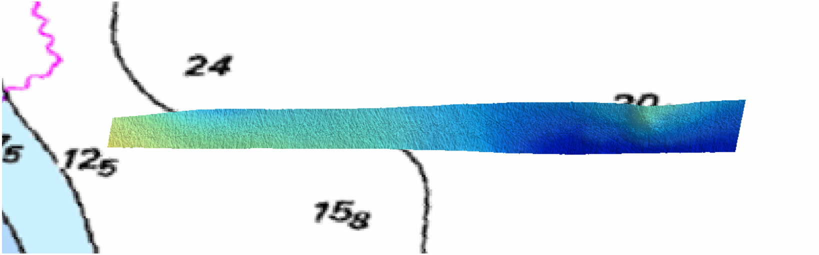
Blue: actual trajectory corrected in forward filtering.
Red: GNSS positions.

Towards SLAM

SLAM approach: observations of the seabed

Use of a Multi-Beam Echo-Sounder (MBES)

- accurate sensing of the seabed
- main usage: building maps (Digital Elevation Models)

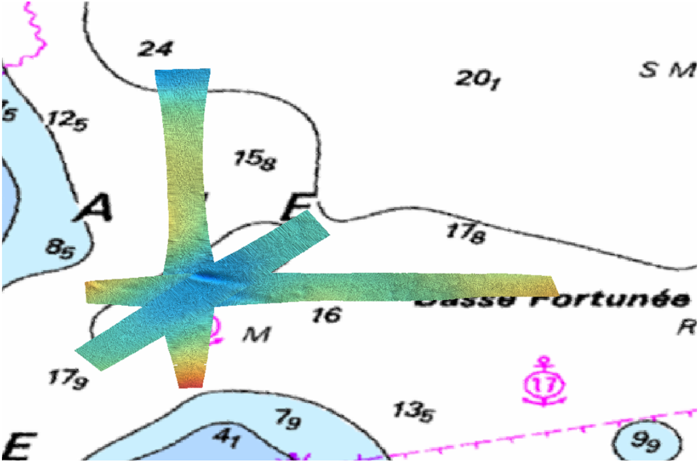


Example of one acquisition track, during the Daurade mission.

Towards SLAM

SLAM approach: observations of the seabed

Goal: use bathymetric information for localization purposes

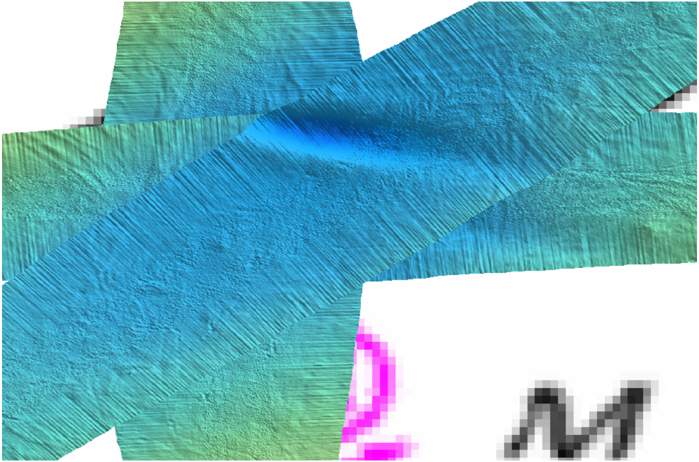


Crossed acquisition tracks (looped trajectories), in case of positioning drift.

Towards SLAM

SLAM approach: observations of the seabed

Goal: use bathymetric information for localization purposes



Crossed acquisition tracks (looped trajectories), in case of positioning drift.

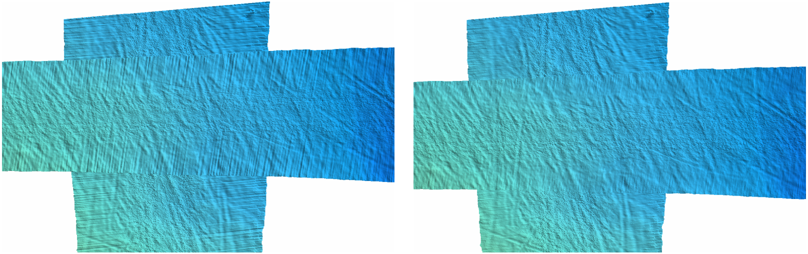
Towards SLAM

SLAM on seabed roughness (micro-relief)

Which information?

- low frequencies: sensitive to tide effects, or flat environments
- high frequencies: artefacts related to electronic noise (borders of the tracks)

→ **approach based on seafloor roughness**, such as ripples, grooves, small rocks...



Left: no correction. Right: the two racks match.

Towards SLAM

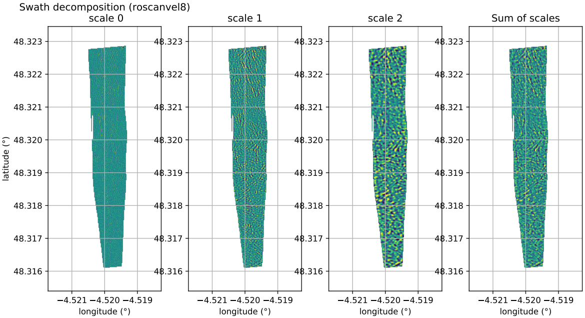
Bathymetric decomposition

- separate resolution scales
- keep only those that are reliable and descriptive

$\rightarrow$ **use of multilevel B-Splines**

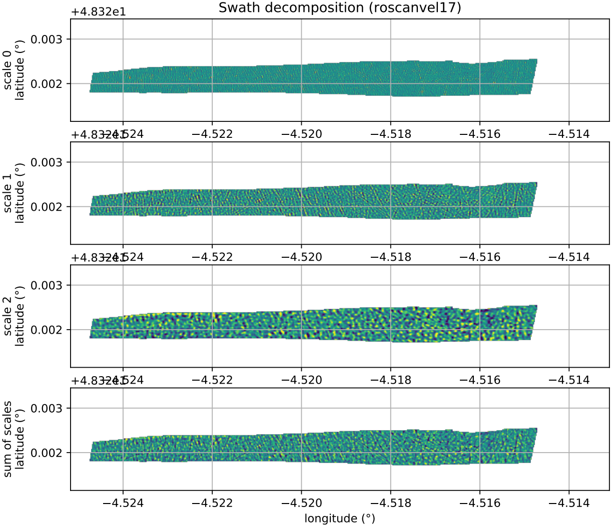
Seungyong Lee, George Wolberg, and Sung Yong Shin,
IEEE Transactions On Vizualisation And Computer Graphics, Vol. 3, No. 3, July-Sept 1997

Application to bathymetry:



Towards SLAM

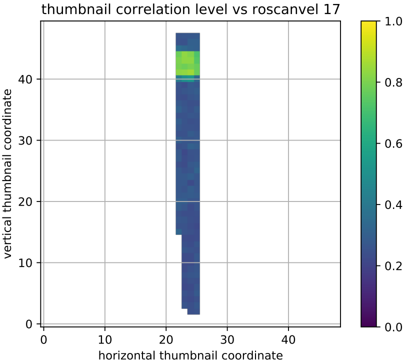
Multilevel B-Splines on bathymetric data

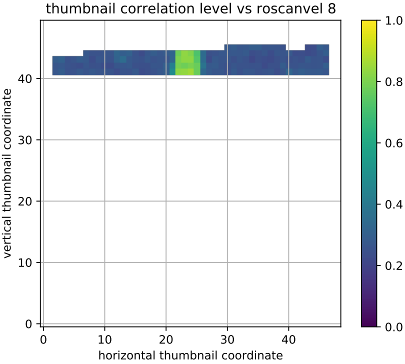


Towards SLAM

Global correlation between two racks (ex: n17 and n8)

Each rack is broken down into cells of 64×64 pixels (32m by 32m)

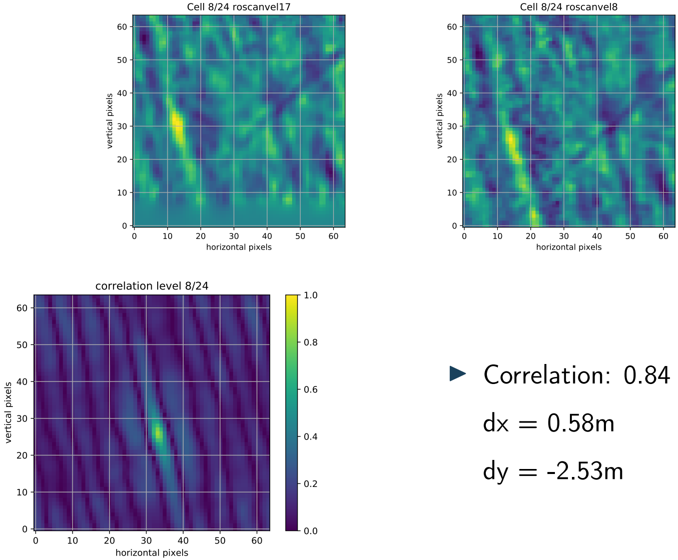




These figures are obtained by correlation of the cells.

Towards SLAM

Correlation of a couple of cells of two racks



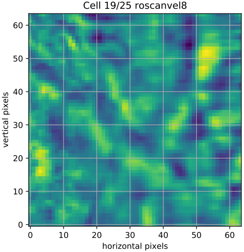
- Correlation: 0.84

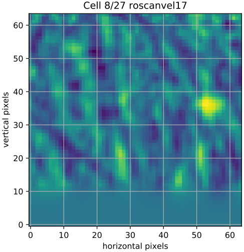
  dx = 0.58m

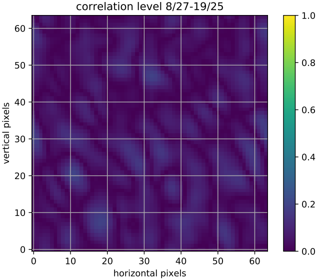
  dy = -2.53m

Towards SLAM

Non-correlation of a couple of cells of two racks



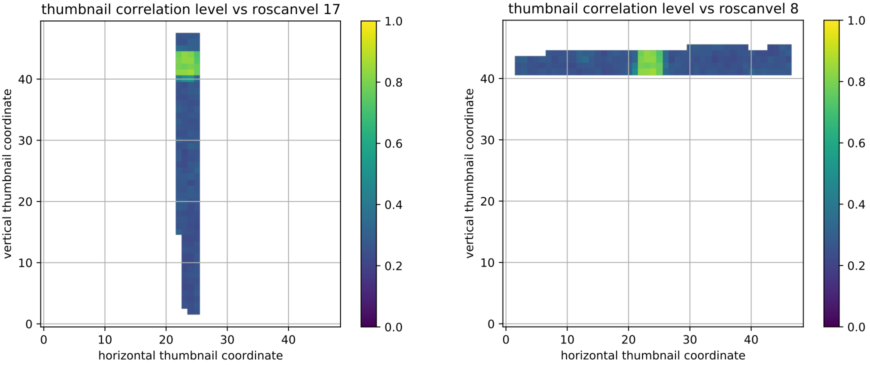




- Correlation: 0.17

Towards SLAM

Global correlation between two racks (ex: n17 and n8)



These figures are obtained by correlation of the cells.

Towards SLAM

Formalism

Inter-temporal measurement in SLAM:

$$\mathbf{x}(t_1) + \mathbf{d}_{1,2} = \mathbf{x}(t_2)$$

$$\dot{\mathbf{x}}(t) = \mathbf{v}(t)$$

Set-membership approach:

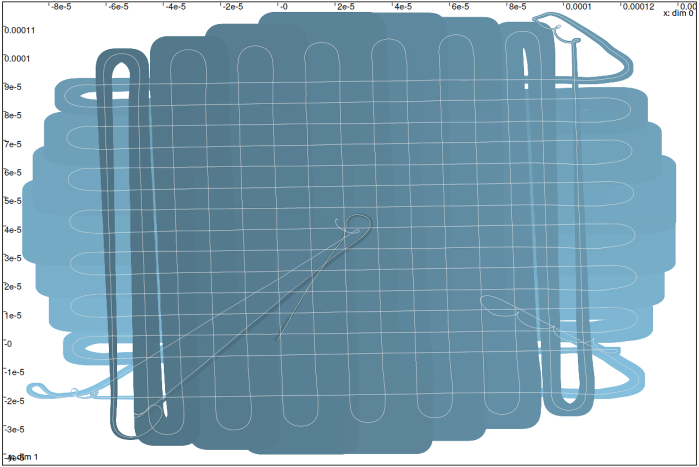
$$\mathbf{x}(\cdot) \in [\mathbf{x}](\cdot), \mathbf{v}(\cdot) \in [\mathbf{v}](\cdot)$$

$$t_1 \in [t_1], t_2 \in [t_2]$$

$$\mathbf{d}_{1,2} \in [\mathbf{d}_{1,2}]$$

Towards SLAM

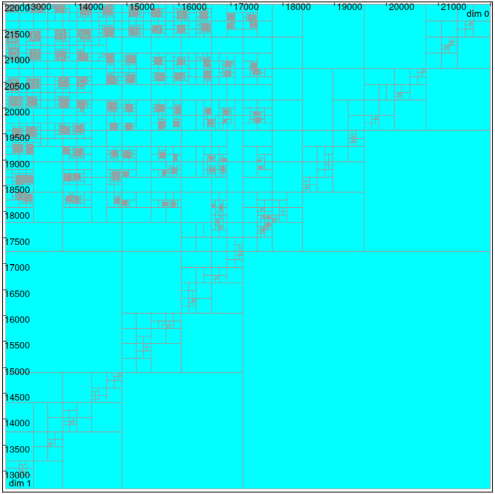
Deadreckoning



Initial tube before SLAM (inertial/DVL odometry + GNSS fixes).

Towards SLAM

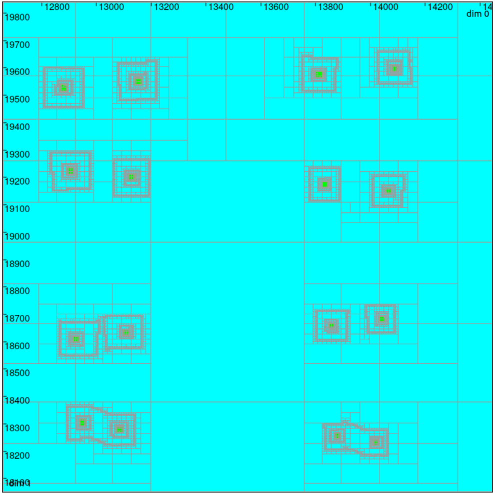
t-plane of the mission (loops summary)



t-plane $[0, t_{\max}]^2$

Towards SLAM

t-plane of the mission (loops summary)



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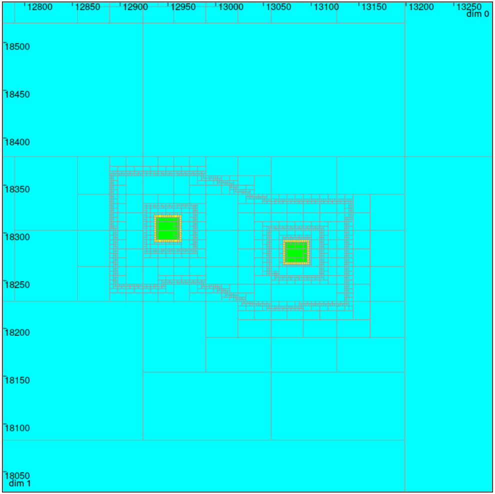
Towards SLAM

t-plane of the mission (loops summary)



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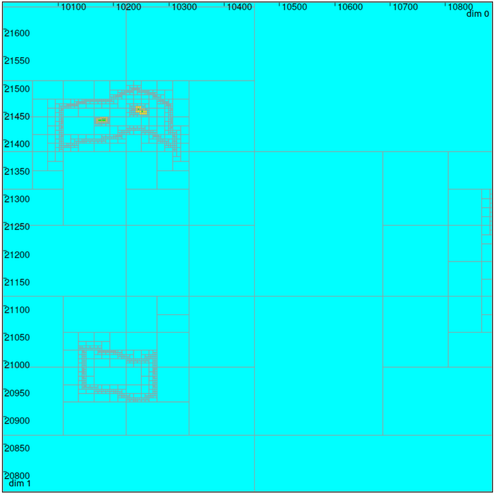

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Towards SLAM

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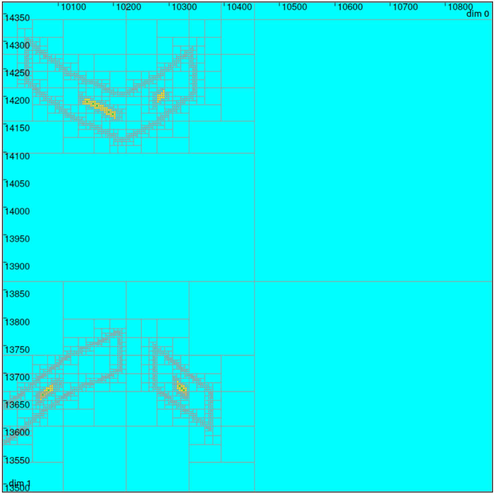

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Towards SLAM

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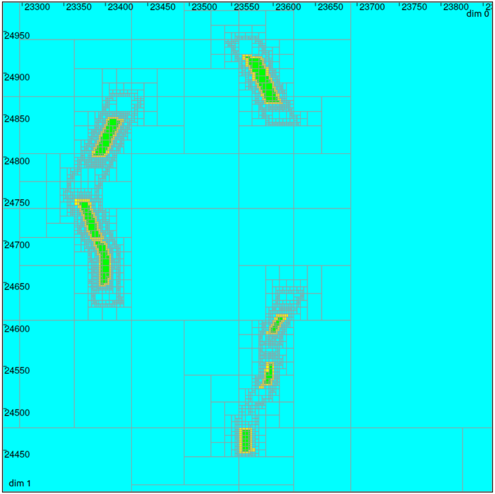
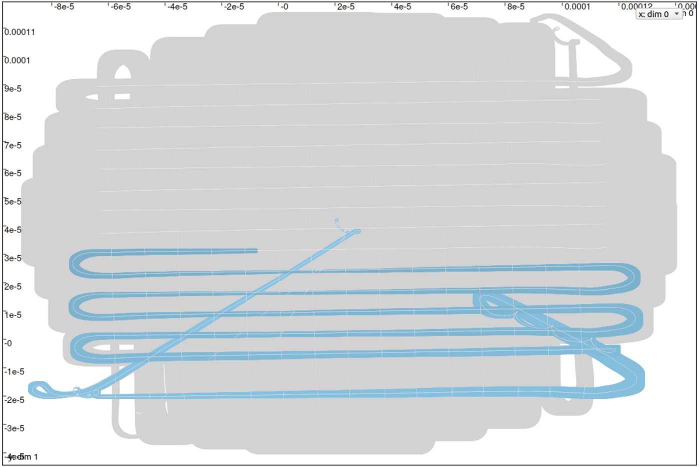


t-plane $[0, t_{\max}]^2$

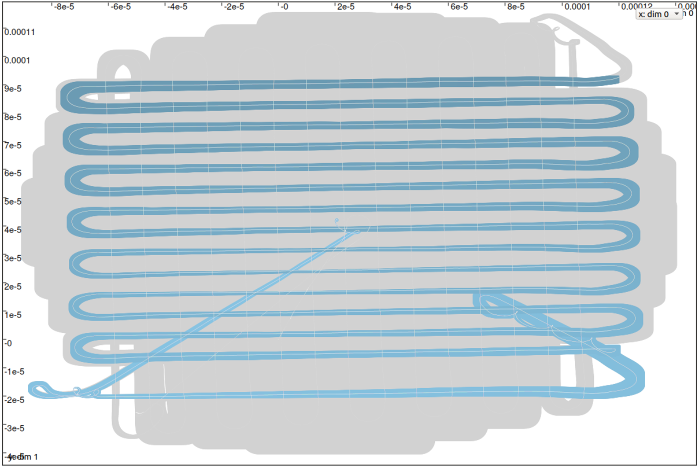
Towards SLAM

SLAM results



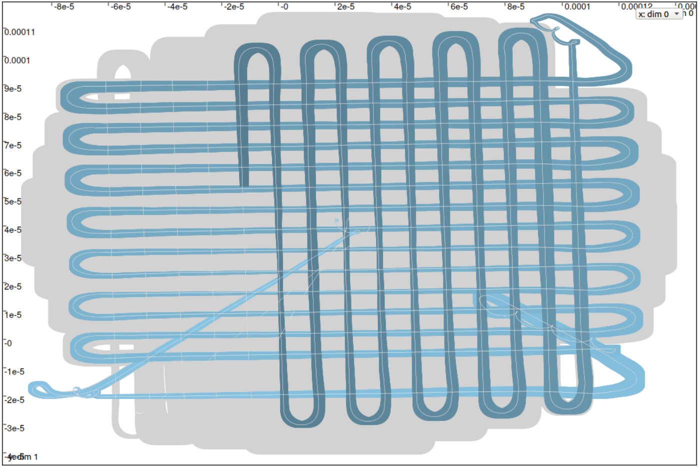
Towards SLAM

SLAM results



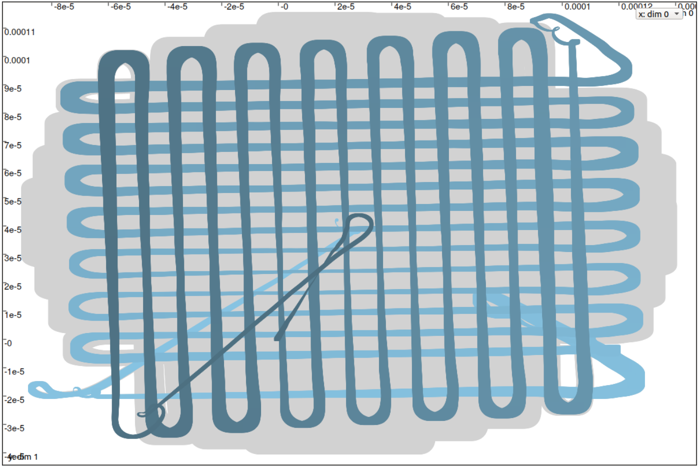
Towards SLAM

SLAM results



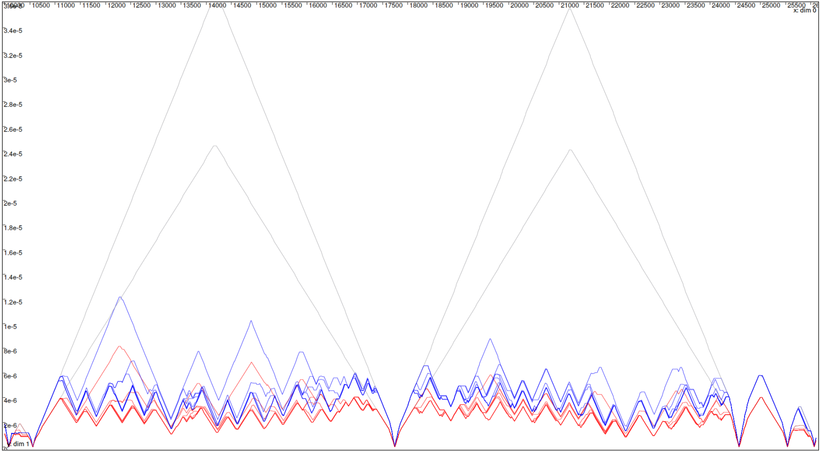
Towards SLAM

SLAM results



Towards SLAM

SLAM results



Tubes thickness along time: $\mathrm{width}(\mathbf{x}(t))$.
In gray: tubes without SLAM (deadreckoning only). In blue/red: with SLAM in x/y.

Towards SLAM

Conclusion

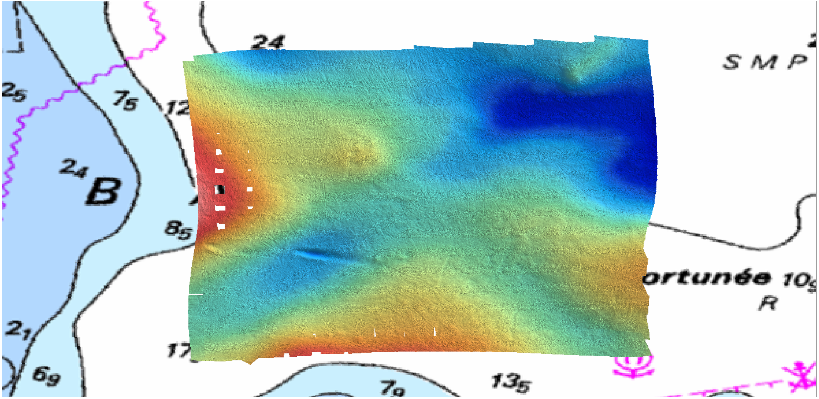
Assets of this approach:

- reliable localization, whatever the roughness/similarities of the seabed
- localization method efficient even over flat seabeds

Remaining points to investigate:

- need to automatically "clean" the MBES data before the correlation step
- online SLAM?

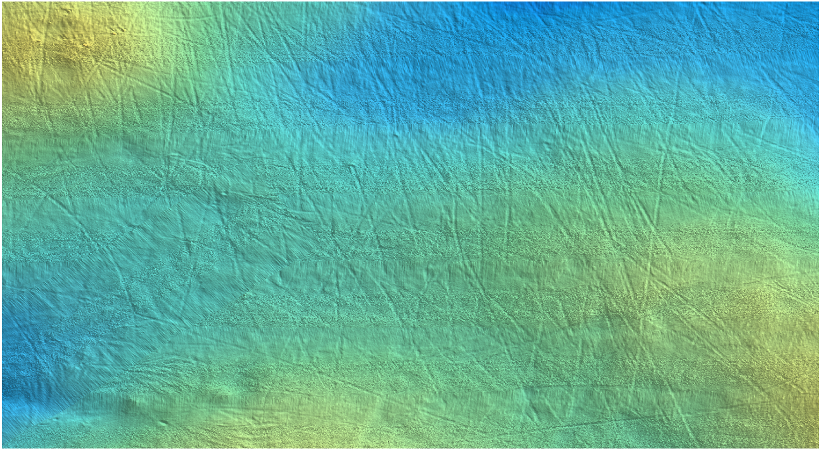
Towards SLAM
Conclusion



todo: Build the final DEM of the Roscanvel Baie.

Towards SLAM

Conclusion



todo: Build the final DEM of the Roscanvel Baie.

Section 9

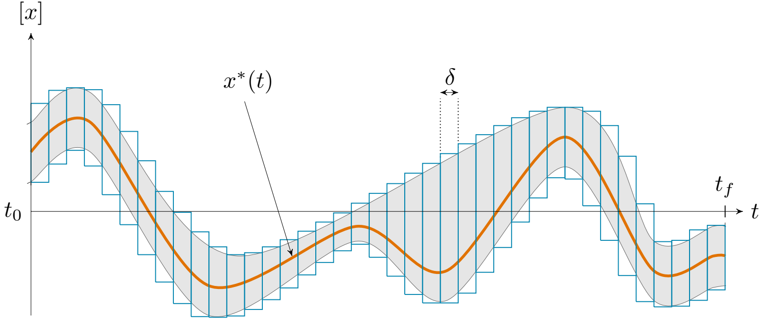
Appendix

Appendix

Tubes: computer representation

Implementation **enclosing** $[x^-(\cdot), x^+(\cdot)]$ inside an interval of step functions $[\underline{x^-}(\cdot), \overline{x^+}(\cdot)]$ such that:

$$\forall t \in \mathbb{R},\ \underline{x^-}(t) \leqslant x^-(t) \leqslant x^+(t) \leqslant \overline{x^+}(t)$$



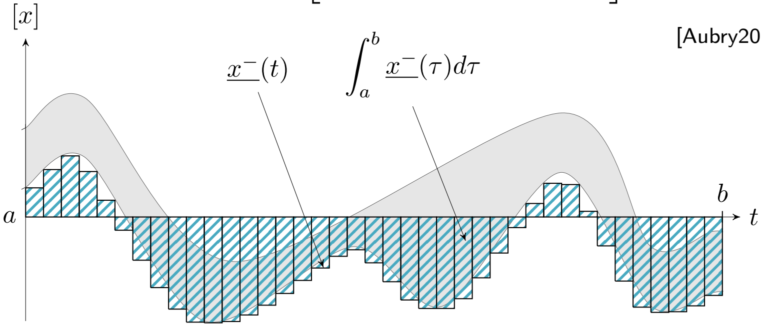
tube implementation with a set of boxes – this outer representation adds pessimism but enables guaranteed and simple computations

Appendix

Tubes integral: implementation

Outer approximation of the integral computed by:

$$\int_a^b [x](\tau)d\tau \subset \left[\int_a^b \underline{x^-}(\tau)d\tau, \int_a^b \overline{x^+}(\tau)d\tau\right]$$



[Aubry2013]

blue area: outer approximation of the lower bound of the tube's integral

Appendix

Reliable approximation of absolute speed $\mathbf{v}^*(\cdot)$

Robot sensors for **absolute speed computation**:

- velocity sensor (DVL)
- inertial measurement unit

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Uncertainties:

- datasheets $\implies$ standard deviation σ for each sensor

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Reliable approximation of absolute speed $\mathbf{v}^*(\cdot)$

Robot sensors for **absolute speed computation:**

- velocity sensor (DVL)
- inertial measurement unit

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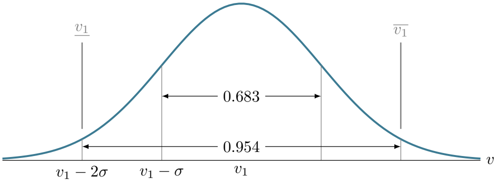
- datasheets $\implies$ standard deviation σ for each sensor
- 95% confidence rate: $v_1^* \in [v_1] = [v_1 - 2\sigma, v_1 + 2\sigma]$



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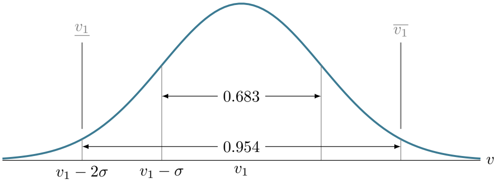

- uncertainties propagated thanks to interval arithmetic