Brunovsky decomposition for dynamic interval localization

Simon Rohou[1], Luc Jaulin[1]

[1] ENSTA Bretagne, Lab-STICC, UMR CNRS 6285, Brest, France
simon.rohou@ensta-bretagne.fr

Keywords: non-linear system, localization, differential equations

Introduction

We present a new set-membership method [1] for estimating the trajectories of dynamical systems $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$, when the states are completely unknown and only observations $\mathbf{y}$ are available under the form of $\mathbf{y} = \mathbf{g}(\mathbf{x})$, with $\mathbf{f}$ and $\mathbf{g}$ non-linear functions.

When the states are completely unknown, conventional methods such as Kalman filters run into difficulties, as it is difficult to find a linearization point, or to perform prediction steps. Particle filters will employ algorithms with high complexity without ensuring a reliable convergence. In contrast, the use of set-membership approaches avoids the need for linearization and is more suited to large uncertainties by not removing consistent solutions. They will however badly behave in our context considering that the states are completely unknown. In order to overcome this problem, interval methods from state-of-the-art apply some branch-and-prune techniques such as shaving methods for reducing the state sets by performing bisections. The counterpart is obviously the increasing complexity of these algorithms. As a consequence, the current existing tools are not sufficient for addressing the considered problem both in a reliable and an efficient way.

Our contribution is to tackle this state estimation efficiently without performing bisections. This can be achieved by rewriting the system into a Brunovsky form.

First part: symbolic Brunovsky decomposition

The first part of the proposed method is symbolic and follows the decomposition of Brunovsky [2], *i.e.*, it rewrites the set of differential equations into two blocks of constraints: one block gathers non-linear equations that do not involve differential operators, while the other block is composed of linear chains of integrators.

For instance, a differential flat system with flat outputs $z_1, \ldots, z_m$ and sensor outputs $\mathbf{y}$, admits the following Brunovsky decomposition:

$$\begin{cases} \mathbf{y} = \mathbf{g}\left(\mathbf{x}\right) \\ \begin{pmatrix} z_1 \\ \dot{z}_1 \\ \vdots \\ z_m^{(\kappa_m)} \end{pmatrix} = \lambda \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix} \\ z_1^{(\kappa_1)} \xrightarrow{\int} \cdots \xrightarrow{\int} \dot{z}_1 \xrightarrow{\int} z_1 \\ \vdots \\ z_m^{(\kappa_m)} \xrightarrow{\int} \cdots \xrightarrow{\int} \dot{z}_m \xrightarrow{\int} z_m \end{cases} \tag{1}$$

The first block with non-linear equations and no differential relations corresponds to the functions λ and $\mathbf{g}$. The second block is only made of chains of integrators, for which an optimal operator will be at hand.

Second part: contractor approach

The second part of the method is numerical and based on a contractor method. It relies on the previous decomposition and encloses the variables into boxes and tubes. Then, contractor operators are used for narrowing the sets of feasible solutions. In particular, a new contractor is provided for dealing with the chains of integrators, that gather all the differential aspects of the dynamical system.

Application to robot localization

A robot measures some distances to known landmarks, in addition to known inputs **u** of the system, but without any prior knowledge about the states. This problem is known to be difficult to solve, and methods from state-of-the-art usually come into bisection procedures of the heading and position values [3], which implies a strong computation burden. We are able to provide a bounded estimate of the trajectory of the states, by using contractors and without performing bisections.

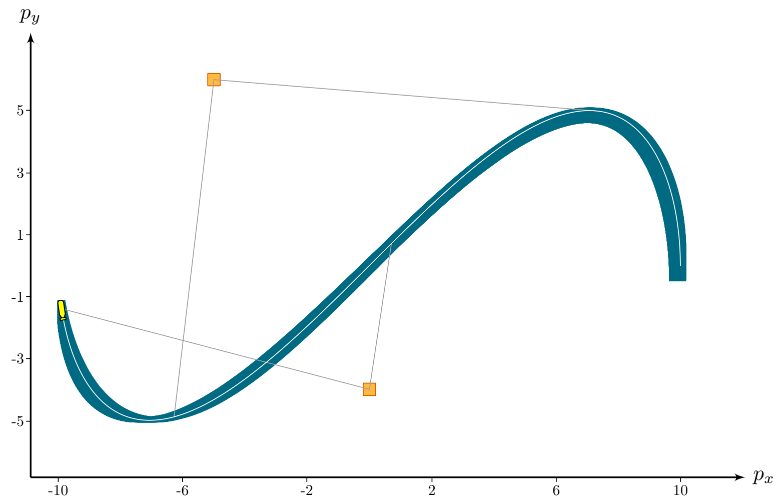

Figure 1: Guaranteed computation of a tube enclosing the feasible trajectories of a robot measuring bounded distances from two landmarks, without prior knowledge about its states (positions, velocity, heading).

References

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