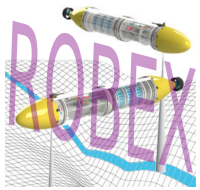
The Codac library

Simon Rohou, Benoît Desrochers

ENSTA Bretagne, Lab-STICC, UMR CNRS 6285, Brest, France

SWIM 2022
19th July 2022



Section 1

Codac in a nutshell

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Domains (wrappers)

- for reals $x \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$: intervals $[x]$ and boxes $[\mathbf{x}]$ (IBEX)
- for trajectories $x(\cdot) : \mathbb{R} \to \mathbb{R}$: tubes $[x](\cdot)$

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- for subsets $\mathbb{X} \subset \mathbb{R}^n$: thicksets $\mathbb{X} \in [\mathbb{X}] = [\mathbb{X}^-, \mathbb{X}^+]$

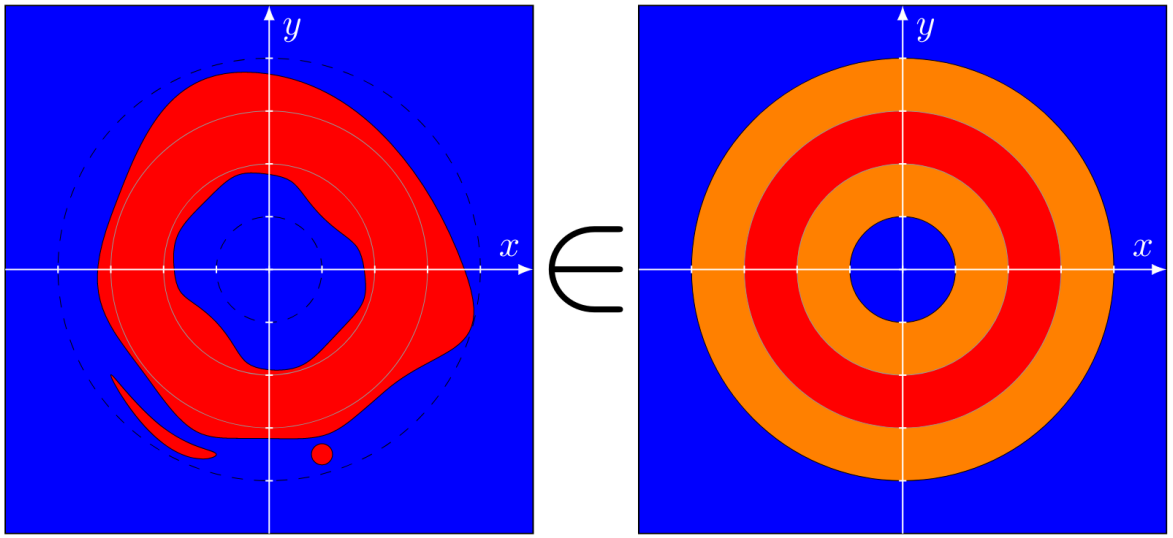

Illustration of a thickset (right-hand side)
for enclosing and uncertain red set (left-hand side)

Thick set inversion
Desrochers, Jaulin. *Artificial Intelligence. Volume 249, Issue C, Pages 1-18*, 2017

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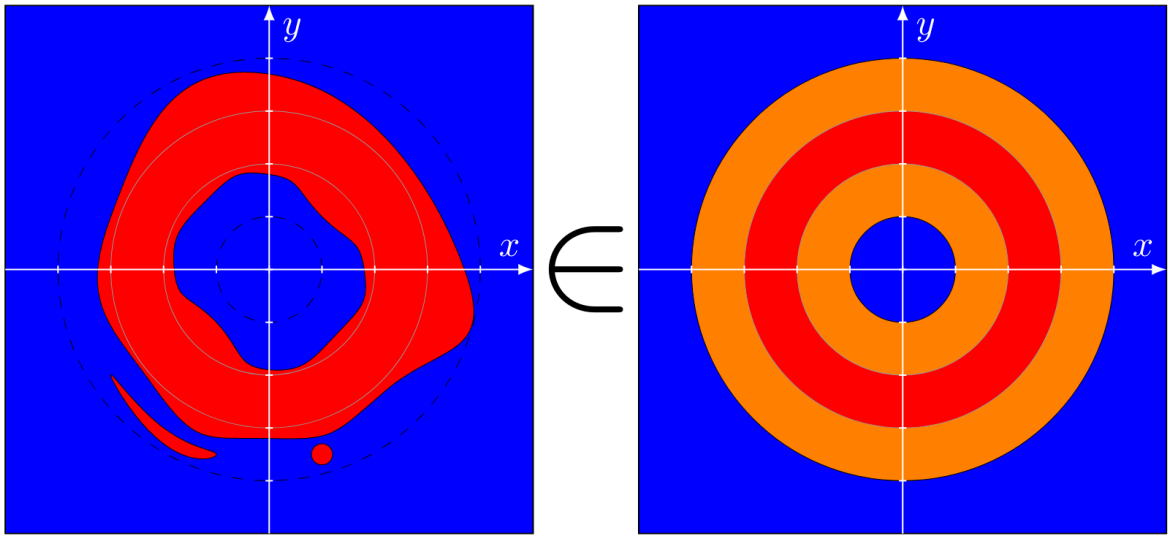
- *etc.*



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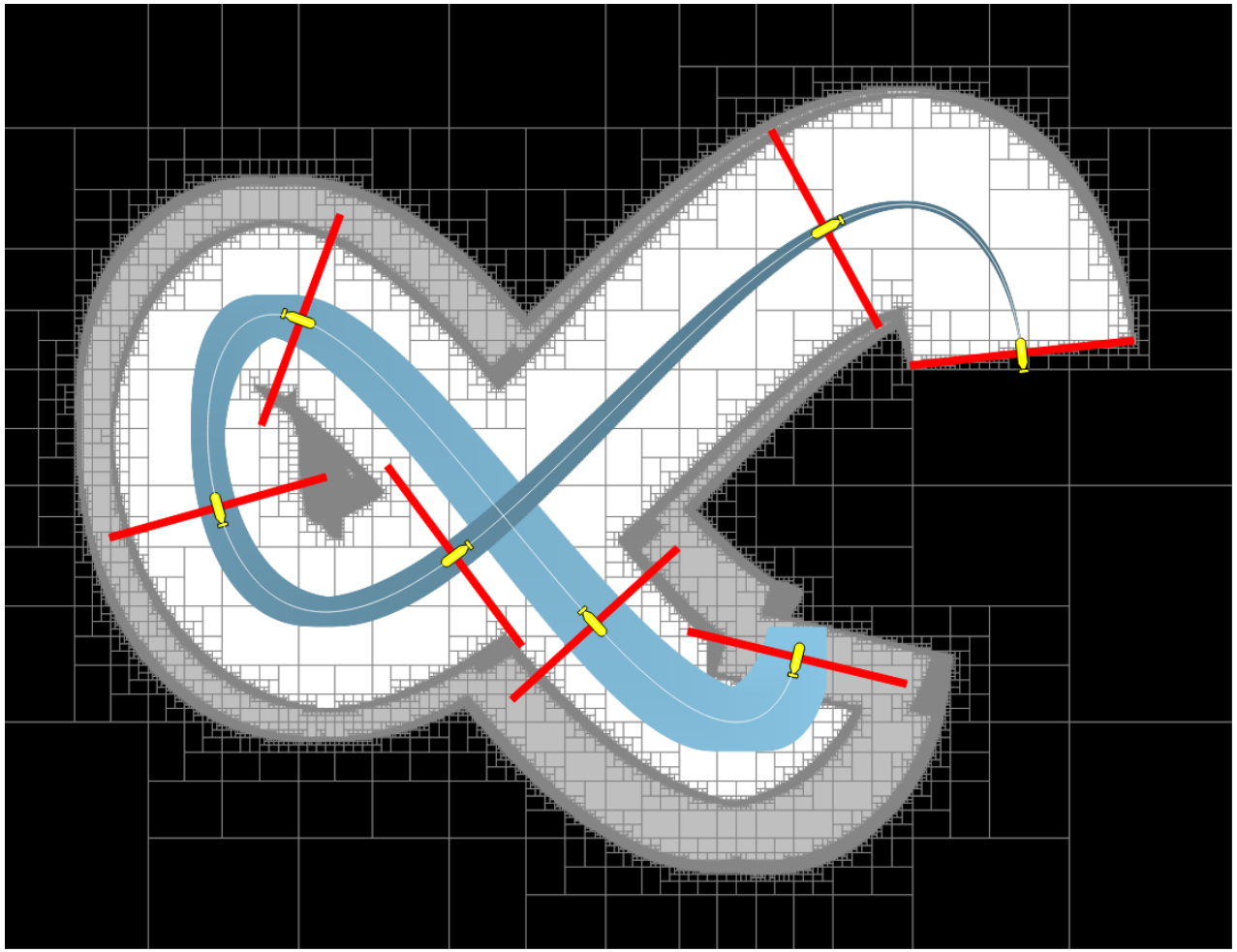
Codac in a nutshell

Example of tubes and thicksets



Computing a Guaranteed Approximation of the Zone Explored by a Robot
Desrochers, Jaulin. *IEEE Transaction on Automatic Control. Volume 62, Issue 1, pages 425-430*, 2017

Codac in a nutshell

Codac: Catalog Of Domains And Contractors

Several types of **domains**:

- `Interval`, `IntervalVector`, `IntervalMatrix` (from IBEX)
- `Tube`, `TubeVector`, `Slice`
- `Thickset`
- ...

Codac in a nutshell

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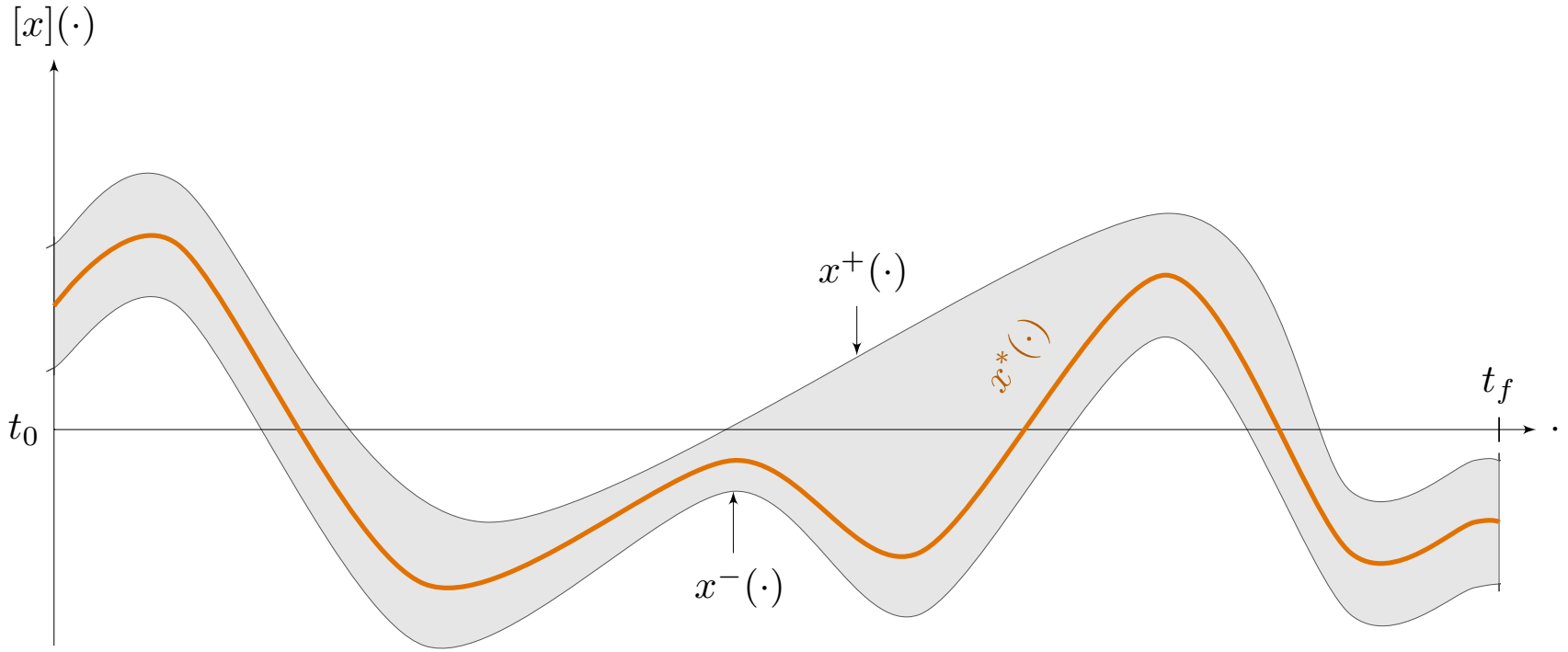
- `Thickset`
- ...

Contractors for various constraints:

- non-linear constraints $\mathbf{f}(\mathbf{x}) = \mathbf{0}$
- geometric constraints: distance, polar equation, circles, ...
- differential equations: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$
- time uncertainties: $\mathbf{y} = \mathbf{x}(t), \quad \text{with } t \in [t]$
- delays: $x(t) = y(t - \tau)$
- ...

Codac in a nutshell

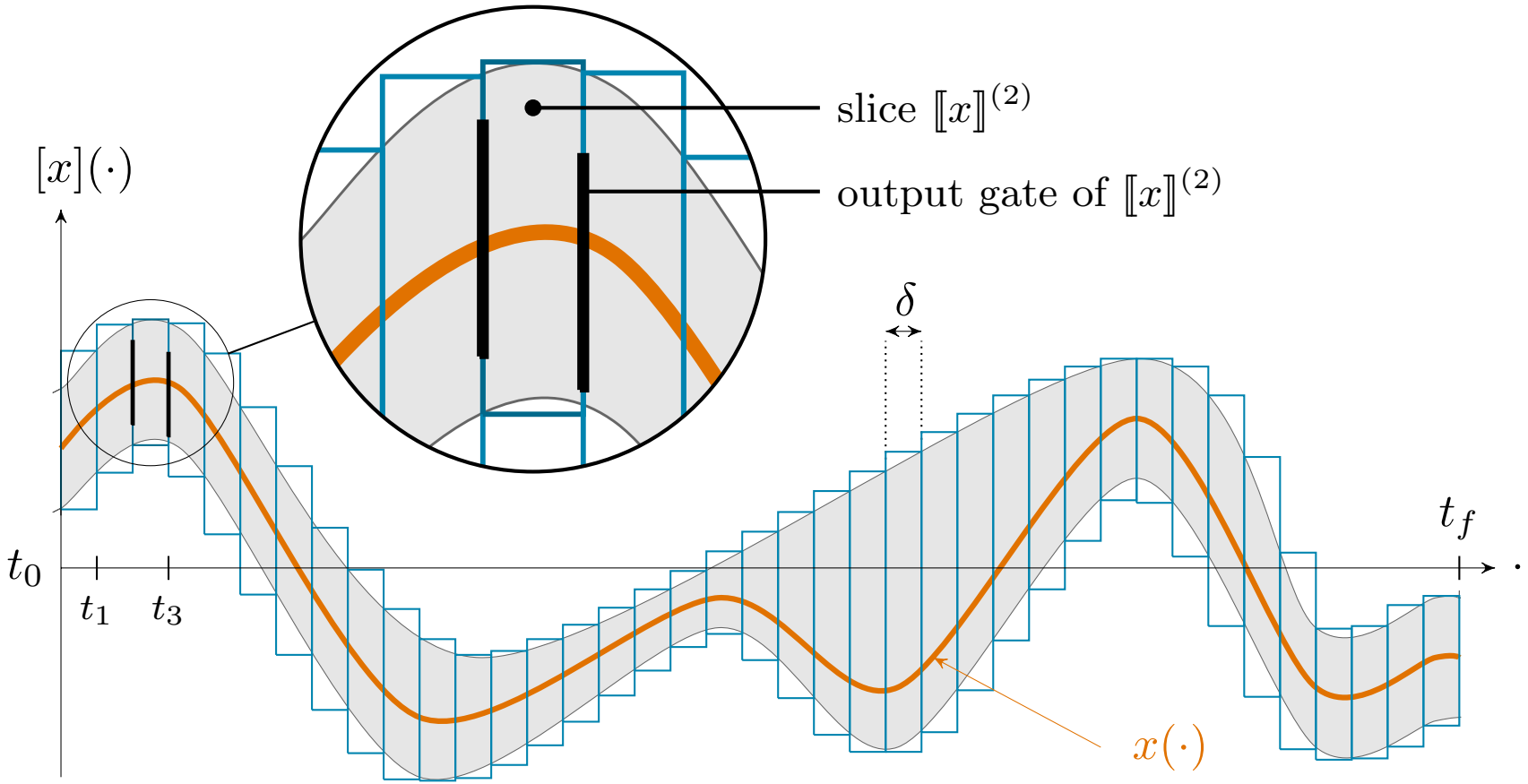
Domains for trajectories: tubes



Example of scalar tube: interval of two trajectories

Codac in a nutshell

Domains for trajectories: tubes



Computer implementation (http://codac.io)

Codac in a nutshell

Example of optimal contractors for the $\mathcal{L}_{\text{polar}}$ constraint

$$\mathcal{L}_{\text{polar}} : \begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \end{cases}$$

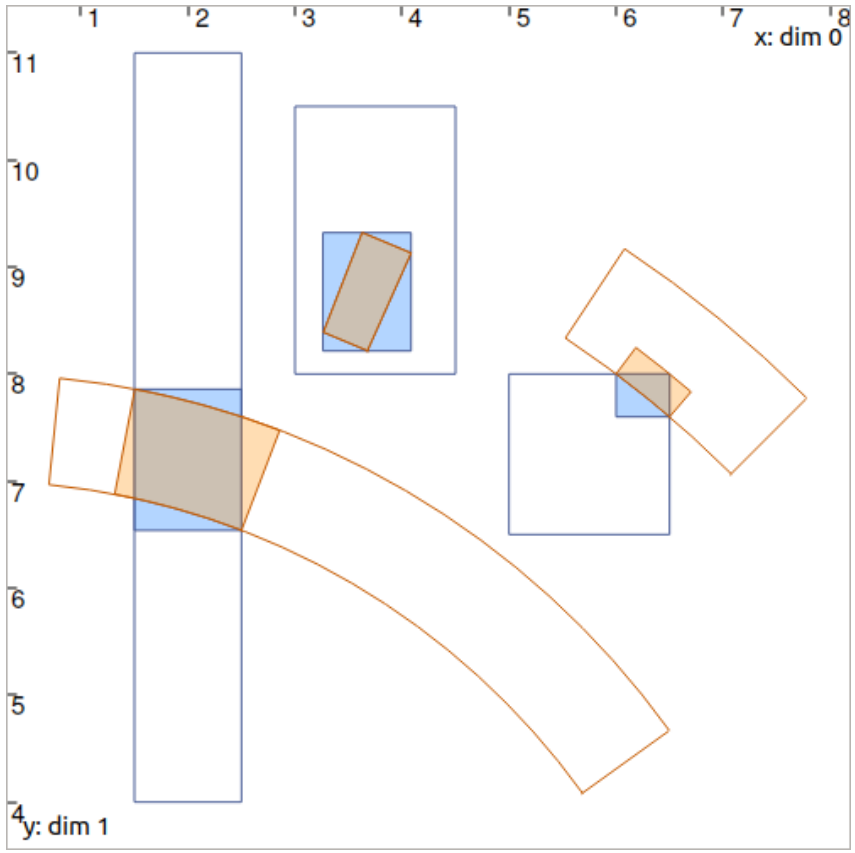
Optimally dealt with by:

$$\mathcal{C}_{\text{polar}}\big([x],[y],[\rho],[\theta]\big)$$

Using Codac:

```
x = Interval(..)
y = Interval(..)
r = Interval(..)
theta = Interval(..)

ctc.polar.contract(x,y,r,theta)
```



A Minimal contractor for the Polar equation: application to robot localization
Desrochers, Jaulin. *Engineering Applications of Artificial Intelligence, 55(Supplement C):83–92*, 2016

Codac in a nutshell

The library is open source and available:

- in Python and C++
- on Linux, Windows, MacOS systems
- from official packages:
  Python package: `pip install codac`
  Debian in progress..: `sudo apt install libcodac`

http://www.codac.io

Section 2

Application: range-only SLAM

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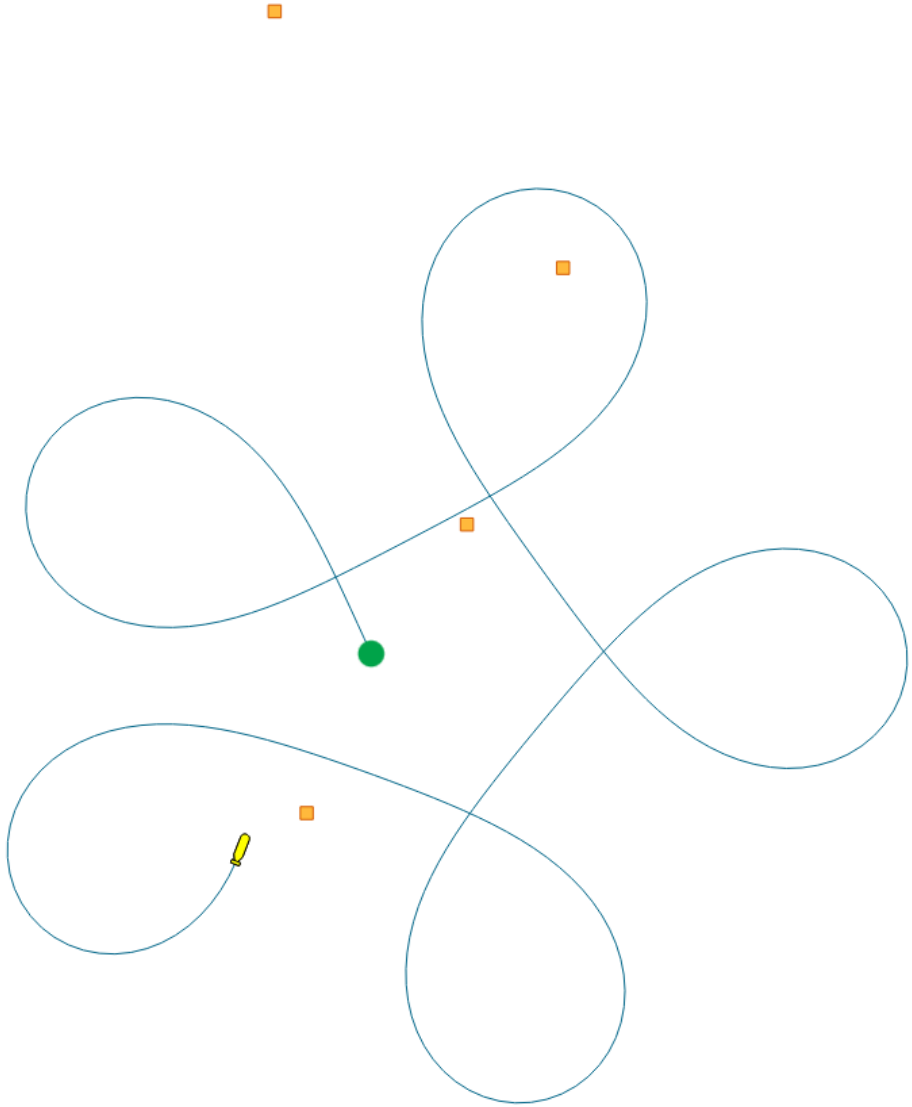
Simultaneous Localization And Mapping



Application: range-only SLAM

Formalization

SLAM: Simultaneous Localization And Mapping.
Classically, we have:

$$\begin{cases} \mathbf{x}(0) = \mathbf{0} & (\text{initial state}) \\ \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) & (\text{evolution}) \end{cases}$$

With:

- $\mathbf{x}$: state vector (position, heading, ...)
- $\mathbf{u}$: input vector (command)
- $\mathbf{f}$: *evolution* function

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With:

- $\mathbf{x}$: state vector (position, heading, ...)
- $\mathbf{u}$: input vector (command)
- $\mathbf{f}$: *evolution* function
- g: *observation* function (scalar, distance equation)
- y_i: scalar measurements (at t_i) (distance values)
- $\mathbf{b}_j$: unknown position of a landmark

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Application: range-only SLAM

Involved variables and domains

Variables:

- reals: $y_i \in \mathbb{R}$
- vectors: $\mathbf{b}_j \in \mathbb{R}^2$
- trajectories: $\mathbf{x}(\cdot) : \mathbb{R} \to \mathbb{R}^n$

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Domains (envelopes) of the variables:

- intervals: $[y_i] \in \mathbb{IR}$
- boxes: $[\mathbf{b}_j] \in \mathbb{IR}^2$
- tubes: $[\mathbf{x}](\cdot) : \mathbb{R} \to \mathbb{IR}^n$

Application: range-only SLAM

Decomposition of the problem

System:

$$\begin{cases} \dot{\mathbf{x}}(\cdot) = \mathbf{f}\big(\mathbf{x}(\cdot)\big) \\ y_i = g\big(\mathbf{x}_{1,2}(t_i), \mathbf{b}_j\big) \end{cases}$$

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Elementary constraints:

- $\mathbf{v}(\cdot) = \mathbf{f}\big(\mathbf{x}(\cdot)\big) \rightarrow$ algebraic constraint $\rightarrow \mathcal{L}_{\mathbf{f}}\big(\mathbf{x}(\cdot), \mathbf{v}(\cdot)\big)$

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Application: range-only SLAM

Decomposition of the problem

System:

$\mathbf{v}(\cdot)$ and $\mathbf{p}_i$ are intermediate variables

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Note: some symbolic solver could break down such problem automatically.

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Application: range-only SLAM

Using Codac

1. Defining the domains:

```
# Creating the state tube:
dt = 0.01
tdomain = Interval(0,6)          # temporal domain
x = TubeVector(tdomain, dt, 4) # dim. 4
# etc.
```

.

Application: range-only SLAM

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# etc.
```

2. Defining contractors:

```
ctc.deriv   # Some contractors are objects
ctc.eval    # already defined in the library
ctc.polar
ctc.dist

# Other contractors can be built from analytical expressions:
ctc_plus = CtcFunction(Function("x","y","z","x+y-z"))
# CtcFunction is related to constraints under the form f(x,..)=0
```

Application: range-only SLAM

Using Codac

3. Build a Contractor Network

System:

$$\begin{cases} \dot{\mathbf{x}}(\cdot) = \mathbf{f}\big(\mathbf{x}(\cdot)\big) \\ y_i = g\big(\mathbf{x}_{1,2}(t_i), \mathbf{b}_j\big) \end{cases}$$

Contractor programming:

1. $\mathcal{C}_{\mathbf{f}}\big([\mathbf{x}](\cdot), [\mathbf{v}](\cdot)\big)$
2. $\mathcal{C}_{\text{deriv}}\big([\mathbf{x}](\cdot), [\mathbf{v}](\cdot)\big)$
3. $\mathcal{C}_{\text{eval}}\big([t_i], [\mathbf{p}_i], [\mathbf{x}_{1,2}](\cdot)\big)$
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Using Codac:

```
cn = ContractorNetwork()

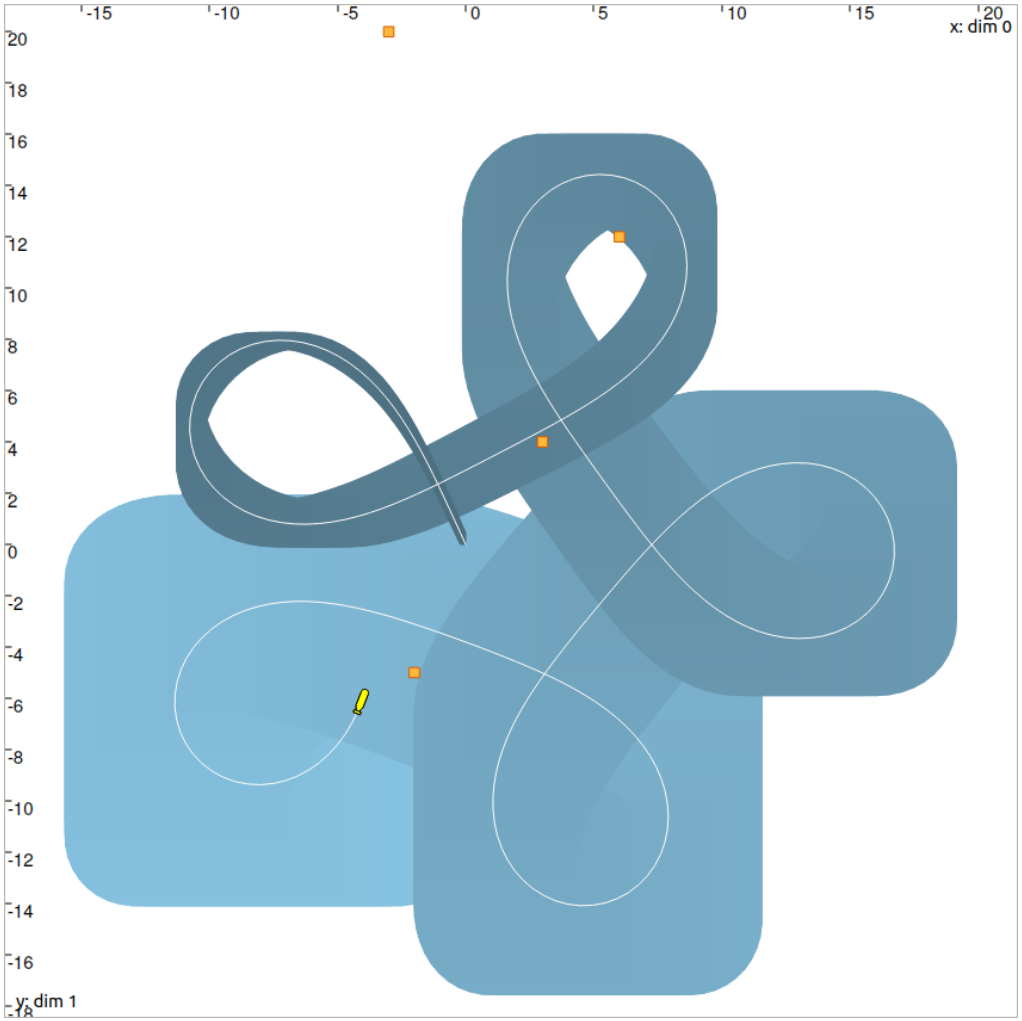
cn.add(ctc.polar, [v[0],v[1],x[3],x[2]])  #1
cn.add(ctc.deriv, [x,v])                  #2

for i in range(len(v_t)):
  pi = IntervalVector(4)
  cn.add(ctc.eval, [t[i],pi,x])           #3
  cn.add(ctc.dist, [pi,b[j],y[i]])        #4

cn.contract()                             -
```

Application: range-only SLAM

Using Codac

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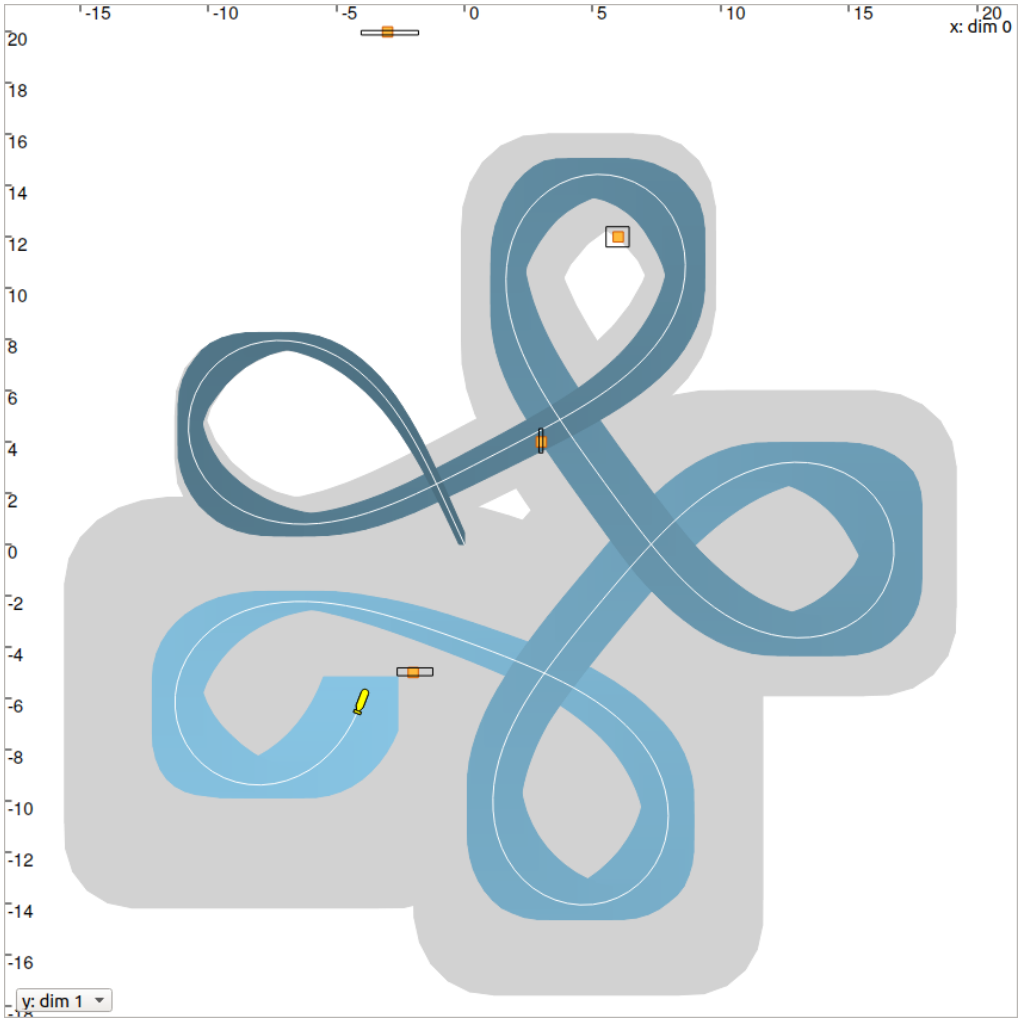


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```

2399 contractors, 2410 dom.
Computation time: 0.25s

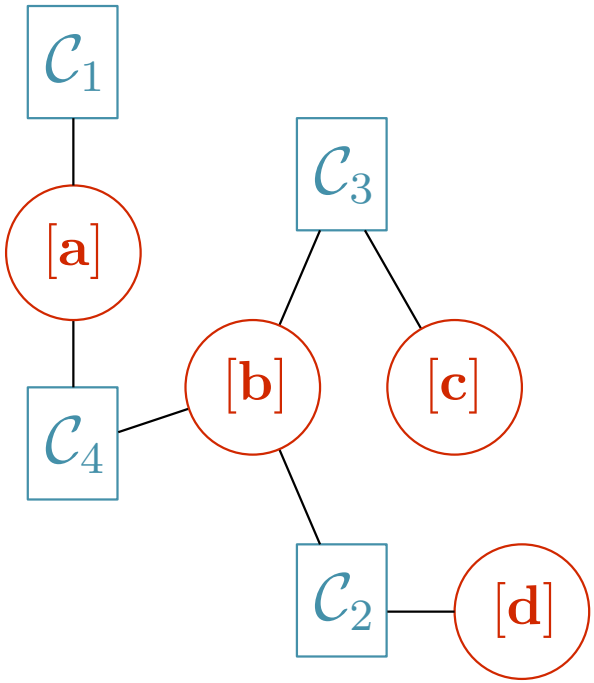
Section 3

Contractor Networks

Contractor Networks

Propagation

- Contractor Network = graph of Domains and Contractors
- The graph is possibly directed
- The network allows accurate propagation of the contractions
- When the stack of contractor calls is empty, a fixed point is reached



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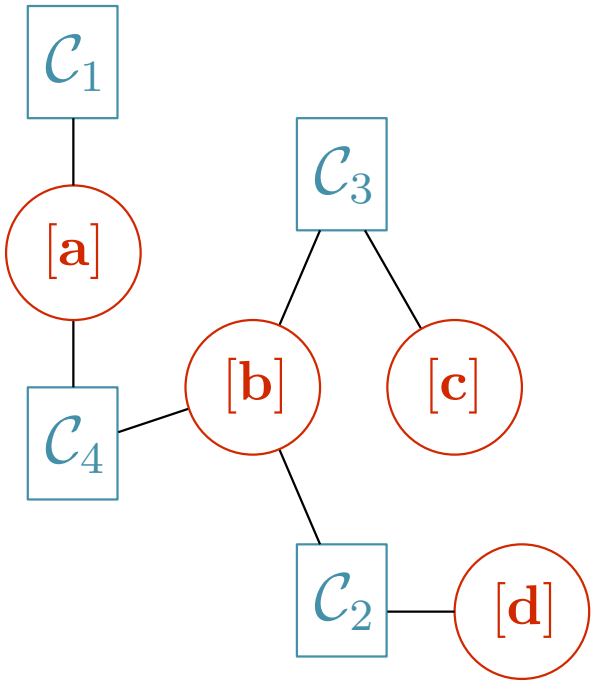

Propagation sequence:

1. $[\mathbf{c}]$ contracted/triggered
2. adding $\mathcal{C}_3$ in stack
3. calling $\mathcal{C}_3$
4. $[\mathbf{b}]$ contracted
5. adding $\mathcal{C}_2$, $\mathcal{C}_3$, $\mathcal{C}_4$ in stack
6. calling $\mathcal{C}_4$
7. $[\mathbf{b}]$ contracted
8. adding $\mathcal{C}_4$ in stack
9. calling $\mathcal{C}_2$
10. $[\mathbf{d}]$ contracted
11. calling $\mathcal{C}_3$, $\mathcal{C}_4$
12. fixed point

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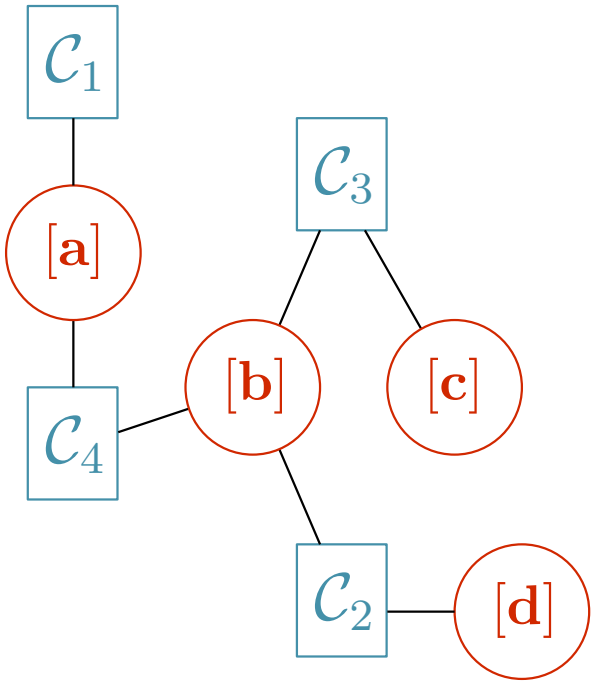

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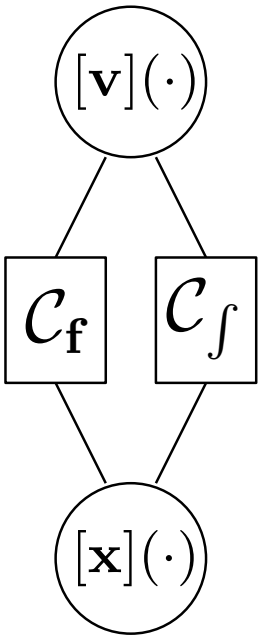
10. $[\mathbf{d}]$ contracted
11. calling $\mathcal{C}_3$, $\mathcal{C}_4$
12. fixed point

$\Rightarrow$ **heuristic of propagation**

Contractor Networks

Contractor Network involving Tubes

Example, dealing with: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \iff \dot{\mathbf{x}} = \mathbf{v}\ ,\ \mathbf{v} = \mathbf{f}(\mathbf{x})$



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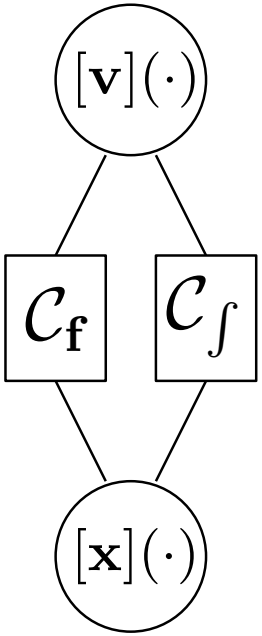
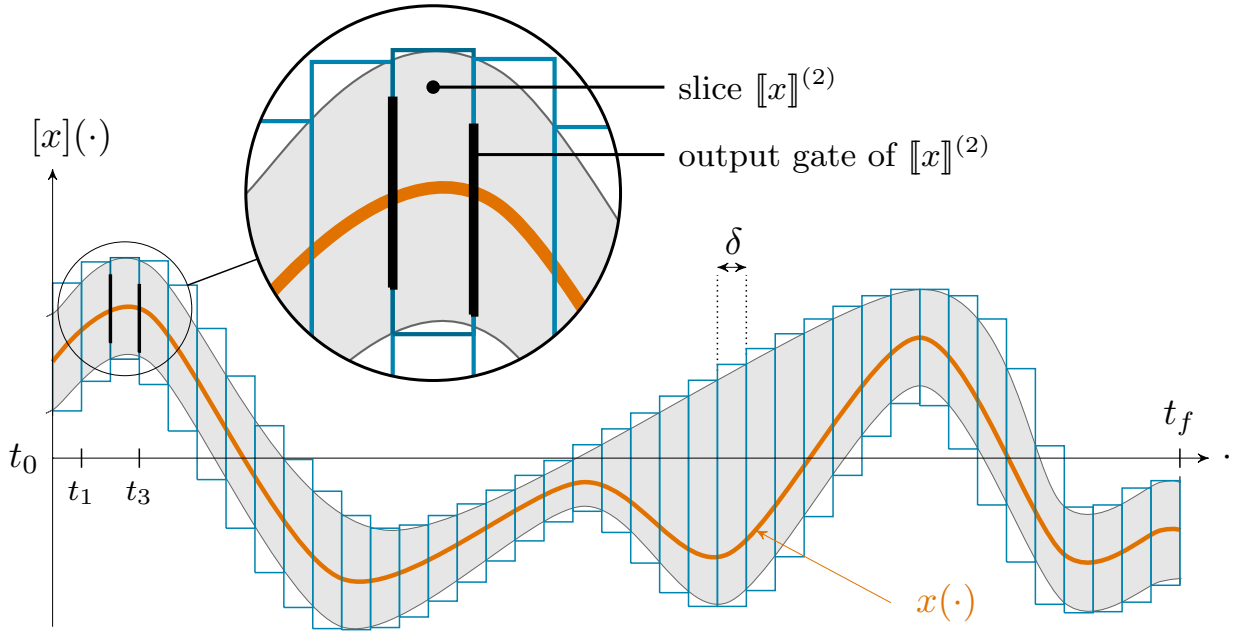
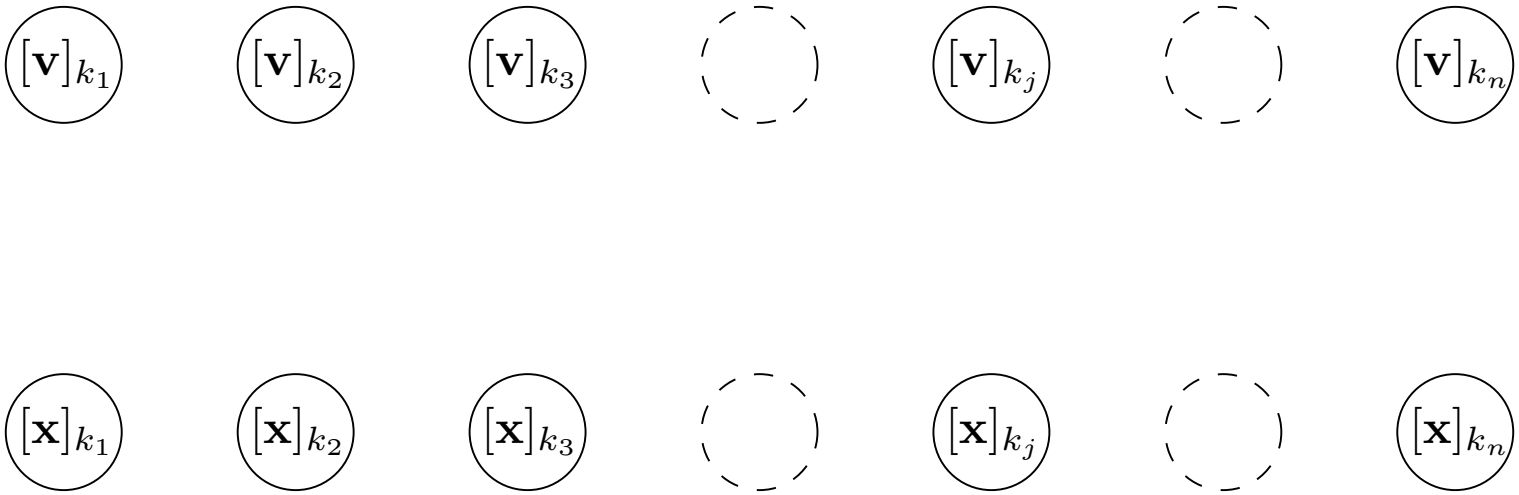




- break down contractors into *micro-contractors* at the level of slices
- $\Rightarrow$ strong densification of the graph

Contractor Networks

Contractor Network involving Tubes



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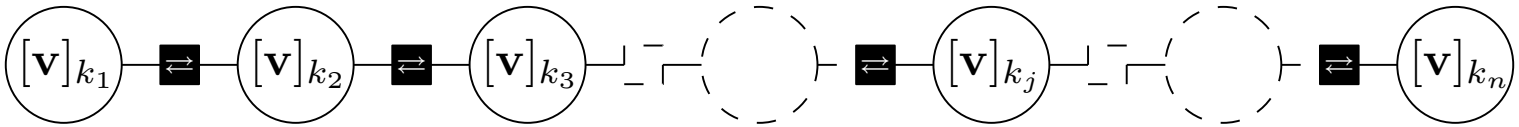

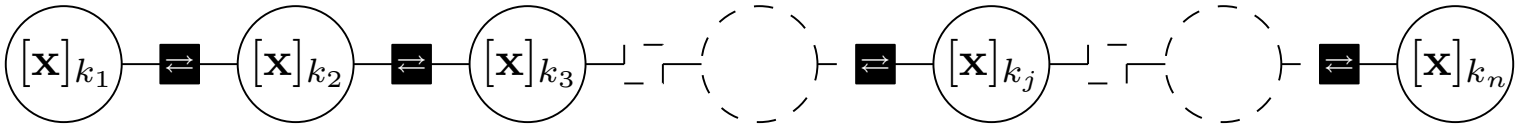

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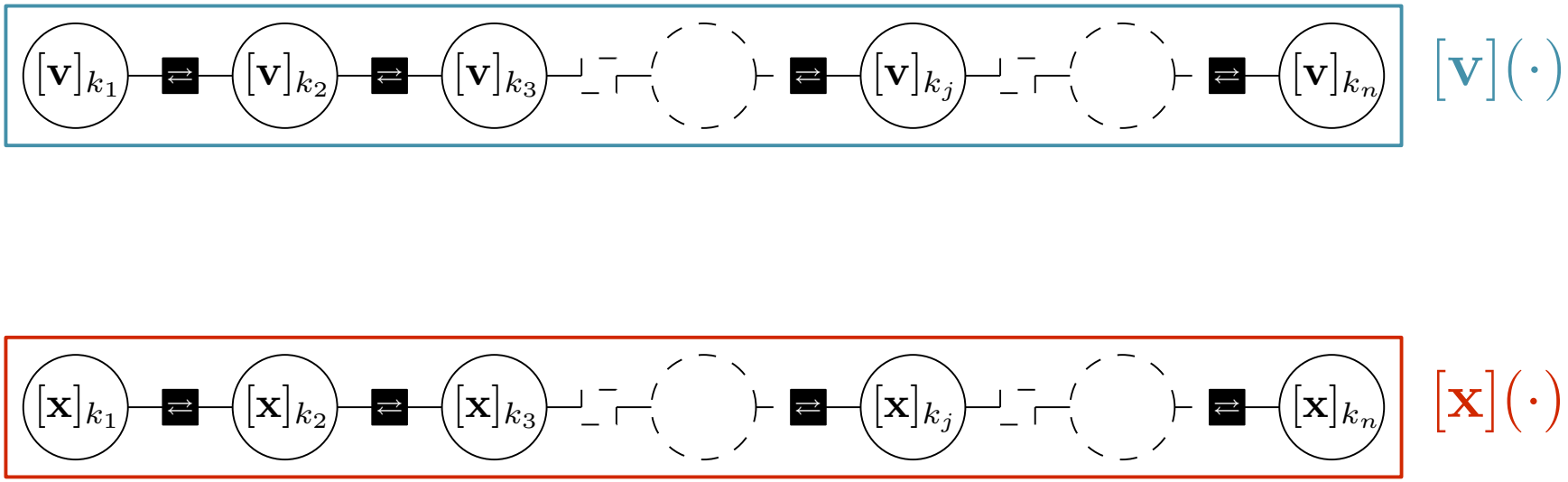

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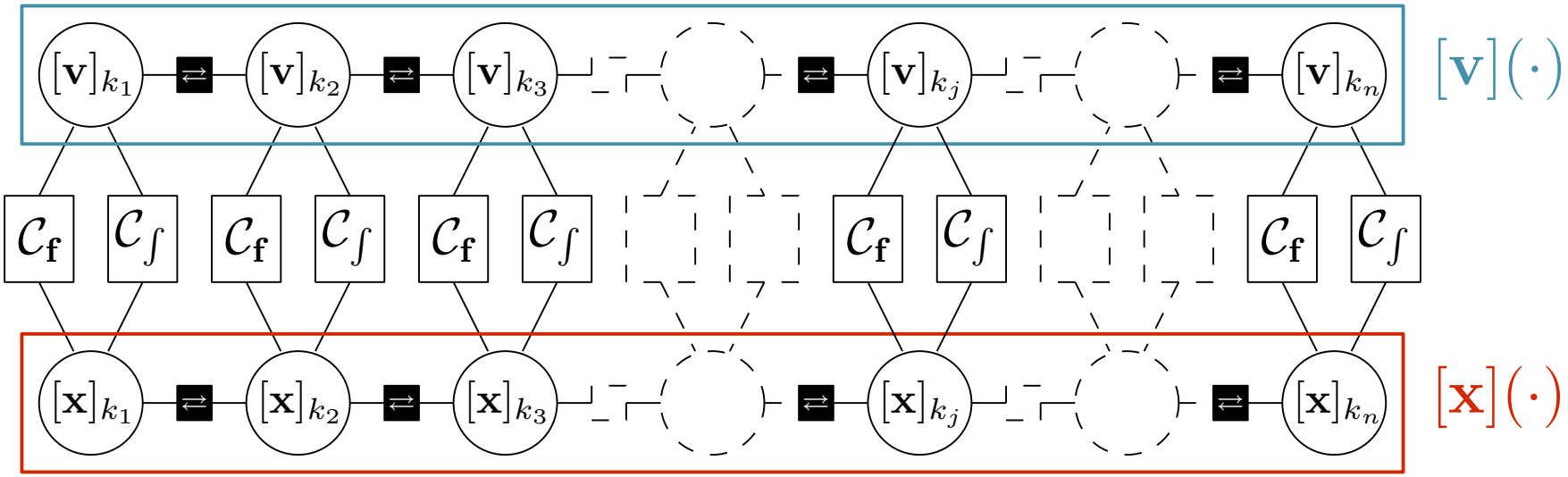
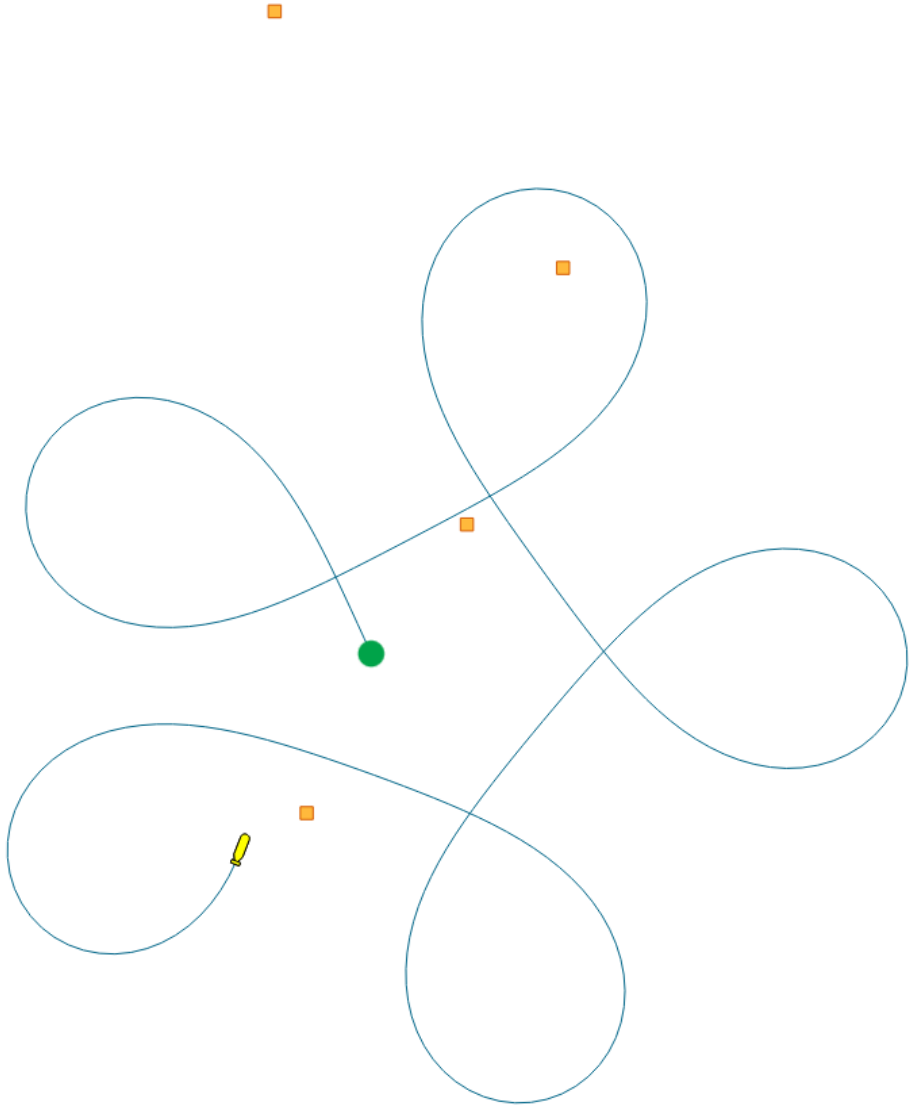


Contractor Networks

Contractor Network for SLAM: SLAM-CN



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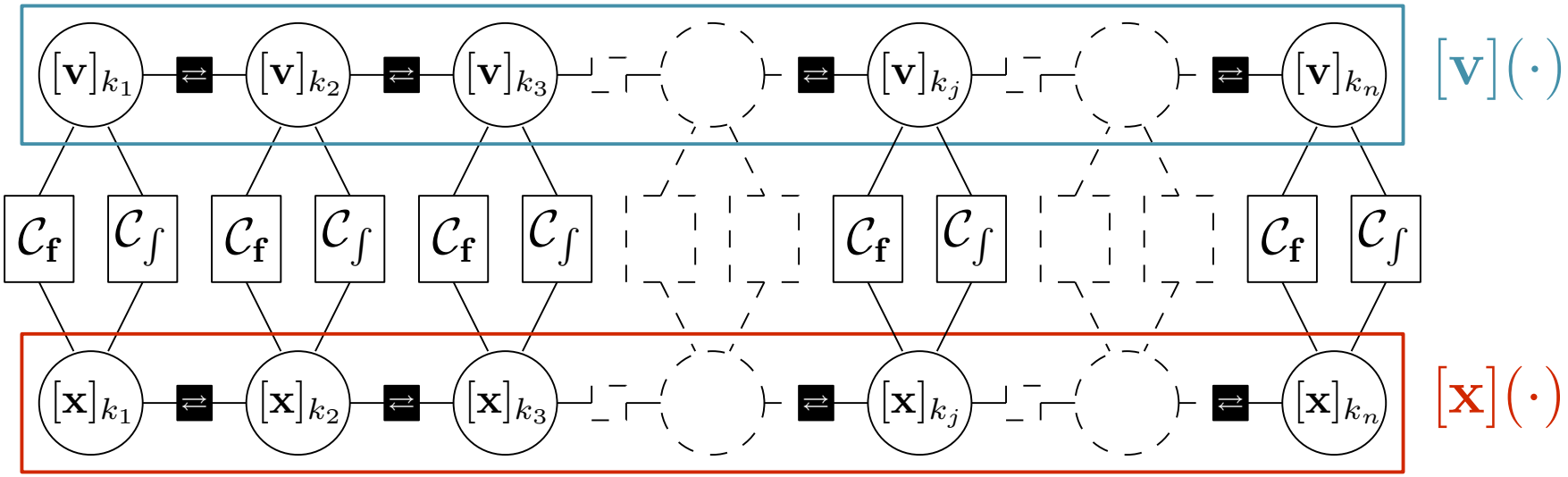

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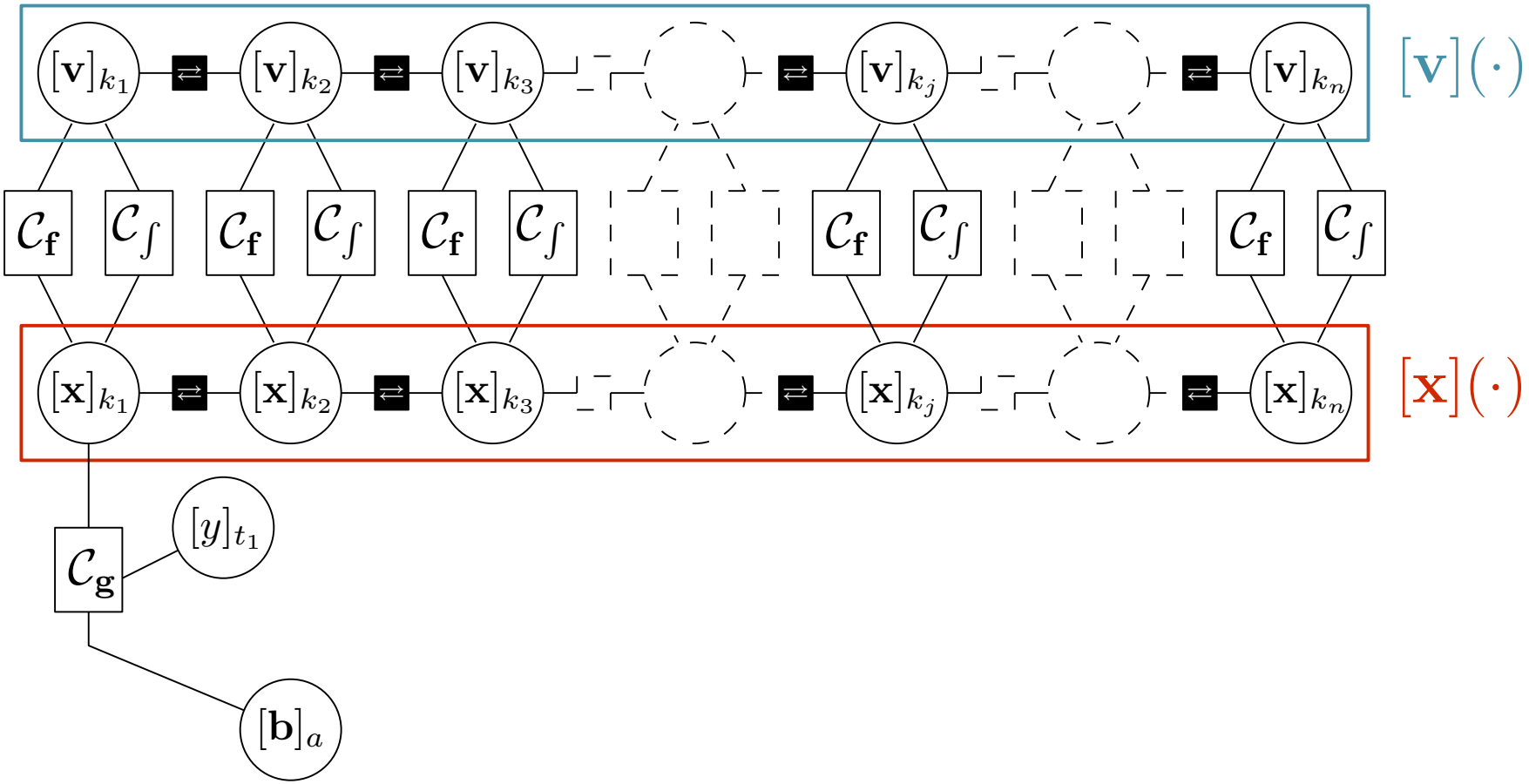

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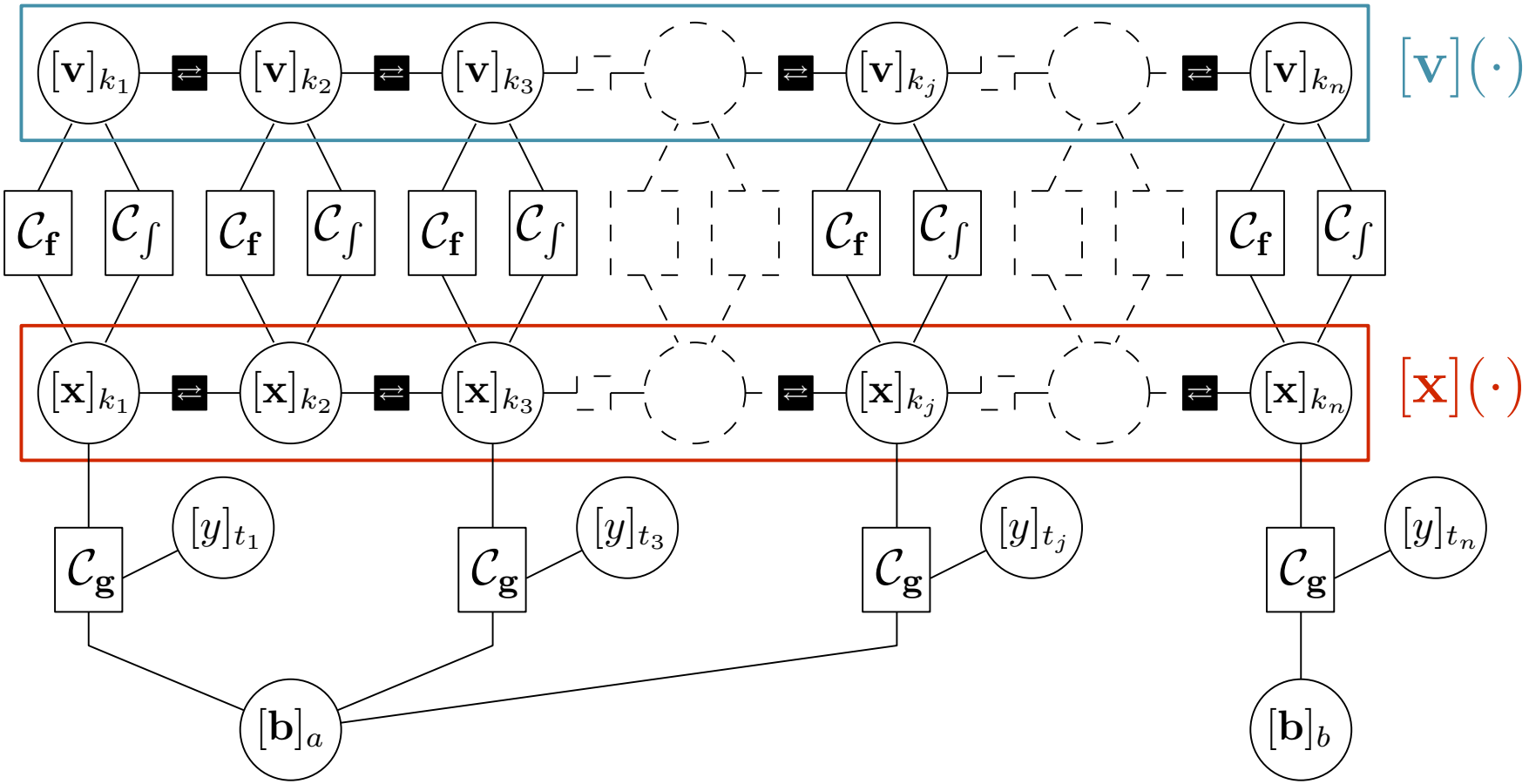

Contractor Networks

Programming a SLAM-CN

1. Define domains:
   - intervals, boxes, tubes, ...
   - related to measurements or initialized as $[-\infty, \infty]$

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3. Build the Contractor Network:

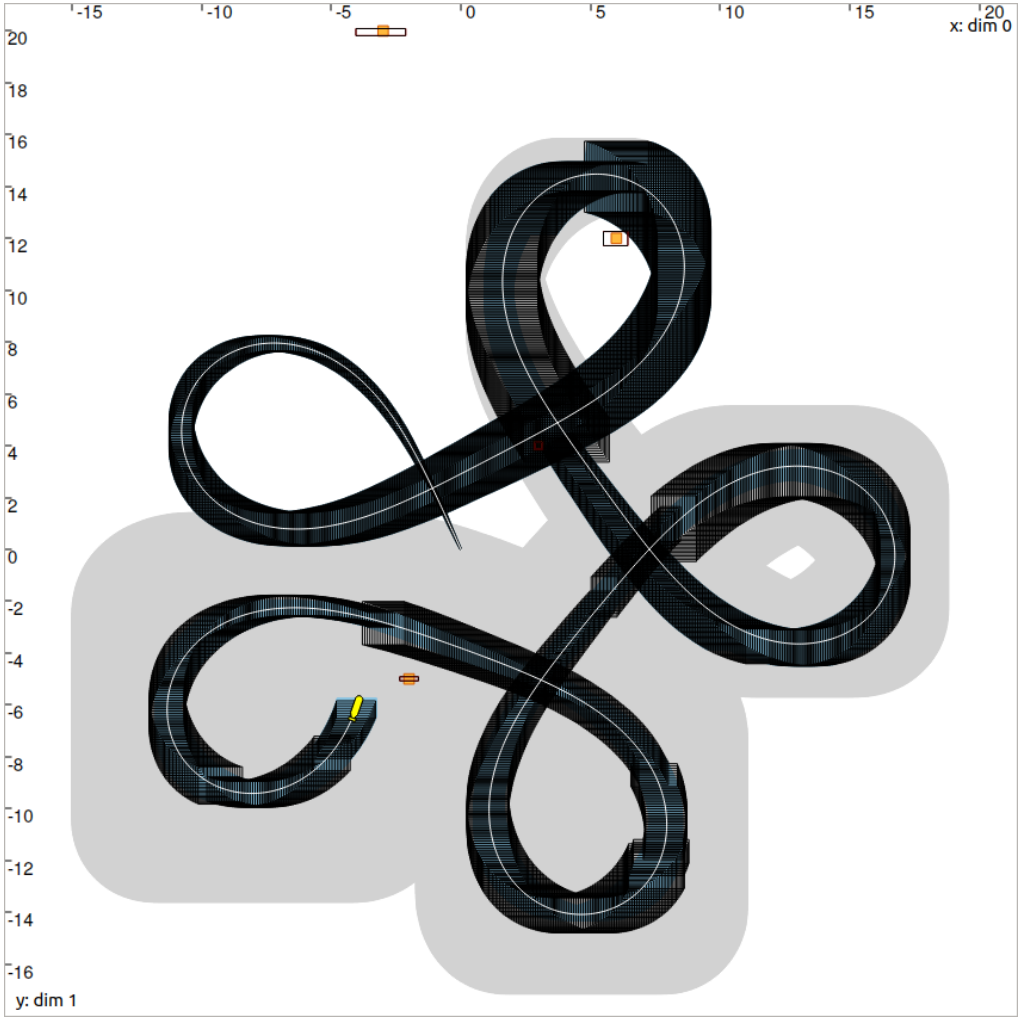
```
cn = ContractorNetwork()
cn.add(ctc_f, [x,v])
cn.add(ctc.deriv, [x,v])

for i in range(len(v_t)):
  pi = IntervalVector(4)
  cn.add(ctc.eval, [t[i],pi,x]
  cn.add(ctc.dist, [y[i],pi,b[i]])

cn.contract()
```

Contractor Networks

SLAM-CN: realtime application



Contractor Networks

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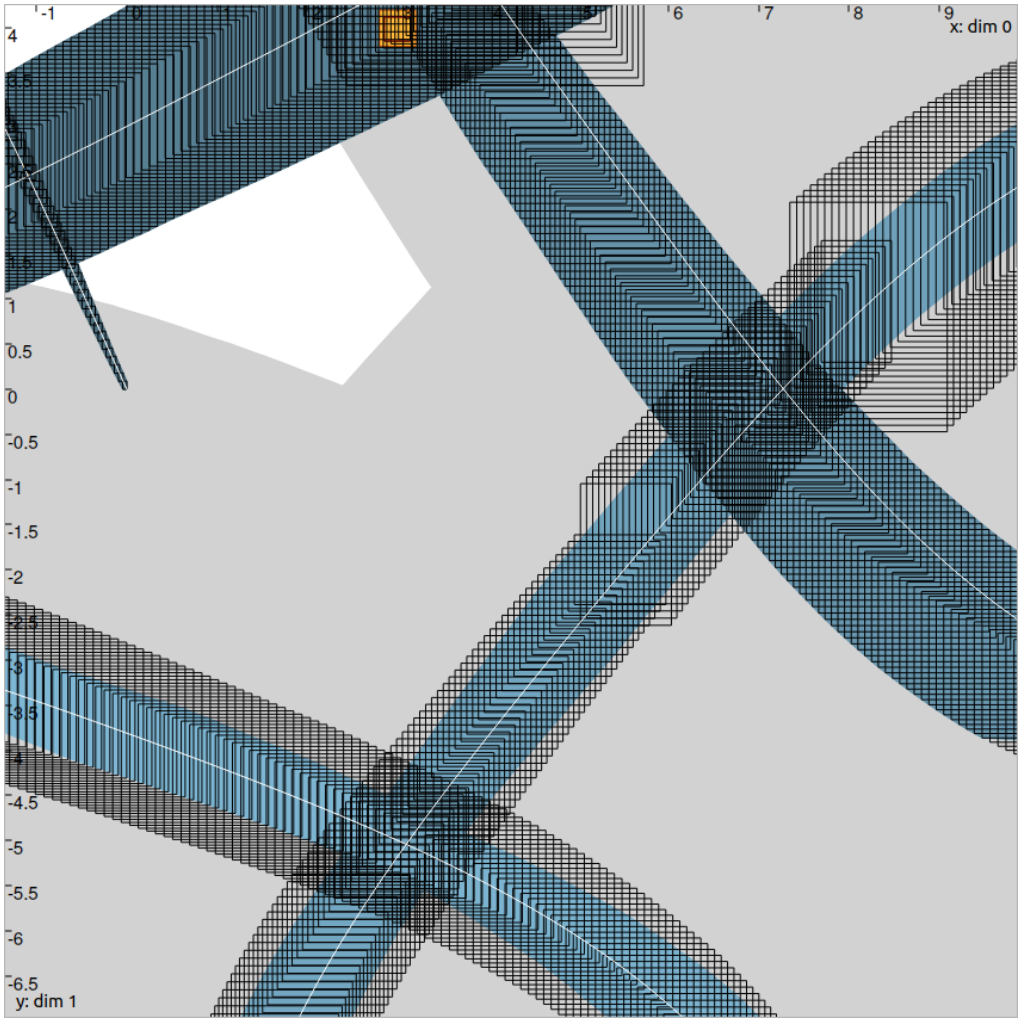

Section 4

Conclusions

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Assets of constraint programming coupled with interval analysis:

- **simplicity** of the approach

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Future work related to Codac:

- extend the *catalog* of contractors
- implement other domains and intermediate wrappers
- ..towards real-time implementations

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Future work related to Codac:

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Codac library: open-source C++/Python library providing tools for constraint programming over reals, trajectories and sets

http://www.codac.io

Conclusions

Conclusion

Thanks to the current (and soon) contributors!

- Benoît Desrochers
- Luc Jaulin
- Gilles Chabert
- Auguste Bourgois
- Julien Damers
- Thomas Le Mézo
- Raphael Voges
- Cyril Bouvier
- Andreas Rauh
- Fabrice Le Bars
- Quentin Brateau
- Damien Massé
- Bertrand Neveu
- Peter Franek
- Gilles Trombettoni
- Aaronkumar Ehambram
- Verlein Radwan
- Mohamed Saad Ibn Seddik
- Jonas Soueidan

Section 5

Appendices

Appendices

"Static" constraints

Static constraint:

- $\forall t,\ f(a(t), b(t), \dots) = 0$
- non differential
  (not in the form
  $\dot{a}(t) = b(t)$)
- non inter-temporal
  (not in the form
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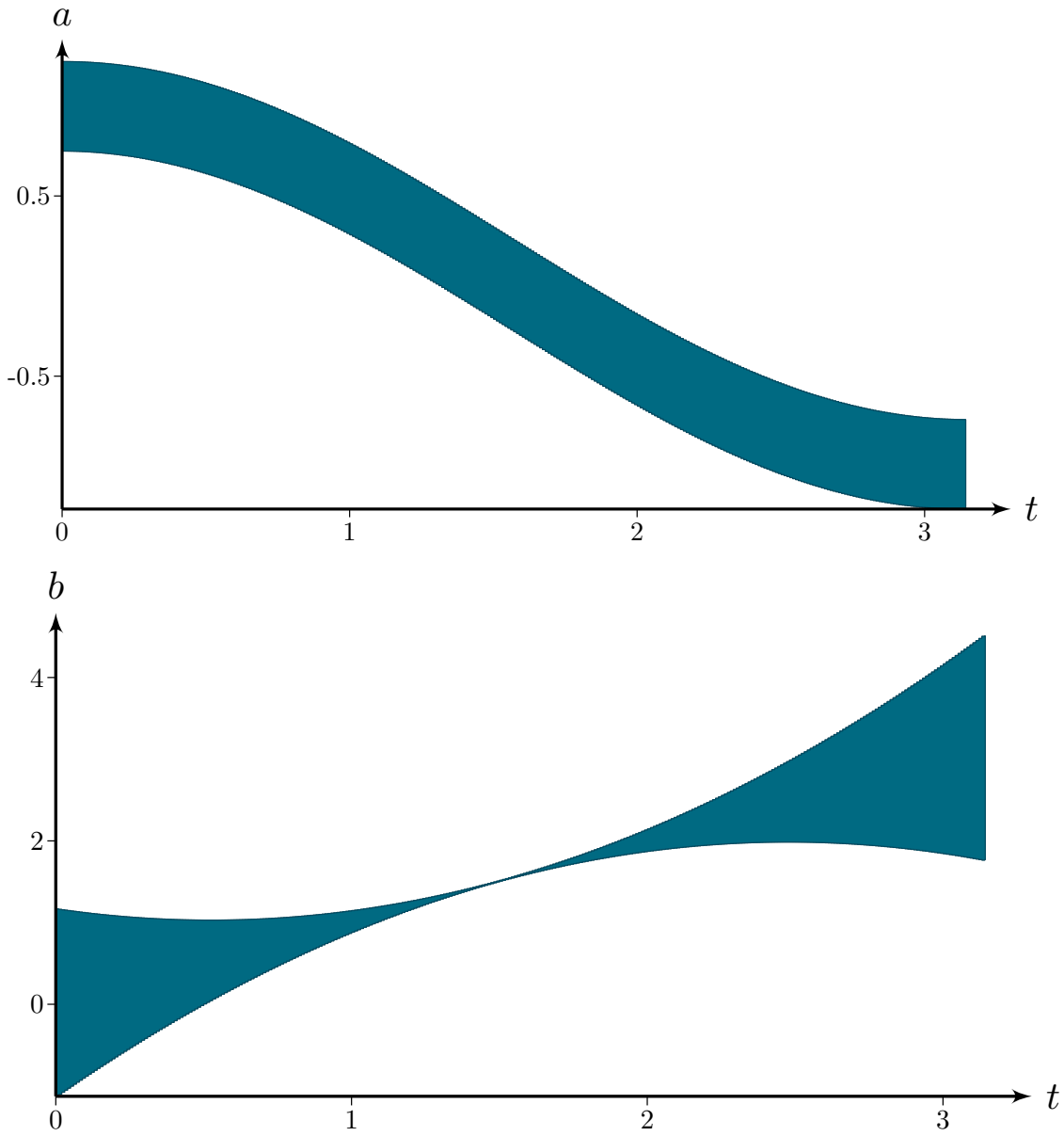

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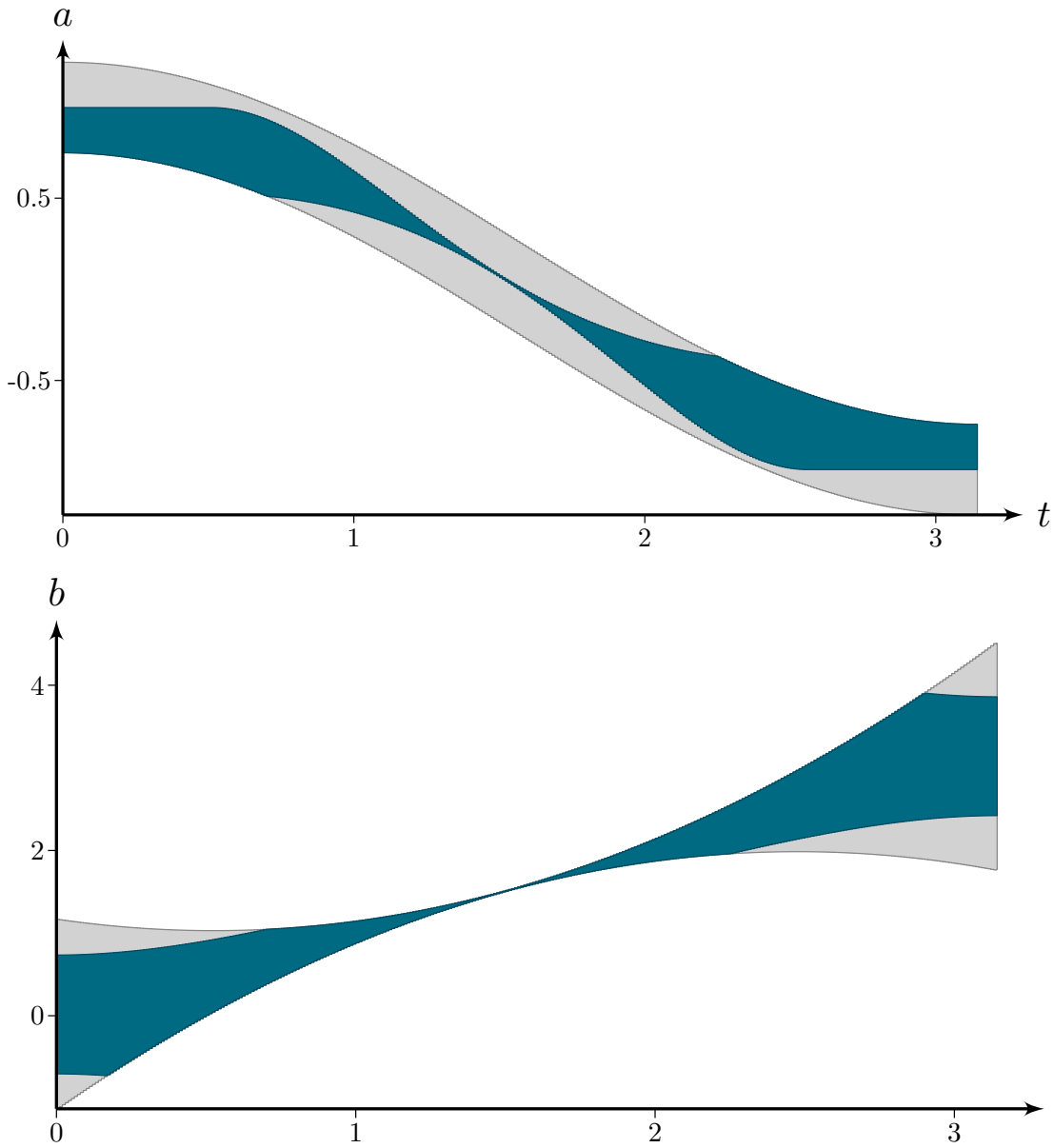
Example with $a(\cdot) = \cos\big(b(\cdot)\big)$:

$$\mathcal{C}_{\cos}\big([a](\cdot), [b](\cdot)\big)$$



Appendices

"Static" constraints

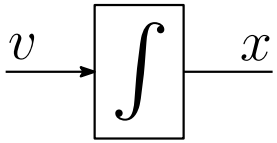
A definition of the $\mathcal{C}_+$ operator for tubes for the constraint $a(\cdot) = x(\cdot) + y(\cdot)$:

$$\mathcal{C}_+\big([a](\cdot),[x](\cdot),[y](\cdot)\big)$$

$$\begin{pmatrix} [a](t) \\ [x](t) \\ [y](t) \end{pmatrix} \xmapsto{\mathcal{C}_+} \begin{pmatrix} [a](t) \cap \big([x](\cdot) + [y](\cdot)\big) \\ [x](t) \cap \big([a](\cdot) - [y](\cdot)\big) \\ [y](t) \cap \big([a](\cdot) - [x](\cdot)\big) \end{pmatrix}$$

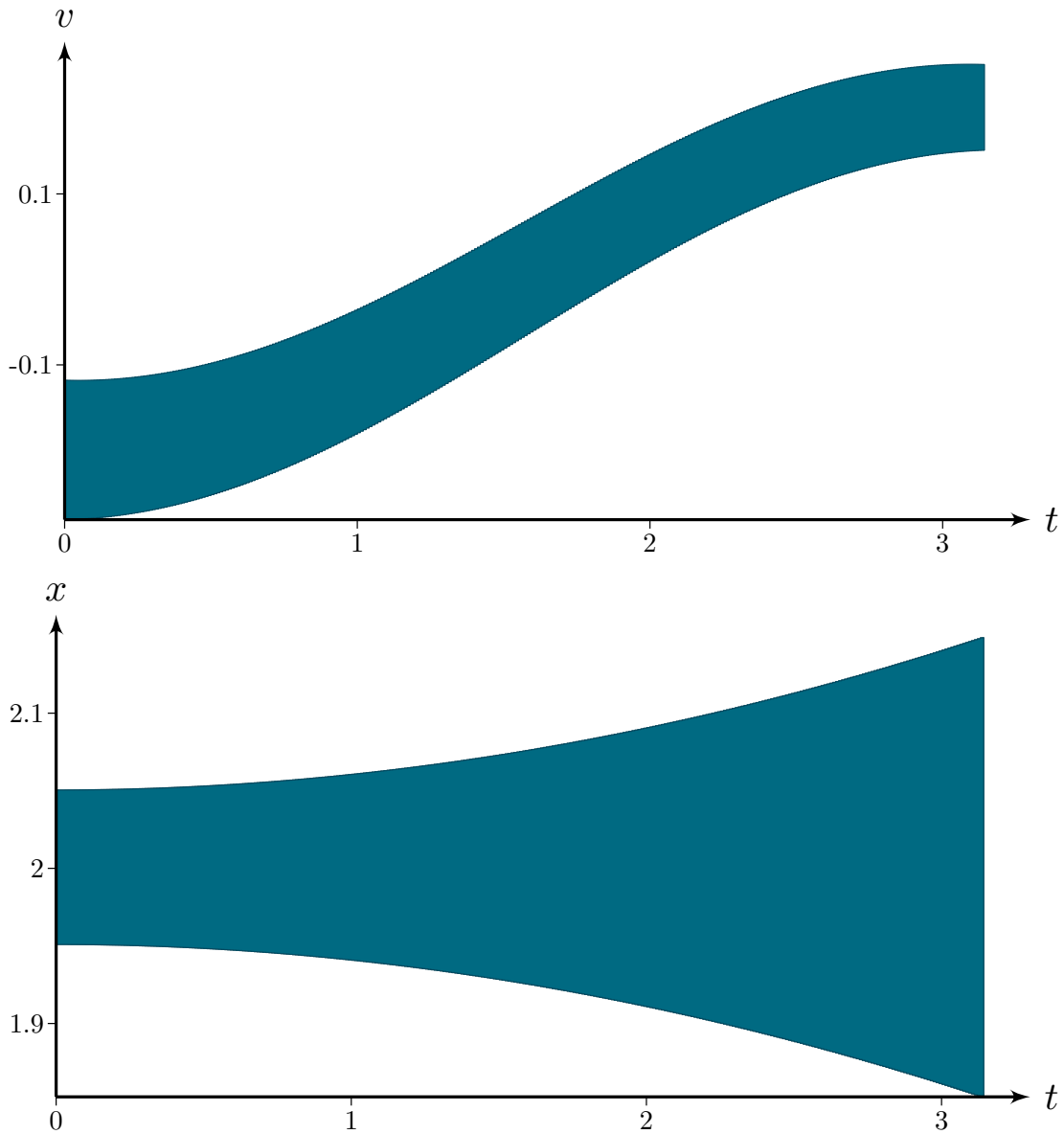
Appendices

Derivative constraint



Differential constraint:

- $\dot{\mathbf{x}}(\cdot) = \mathbf{v}(\cdot)$
- one trajectory and its derivative



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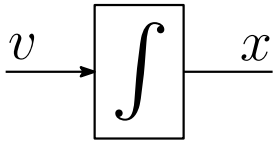

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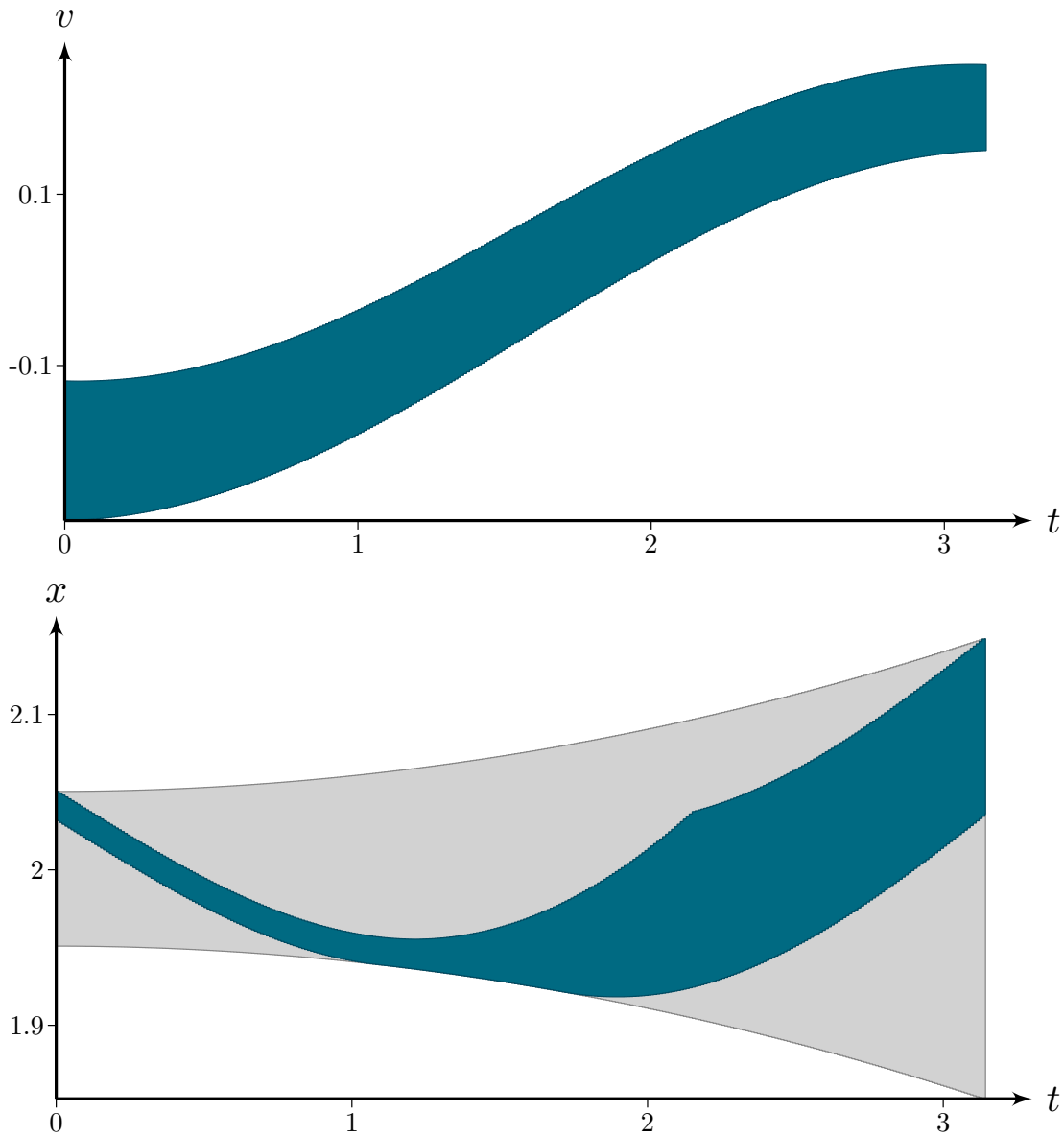
Contractor on tubes:

$$\mathcal{C}_{\frac{d}{dt}}\left([\mathbf{x}](\cdot), [\mathbf{v}](\cdot)\right)$$



Guaranteed computation of robot trajectories

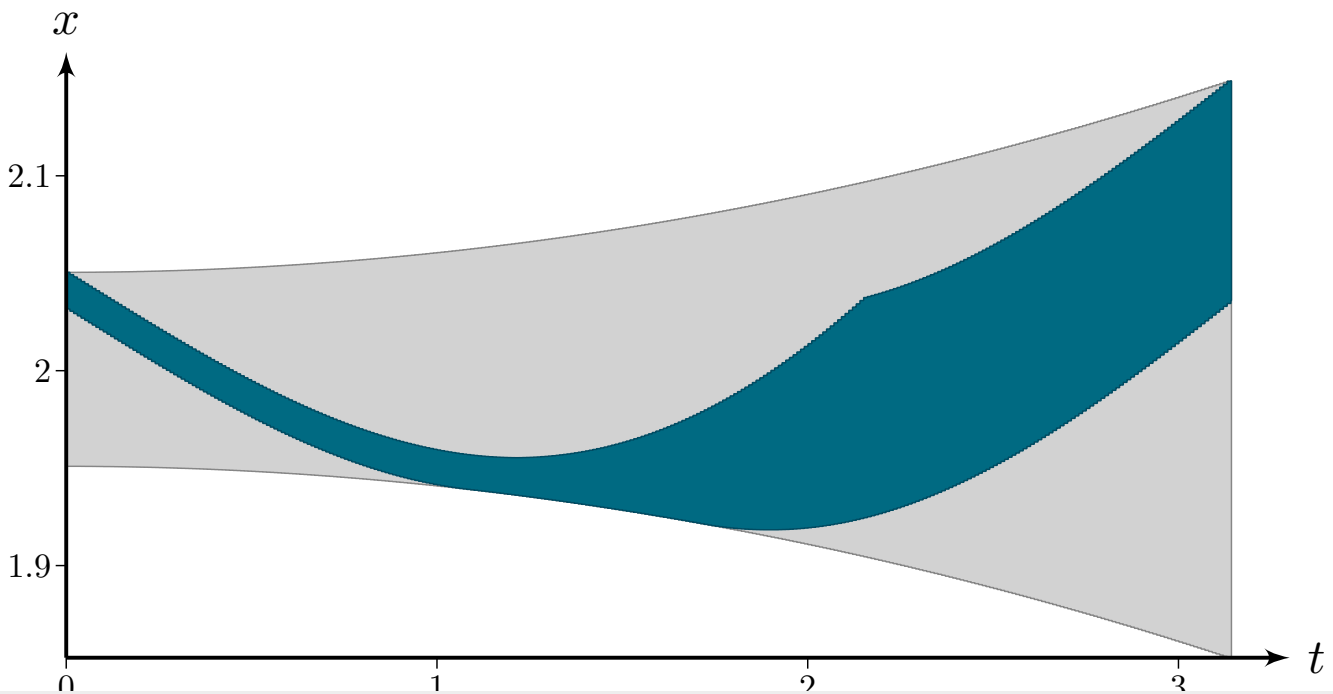
Rohou, Jaulin, Mihaylova, Le Bars, Veres

Appendices

Derivative constraint

Definition of the $\mathcal{C}_{\frac{d}{dt}}$ operator:

$$\begin{pmatrix} [x](t) \\ [v](t) \end{pmatrix} \xmapsto{\mathcal{C}_{\frac{d}{dt}}} \begin{pmatrix} \bigcap_{t_1=t_0}^{t_f} \left([x](t_1) + \int_{t_1}^{t} [v](\tau) d\tau \right) \\ [v](t) \end{pmatrix} \quad (1)$$



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Decomposition of $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$

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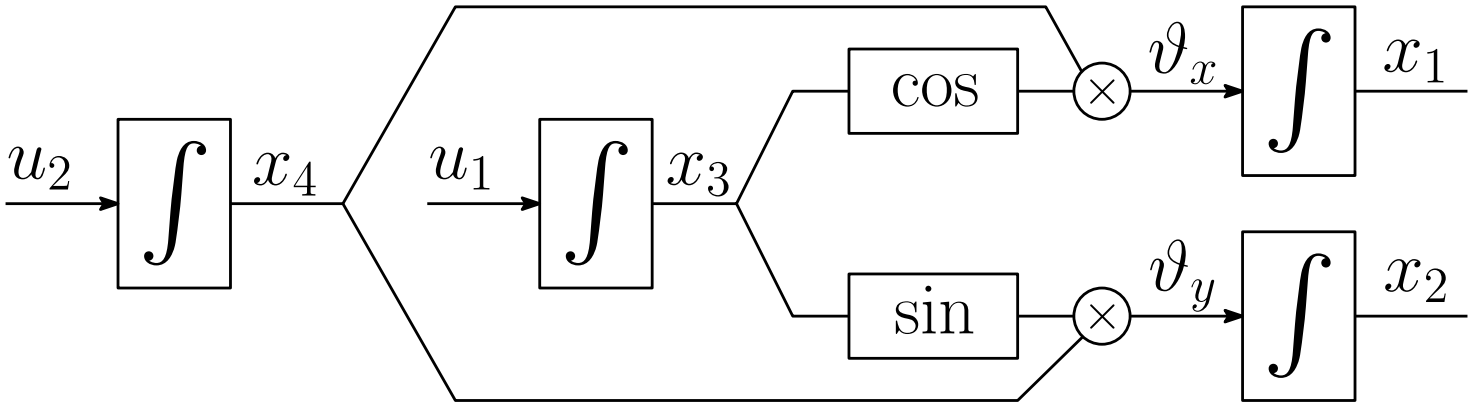
State equation:

$$\begin{cases} (i) & \dot{x}_1 = x_4 \cos(x_3) \\ (ii) & \dot{x}_2 = x_4 \sin(x_3) \\ (iii) & \dot{x}_3 = u_1 \\ (iv) & \dot{x}_4 = u_2 \end{cases}$$



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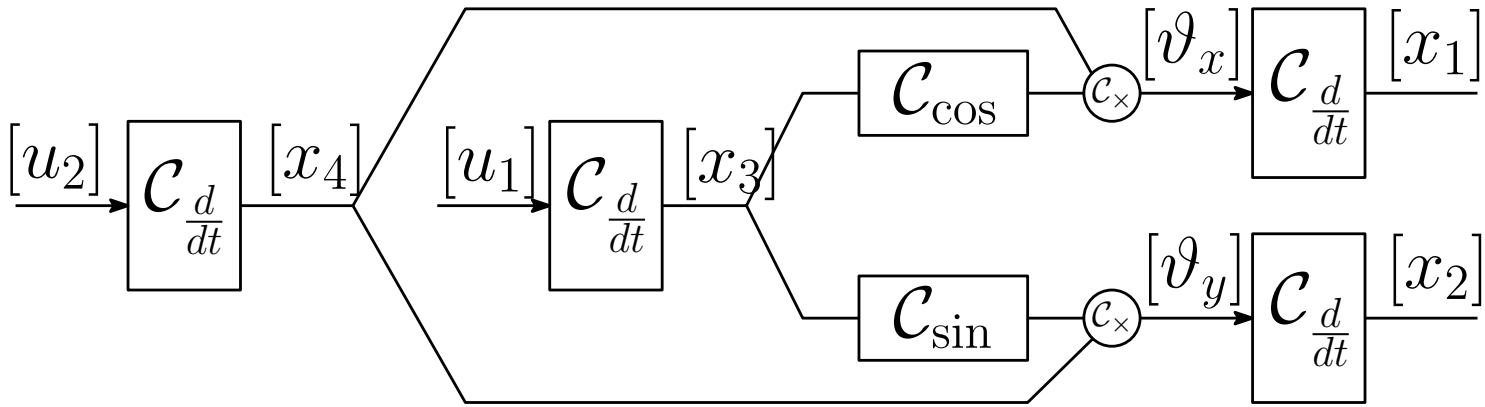

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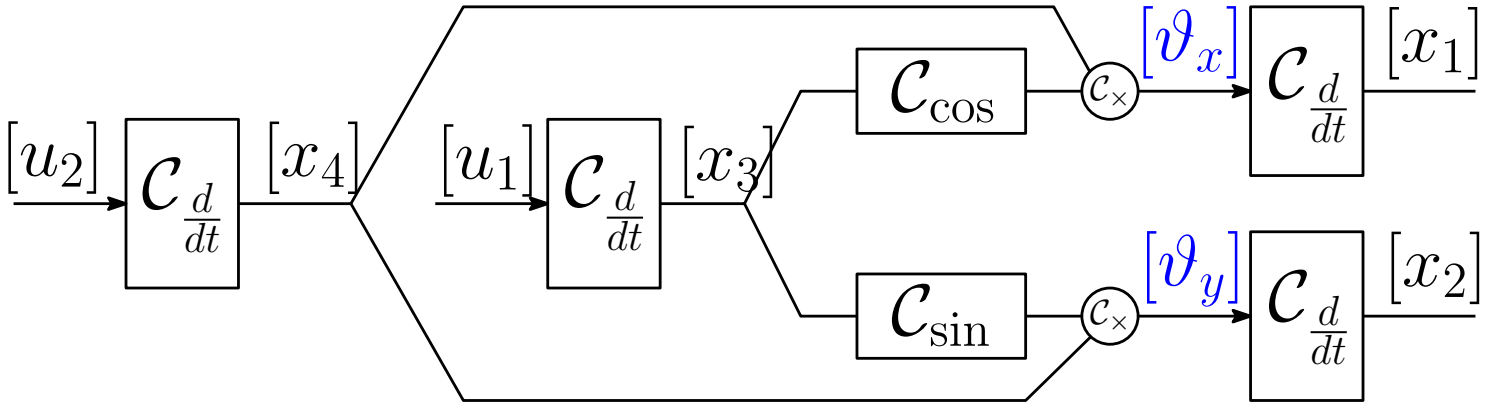
$$\left\{\begin{array}{rlcll} (i) & \dot{x}_1 & = & x_4\cos(x_3) & \Leftrightarrow \ \{\dot{x}_1 = \vartheta_x \ ; \ \vartheta_x = x_4\cos(x_3)\} \\ (ii) & \dot{x}_2 & = & x_4\sin(x_3) & \Leftrightarrow \ \{\dot{x}_2 = \vartheta_y \ ; \ \vartheta_y = x_4\sin(x_3)\} \\ (iii) & \dot{x}_3 & = & u_1 & \\ (iv) & \dot{x}_4 & = & u_2 & \end{array}\right.$$



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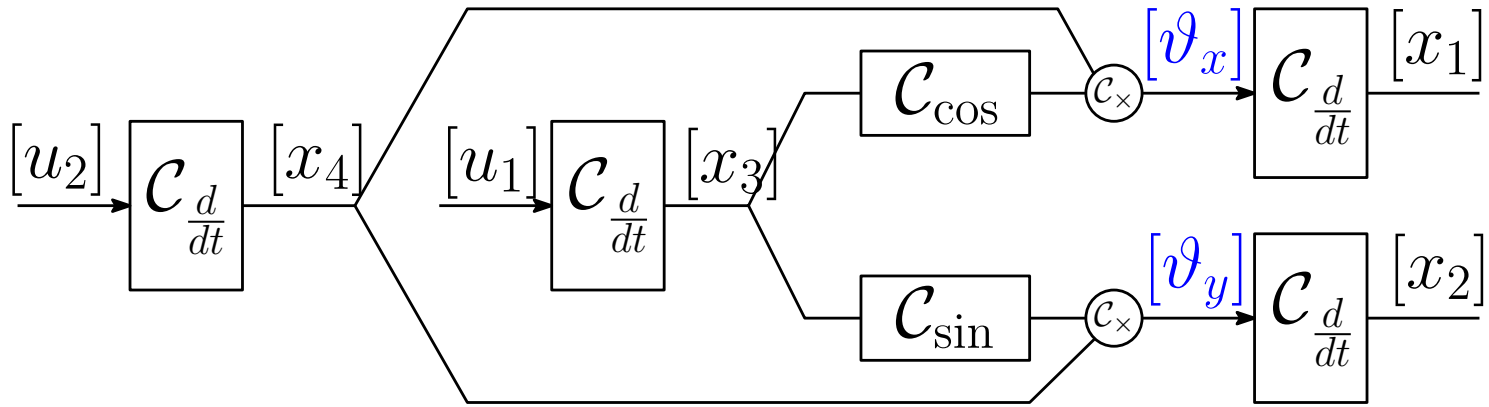

Involved operators: $\mathcal{C}_{\times}$, $\mathcal{C}_{\cos}$, $\mathcal{C}_{\sin}$, $\mathcal{C}_{\frac{d}{dt}}$

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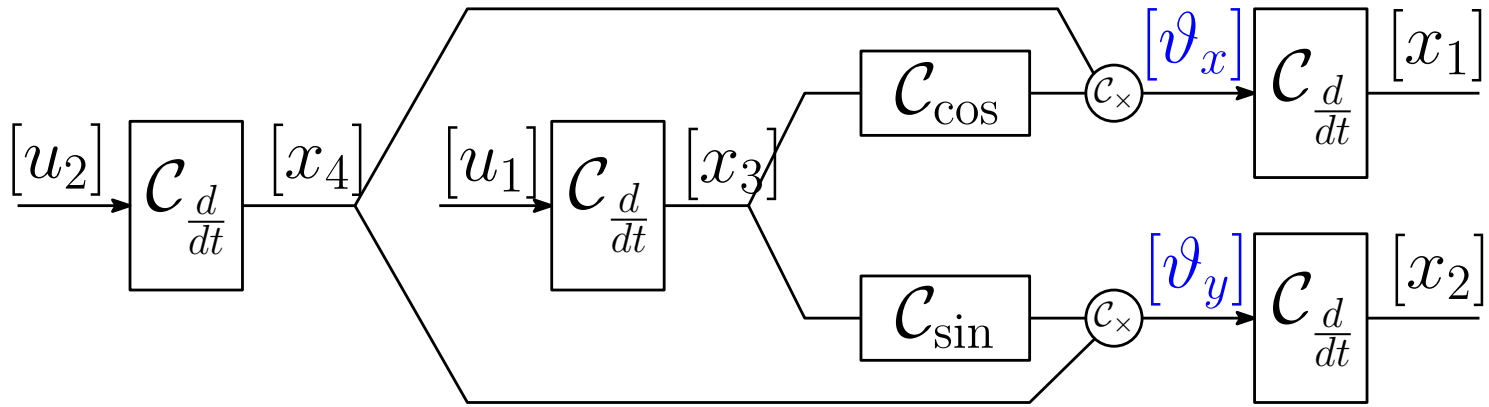

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Involved sets: $[\mathbf{x}](\cdot)$, $[\mathbf{u}](\cdot)$, $[\vartheta_x](\cdot)$, $[\vartheta_y](\cdot)$

Appendices

Decomposition and wrapping effects

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$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \iff \left\{ \begin{array}{l} \mathbf{v} = \mathbf{f}(\mathbf{x},\mathbf{u}) \\ \dot{\mathbf{x}} = \mathbf{v} \end{array} \right. \implies \left\{ \begin{array}{l} \mathcal{C}_{\mathbf{f}}\big([\mathbf{v}](\cdot),[\mathbf{x}](\cdot),[\mathbf{u}](\cdot)\big) \\ \mathcal{C}_{\frac{d}{dt}}\big([\mathbf{x}](\cdot),[\mathbf{v}](\cdot)\big) \end{array} \right.$$

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See also: more efficient contractors without decomposition, *e.g.* $\mathcal{C}_{\text{Lohner}}$ for dealing with $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$

- Safe and collaborative autonomous underwater docking

  Auguste Bourgois *PhD thesis*, 2021