Set-membership state estimation by solving data association

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Contredo, ENPC
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ICRA 2020

ICRA Robotics conference: May 31 – June 4

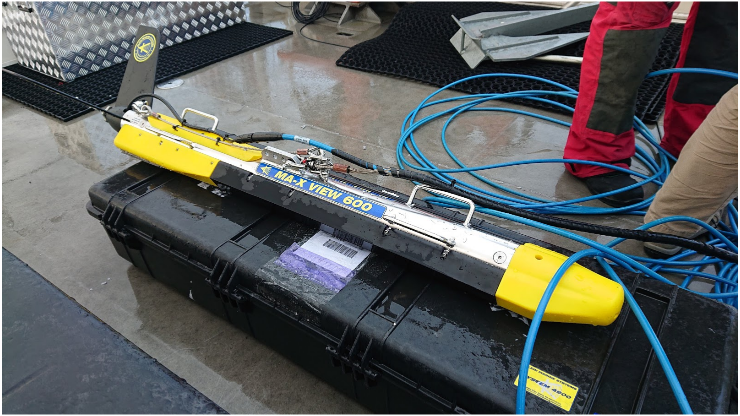
- **tutorial session of Tubex** (1 day)
- **paper presentation**

Program

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
SUNDAY, MAY 31	MONDAY, JUNE 1	TUESDAY, JUNE 2	WEDNESDAY, JUNE 3	THURSDAY, JUNE 4
Workshop & Tutorial sessions	Main Conferences Exhibition Competitions	Main Conferences Exhibition Competitions	Main Conferences Exhibition Competitions	Workshop & Tutorial sessions
Welcome Reception		Gala Evening	Farewell Reception	

Underwater robotics: sonar sensors

Side-scan sonars: to perceive objects on the seabed



A side-scan sonar Klein Max-View 600 during a demo in Brest.

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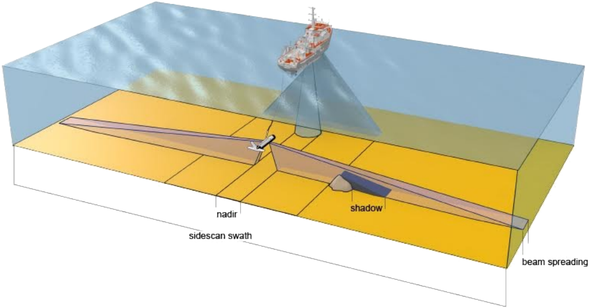

Schematic drawing illustrating the principles of a side-scan sonar.

Image from www.ga.gov.au

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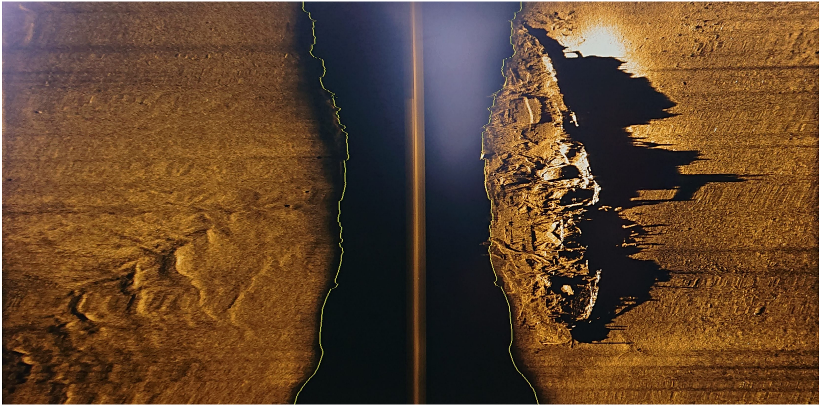
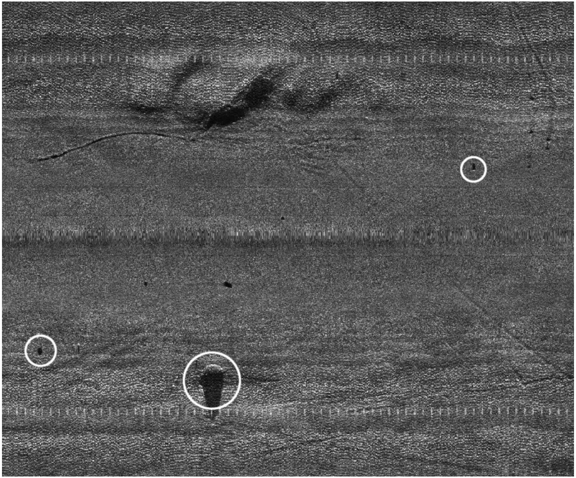


Perception of a wreck with the Klein Max-View 600.

Underwater robotics: sonar sensors

Now, onboard of an **Autonomous Underwater Vehicle** (AUV):



Detection of unidentifiable/indistinguishable rocks on the seabed.

Localization with data association: assumptions

1. **the map is static and made of point landmarks**

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state estimation $\rightleftarrows$ data association

State estimation with landmark perception

Classical localization problem:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) & \text{(evolution equation)} \\ \mathbf{g}\big(\mathbf{x}(t_i)\big) \in [\mathbf{y}](t_i) & \text{(observation equation)} \end{cases}$$

with:

- $\mathbf{x}$, unknown state vector
- $\mathbf{y}$, an output measurement vector
  related to the perception of a landmark
- $\mathbf{u}$, an input measurement vector

It is also known that $\forall t, \mathbf{u}(t) \in [\mathbf{u}](t)$ and $\mathbf{y}(t) \in [\mathbf{y}](t)$.

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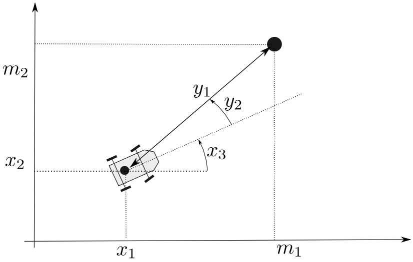
State estimation with landmark perception: example

Example:

- a robot at position $(x_1, x_2)^\intercal$ with a heading x_3
- a landmark $\mathbf{m}$ located at $(4, 5)$
- the corresponding measurement vector is composed of
  - the distance y_1
  - the bearing y_2



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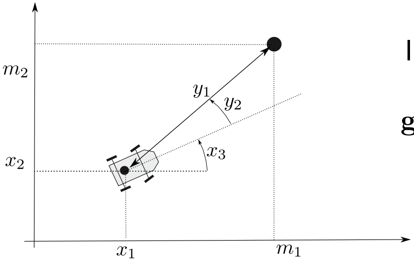

In such case, we have:

$$\mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{pmatrix} x_1 + y_1 \cdot \cos(x_3 + y_2) - 4 \\ x_2 + y_1 \cdot \sin(x_3 + y_2) - 5 \end{pmatrix}$$

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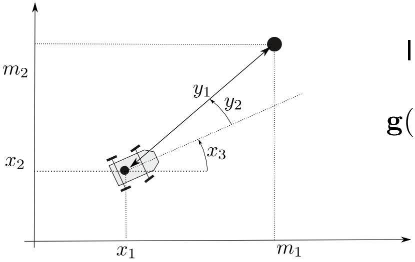

In such case, we have:

$$\mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{m}) = \begin{pmatrix} x_1 + y_1 \cdot \cos(x_3 + y_2) - m_1 \\ x_2 + y_1 \cdot \sin(x_3 + y_2) - m_2 \end{pmatrix}$$

State estimation with landmark perception

In the general case we have:

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with $[\mathbf{m}](t_i)$ the bounded position of the beacon perceived at time t_i.

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Problem: when several landmarks $\mathbf{m}_1,\dots,\mathbf{m}_l$ can be observed,

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$$\big(\mathbf{m}(t_i)\in[\mathbf{m}_1]\big)\vee\dots\vee\big(\mathbf{m}(t_i)\in[\mathbf{m}_\ell]\big)$$

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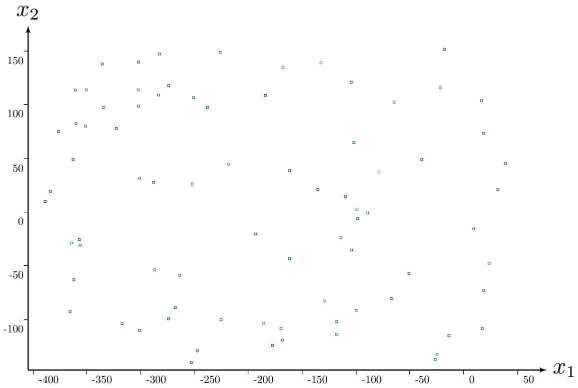
Localization with data association: formalization

$$\left\{\begin{array}{ll}\mathbf{m}(t_i) \in \mathbb{M} & \text{(association constraint)} \\ & \\ & \end{array}\right.$$



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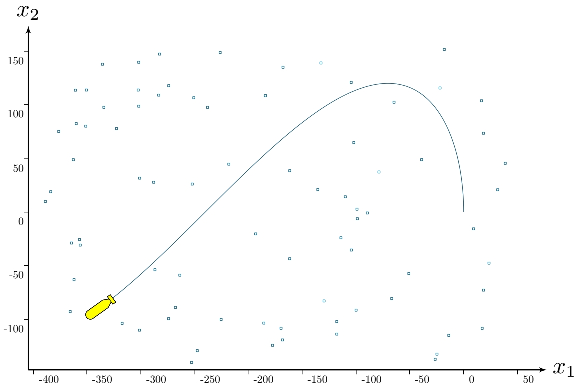

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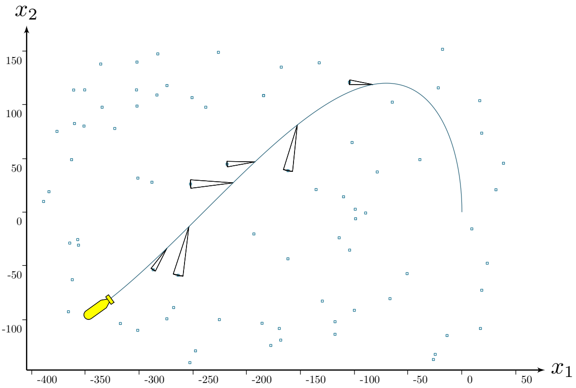

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Provides a test case with **heterogeneous constraints**:

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Decomposition

We recall the problem:

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Association constraint: constellation contractor

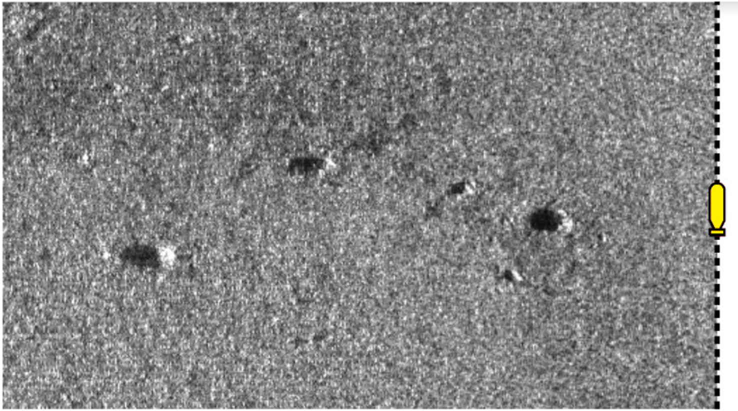
Let us consider a constellation of ℓ points $\mathbb{M} = \{\mathbf{m}_1, \dots, \mathbf{m}_\ell\}$ of $\mathbb{R}^d$ and a box $[\mathbf{x}] \in \mathbb{IR}^d$. We want to compute the smallest box $\mathcal{C}([\mathbf{x}])$ containing $\mathbb{M} \cap [\mathbf{x}]$, or equivalently:

$$\mathcal{C}([\mathbf{x}]) = \bigsqcup_i \big([\mathbf{x}] \cap \{\mathbf{m}_i\} \big), \tag{3}$$

where $\bigsqcup$, called *squared union*, returns the smallest box enclosing the union of its arguments.

Constraint network on data

Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



Perception of the seabed with a side-scan sonar.

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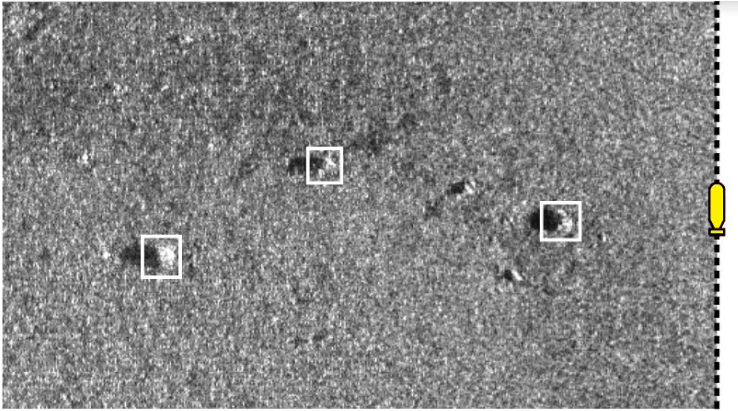

Seamarks are already known with some uncertainty.

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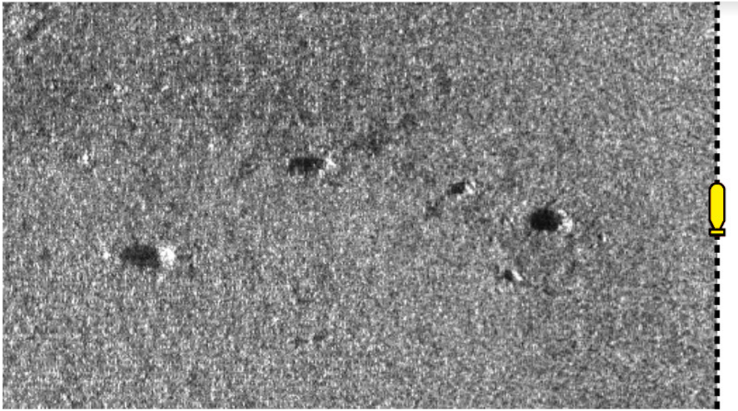
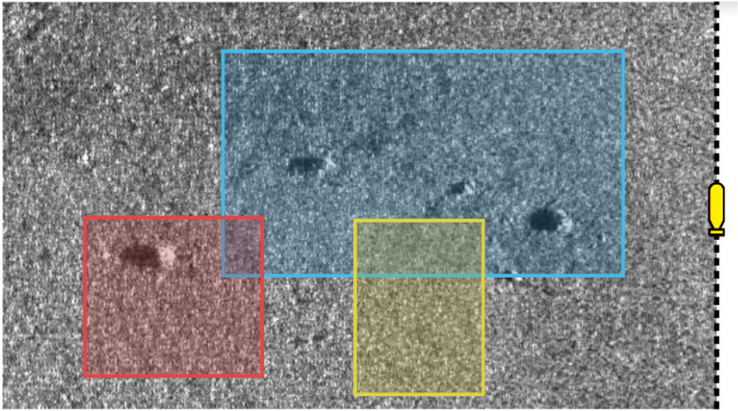
Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



Some of the rocks may be observed by the robot with its sonar.

Constraint network on data

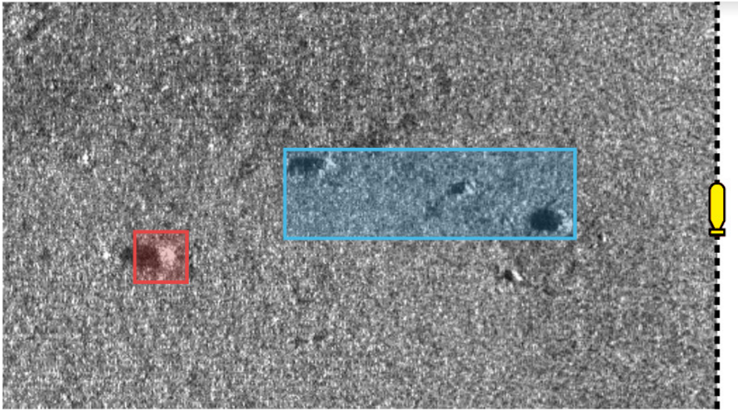
Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



The position of the rock is first estimated from robot's position estimate.

Constraint network on data

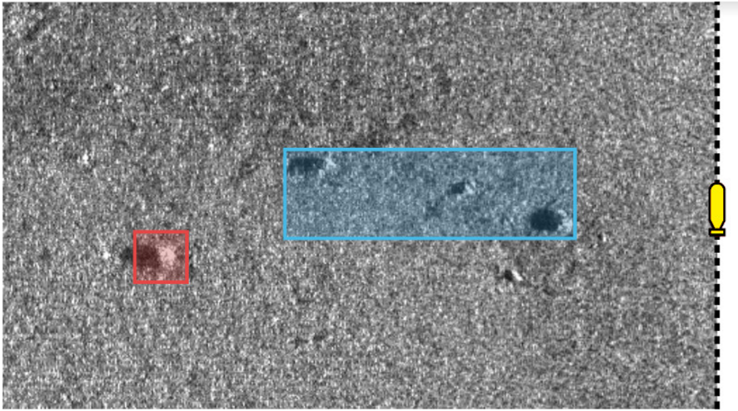
Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



Then the position of the rock is contracted from the known map.

Constraint network on data

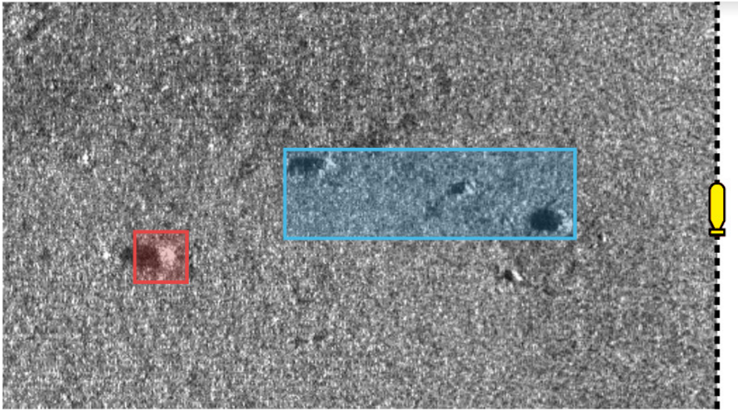
Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



If the boxed-position is a singleton, then the rock is *identified*.

Constraint network on data

Constraint $\mathbf{m}(t_i) \in \mathbb{M}$:
An observation $\mathbf{m}(t_i)$ is related to one of the known seamarks $\mathbb{M}$.



In any cases, the boxed-positions of the rocks allow localization updates.

Application

- Daurade: Autonomous Underwater Vehicle (AUV)
- weight: 1010kg – length: 5m – max depth: 300m

Special thanks to DGA-TN Brest (formerly GESMA)

Application

Video

Results on actual data

Map: 133 objects. 54 detections in sonar images.

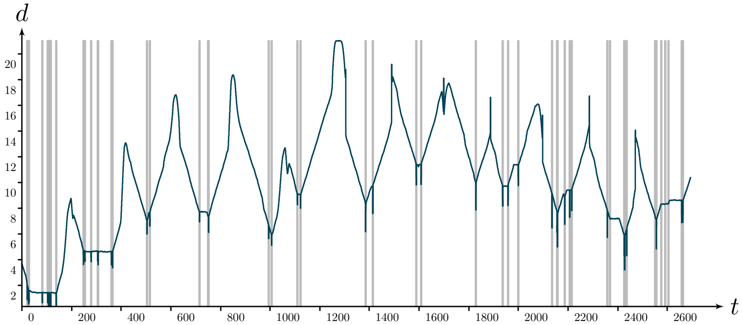
Table: Numerical results of the iterative localization algorithm.

#	time(s)	#min	#max	#ok
1	0.278	133	133	0
2	0.271	14	64	0
3	0.268	5	52	0
4	0.266	1	34	2
5	0.271	1	16	39
6	0.267	1	4	48
7	0.266	1	3	49
8	0.266	1	3	50
9	0.266	1	2	51

- #min: minimal number of objects included in the $[\mathbf{m}](t_i)$
- #max: maximal number of objects included in the $[\mathbf{m}](t_i)$
- #ok: number of correct associations

Results on actual data

The initial position of the robot is **not known before the contractions**, and is finally estimated with an **error of 3.6m** in the worst case:



$d = w([\mathbf{p}])$ when reaching a contracting fixed point. Computation time < 2.5s.

d: diameter of each box $[x_1](t) \times [x_2](t)$, *i.e.* localization error in the very worst case.