Exhaustive Interval-based 2-D Shape Registration Under Similarity Transformation

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Keywords: Procrustes Analysis, Registration, Separators, Interval Analysis

Motivation

The registration of two sets aims at getting all the transformation parameters that map them. This work covers the case of bounded subsets of $\mathbb{R}^2$ and similarity transforms, consisting of a composition of translation, rotation and uniform scaling. The classical Procrustes-like approach [1] to this problem is to switch from the original 4D transformation to a 1D orientation problem. Since rotational symmetries induce several solutions for the orientation problem, state-of-the-art methods based on local optimization are *de facto* unsuitable when they appear.

We propose a set-membership approach capable of approximating all solutions by reproducing the Procrustes-like approach and solving the orientation problem, thanks to set manipulation using descriptive operators called *separators* [2].

A Procrustes set-membership approach

The algorithmic sequence used to perform the registration of two sets $\mathbb{A}$ and $\mathbb{B}$ starts by centering and normalization steps, followed by rotational mapping. The usual centers and scaling factor come from the fact that $\mathbb{A}$ and $\mathbb{B}$ are represented as point clouds in state-of-the-art methods. In our set-membership approach, we have to find appropriate substitute for these (centers and scaling factors) parameters.

Proposition 1. *The minimum enclosing circle [3] provides different but suitable parameters for the centering and normalization.*

Thus, the whole algorithm consists of the following steps:

1. Find the minimum circles centers $\mathbf{c}^{\mathbb{A}}_{1,2}$, $\mathbf{c}^{\mathbb{B}}_{1,2}$ and radii $c^{\mathbb{A}}_3$, $c^{\mathbb{B}}_3$. (Fig. 1.A),
2. Describe the normalized sets $\mathbb{A}_N := \left(\mathbb{A} - \mathbf{c}^{\mathbb{A}}_{1,2}\right)/c^{\mathbb{A}}_3$, $\mathbb{B}_N := \left(\mathbb{B} - \mathbf{c}^{\mathbb{B}}_{1,2}\right)/c^{\mathbb{B}}_3$. (Fig. 1.B–C),
3. Describe the set of possible rotations $\Theta = \{\theta \mid \mathbb{B}_N = \mathrm{R}_\theta \cdot \mathbb{A}_N\}$, where R_θ is the rotation matrix of parameter θ. (Fig. 1.D).

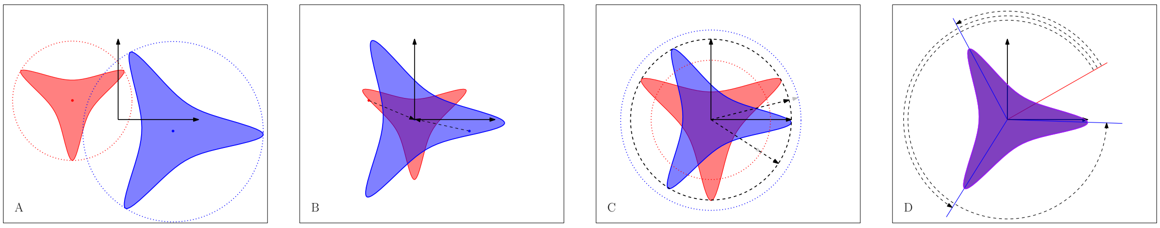

Figure 1: Algorithm in 4 steps: A: Identification of smallest circles (centers and sizes). B: Centering of sets. C: Normalization of sets. D: Identification of rotation parameters.

Approximation using separators

Usually involved in a branch and separate algorithm, a separator [2] is an algorithmic operator capable of representing a set: from an initial domain, it can remove non-solution parts (contraction) as well as parts containing only solutions.

Using the Codac library (`www.codac.io`, [4]) which offers a catalog of separators, it is possible to define a new separator as a sequence of separators handling constraints involving sets and set operations.

To implement the Procrustes approach, we first give the system of constraints that defines the problem, and for each variables we propose a separator consistent with it.

As a result, the algorithm returns separators describing respectively all possible sets Θ, $\mathbb{K}$, $\mathbb{T}$ of rotations θ, (unique) uniform scaling k and translations $\mathbf{t}$.

Finally, by slightly modifying the chain of separators, we will see that the same approach allows similarity transformations to be approximated completely, including reflection symmetries.

References

[1] P.H. Schönemann and R.M. Carroll. Fitting one matrix to another under choice of a central dilation and a rigid motion. *Psychometrika*, vol 35, 245–255, 1970.

[2] L. Jaulin and B. Desrochers. Introduction to the Algebra of Separators with Application to Path Planning. *Engineering Applications of Artificial Intelligence*, vol 33, 141–147, 2014.

[3] J.J. Sylvester. A question in the geometry of situation. *Quarterly Journal of Pure and Applied Mathematics*, vol 1, 79–80, 1857.

[4] S. Rohou and B. Desrochers and F. Le Bars. The Codac Library. *Acta Cybernetica, Special Issue of SWIM 2022*, vol 26, 871–887, 2024.