Verifying the existence of loops in robot trajectories

Simon Rohou*[1], Peter Franek[2], Clément Aubry[3], and Luc Jaulin[1]

[1]ENSTA Bretagne, Lab-STICC, UMR CNRS 6285, Brest, France
[2]IST Austria, Klosterneuburg, Austria
[2]ISEN Brest, France

Keywords: mobile robotics, SLAM, loop detection, interval analysis, topological degree, tubes

Introduction

We present a reliable method to verify the existence of loops along the uncertain trajectory of a robot, based on proprioceptive measurements only [1], within a bounded-error context.

The loop closure detection is one of the key points in Simultaneous Localization And Mapping (SLAM) methods, especially in homogeneous environments with difficult scenes recognitions.

The approach we propose [3] is fast, reliable and could be coupled with conventional SLAM algorithms to reliably reduce their computing burden, thus improving the localization and mapping processes in the most challenging environments such as unexplored underwater extents.

Loops

An example of loop is given in Figure 1, with a mobile robot that came back at time t_2 to a previous position reached at t_1. In this work, for a given trajectory, a loop is defined temporally as a 2d vector $\mathbf{t} = (t_1, t_2)^\intercal$ such that $\mathbf{f}(\mathbf{t}) = \mathbf{0}$ with

$$\mathbf{f}(\mathbf{t}) = \int_{t_1}^{t_2} \mathbf{v}(\tau)d\tau,$$

a function describing robot's move from t_1 to t_2, based on its absolute velocities $\mathbf{v}(t) \in \mathbb{R}^2$.

*Corresponding author.

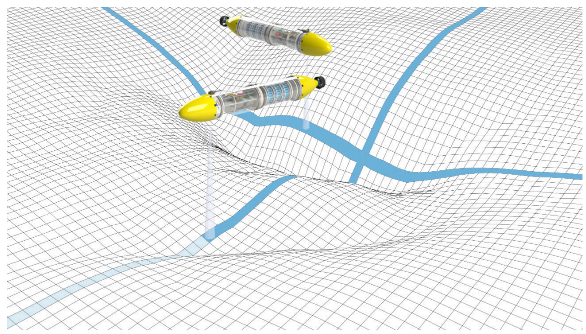

Figure 1: An underwater robot exploring its environment, before and after performing a loop. The robot trajectory is projected in blue on the sea-floor.

Detection *vs.* verification

A distinction has to be made between the *detection* and the *verification* of a loop $\mathbf{t}$. Considering a set of feasible trajectories, some of them may cross themselves at some point; this will lead to a *detection*. In addition, when we verify that all the feasible trajectories are looped, then we can speak about a *loop proof* since a loop occurs whatever the considered uncertainties coming from the sensors. Figure 2 provides an illustration of this distinction.

In a reliable context, any feasible trajectory has to be considered, based on the uncertainties coming from the measurements of $\mathbf{v}(t)$. Tubes are used for this purpose.

The set-membership method we propose stands on tubes $[\mathbf{x}](\cdot)$, see Figure 3, that are intervals of trajectories $\mathbf{x}^-(\cdot)$ and $\mathbf{x}^+(\cdot)$ such that $\mathbf{x}^-(t) \leqslant \mathbf{x}^+(t)\ \forall t$. Most of the classical mathematics operations we know on intervals can be extended to tubes. In this work, in-

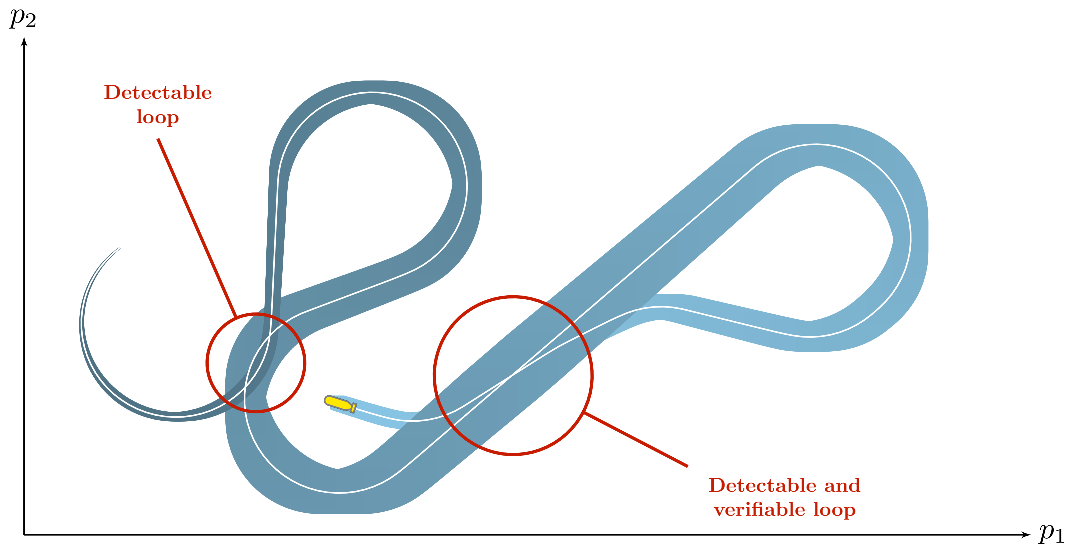

Figure 2: Only one loop can be *verified* in this set of trajectories, while at least two feasible loops are *detected*. Indeed, there exist trajectories that loop only once.

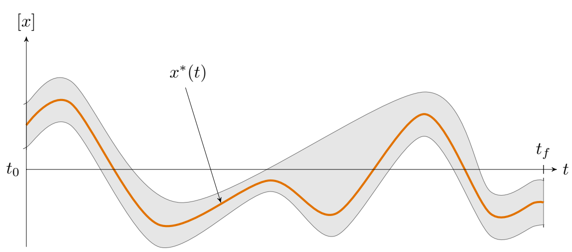

Figure 3: A one-dimensional tube enclosing an uncertain trajectory.

tegral computations of tubes will allow to approximate all feasible loops $\mathbf{t}$: so-called *loop sets* denoted by $\mathbb{T}$. From tubes, we can compute reliable inclusion functions $[\mathbf{f}]$ of $\mathbf{f}$. Then:

$$\mathbb{T} = \{\mathbf{t} \mid \mathbf{0} \in [\mathbf{f}](\mathbf{t})\}\,.$$

Formally, we want to verify that $\forall \mathbf{f} \in [\mathbf{f}], \exists \mathbf{t} \in \mathbb{T}$ such that $\mathbf{f}(\mathbf{t}) = \mathbf{0}$, which is equivalent to verifying a zero of an unknown function $\mathbf{f} \in [\mathbf{f}]$ on $\mathbb{T}$.

Topological degree

For this zero verification, we employ the notion of *topological degree* that originates in the field of differential topology. An algorithm exists [2] to verify a zero of an uncertain function $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ known to belong to an inclusion function $[\mathbf{f}] : \mathbb{IR}^2 \to \mathbb{IR}^2$.

We will show its use as a powerful verification tool for proving robot loops. This will be demonstrated on actual datasets from real missions involving autonomous underwater vehicles at sea.

References

[1] C. Aubry, R. Desmare, and L. Jaulin. Loop detection of mobile robots using interval analysis. *Automatica*, 49(2):463–470, Feb. 2013.

[2] P. Franek and S. Ratschan. Effective topological degree computation based on interval arithmetic. *Mathematics of Computation*, 84(293):1265–1290, Sept. 2014.

[3] S. Rohou, P. Franek, C. Aubry, and L. Jaulin. Proving the existence of loops in robot trajectories. *The International Journal of Robotics Research*, 37(12):1500–1516, 2018.