

Reliable propagation of time uncertainties in dynamical systems

Simon Rohou¹, Luc Jaulin², Lyudmila Mihaylova³, Fabrice Le Bars² and Sandor M. Veres³

¹ IMT Atlantique, LS2N, Nantes, France
`simon.rohou@imt-atlantique.fr`

² ENSTA Bretagne, Lab-STICC, Brest, France

³ The University of Sheffield, Sheffield, United Kingdom

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Problem:

We consider the guaranteed non-linear state estimation of a robot described by the following state equations:

$$\begin{cases} \dot{\mathbf{x}}(\cdot) = \mathbf{f}(\mathbf{x}(\cdot), \mathbf{u}(\cdot)), & (1a) \\ \mathbf{z}_i = \mathbf{g}(\mathbf{x}(t_i)), & (1b) \end{cases}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector representing the system at time t and $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ a non-linear function depicting the evolution of the system based on input vectors $\mathbf{u}(t) \in \mathbb{R}^m$. The observation function $\mathbf{g} : \mathbb{R}^n \rightarrow \mathbb{R}^p$ may also be non-linear. The $t_i \in \mathbb{R}$, $i \in \mathbb{N}$, are measurement times and the $\mathbf{z}_i \in \mathbb{R}^p$ are the related outputs. In this problem, the t_i are subject to strong uncertainties, as well as the other variables \mathbf{z}_i , $\mathbf{x}(\cdot)$, $\mathbf{u}(\cdot)$.

Approach:

We will deal with this dynamical system by using a constraint programming approach. In a nutshell, the method consists in breaking

the Equations (1a),(1b) into a set of elementary constraints that must be satisfied by the variables of the problem. In our case, the constraints may be non-linear or differential equations and the variables are vectors (e.g. $\mathbf{z}_i \in \mathbb{R}^p$) or trajectories (e.g. $\mathbf{x}(\cdot) \in \mathbb{R} \rightarrow \mathbb{R}^n$).

The variables are known to belong to some domains. For vectors of \mathbb{R}^n , we will use boxes in $\mathbb{I}\mathbb{R}^n$. For trajectories, we will use *tubes* denoted by $[\mathbf{x}](\cdot) : \mathbb{R} \rightarrow \mathbb{I}\mathbb{R}^n$. Constraints will be applied on these domains by means of operators called *contractors* \mathcal{C} [2].

Contribution:

The problem involves algebraic and differential constraints on trajectories such as $a(\cdot) = \sin(b(\cdot))$ or $\dot{x}(\cdot) = v(\cdot)$. The related contractors have been the subject of some work. It remains to deal with the following elementary constraint denoted by $\mathcal{L}_{\text{eval}}$:

$$\mathcal{L}_{\text{eval}}(t, z, y(\cdot), w(\cdot)) : \{z = y(t), \dot{y}(\cdot) = w(\cdot)\} \quad (2)$$

with $t \in [t], z \in [z], y(\cdot) \in [y](\cdot), w(\cdot) \in [w](\cdot)$. Here, $w(\cdot)$ is the derivative of the signal to be evaluated. The problem is complex as the uncertainties related to t are difficult to propagate through the differential equation.

We propose the related contractor $\mathcal{C}_{\text{eval}}$, see [1], that will reliably reduce the sets of feasible solutions by contracting the bounds of the tube $[y](\cdot)$ and the intervals $[t]$ and $[z]$. Figure 1 provides an illustration of the evaluation of a trajectory in a bounded-error context.

References

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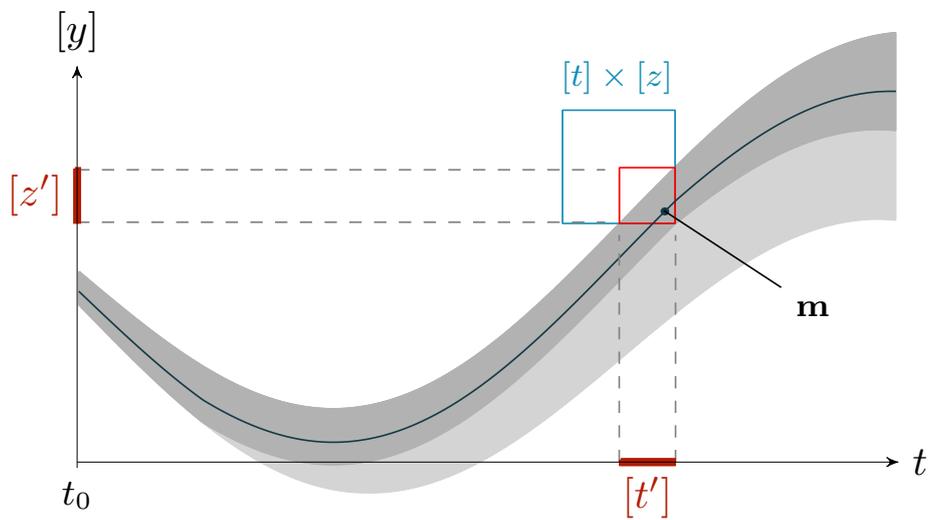


Figure 1: Evaluation on a tube $[y](\cdot)$. A given measurement $\mathbf{m} \in \mathbb{R}^2$, pictured by a black dot, is known to belong to the blue box $[t] \times [z]$. The tube is contracted by means of $\mathcal{C}_{\text{eval}}$; the contracted part is depicted in light gray. Meanwhile, the bounded observation itself is contracted to $[t'] \times [z']$ with $[t'] \subseteq [t]$ and $[z'] \subseteq [z]$. This is illustrated by the red box. The dark line is an example of a compliant trajectory.