Tube Programming
Applied to State Estimation

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Problem. We consider the guaranteed non-linear state estimation of a robot described by its state $\mathbf{x} \in \mathbb{R}^n$ and measuring distances to beacons, see Fig. 1. $\mathbf{x}^\intercal = \{x, y, \theta, v\}$. System's description is given by:

$$\mathcal{R}\left\{\begin{array}{rcl}\dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x},\mathbf{u}) \\ r_i(t_i) & = & g_j(\mathbf{x}(t_i))\end{array}\right. \tag{1}$$

With $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, the continuous *evolution* function based on robot's state $\mathbf{x}$ and input $\mathbf{u} \in \mathbb{R}^m$, and $g_j : \mathbb{R}^n \to \mathbb{R}$, a sporadic *observation* function giving a range value $r_i \in \mathbb{R}$ between the robot and the j-th beacon at time t_i.

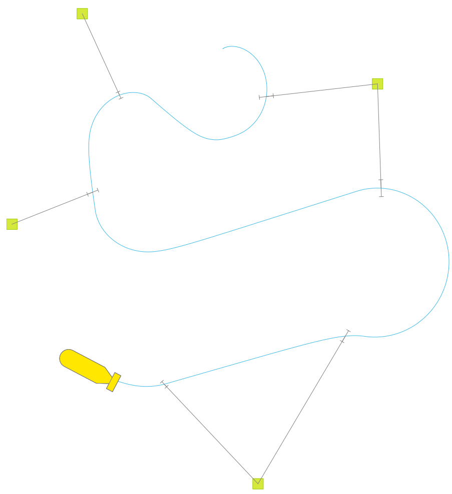

Figure 1: Localization of robot $\mathcal{R}$ among beacons

Main approach. In a set membership approach, $\mathbf{x}$ and $\dot{\mathbf{x}}$ respectively belong to bounded signals evolving with time: $\forall t\ ,\ \mathbf{x}(t) \in [\mathbf{x}](t)\ ,\ \dot{\mathbf{x}}(t) \in [\dot{\mathbf{x}}](t)$. These *trajectories* are represented with *tubes* [1], see Fig. 2. The more uncertain a trajectory is, the thicker the tube containing it will be. When trajectory's estimation is improved, the tube needs to be contracted, thus becoming thinner. To this end, we propose to break down the problem into several elementary constraints involving *contractors* on tubes.

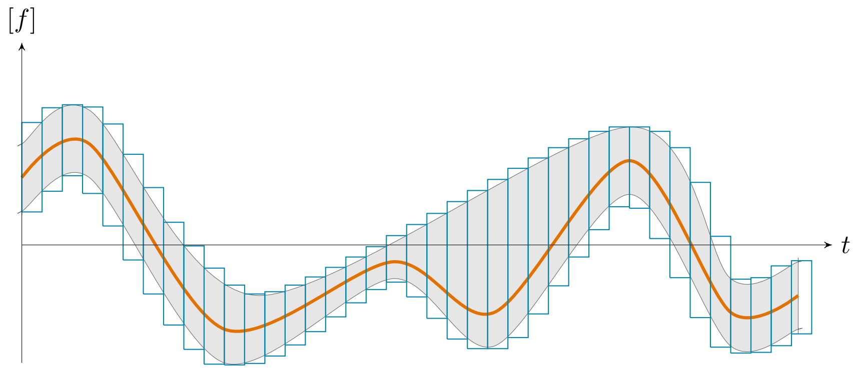

Figure 2: A tube $[f](\cdot)$ implemented with a set of slices

Break down. Let us consider the given equations: $\dot{x} = v \cdot cos(\theta)$, $\sqrt{x(t_1)^2 + y(t_1)^2} = r_1$, the following constraints can be established:

$$\text{continuous constraints} \begin{cases} a & = & cos(\theta) \\ b & = & v \cdot a \\ x & = & \int b \end{cases}$$

$$\text{sporadic constraint} \left\{ \; x(t_1) \quad = \quad \sqrt{r_1^2 - y(t_1)^2} \right.$$

Tube contraction. Our contribution is to perform a reliable state estimation with a simple and general method involving continuous or fleeting [2] constraints on tubes. In the continuous case, constraints are managed with tube arithmetic: variables a and b are tubes too. $[x](\cdot) = [x](\cdot) \cap \int [b](\cdot)$. In the sporadic case, we will show that to be compliant with tube's representation $[x](\cdot)$, such a contraction can only be done considering the derivative tube $[\dot{x}](\cdot)$.

References:

[1] A. Bethencourt, L. Jaulin, Solving non-linear constraint satisfaction problems involving time-dependant functions, *Mathematics in Computer Science*, 2014.

[2] F. Le Bars, J. Sliwka, O. Reynet, L. Jaulin, State estimation with fleeting data, *Automatica*, 2012.