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Robust Polygon-Based Localization

Guilherme S. Franco¹

¹ENSTA Bretagne, Lab-STICC

11 December 2017

Dynamic Localization

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Outline

D Motivation

- 2 Static Localization
 - Map Representation
 - Polygon-Based Localization
 - Experiments
- Oynamic Localization
 - Constraints Network
 - Experiments

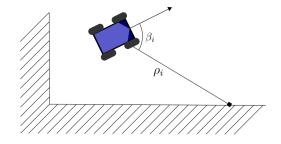


Dynamic Localization

Motivation

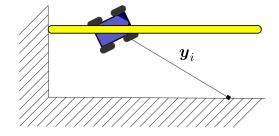
Localization problem.

Consider a pose estimation problem based on rangefinder readings.



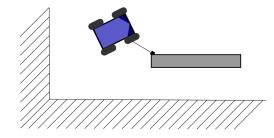
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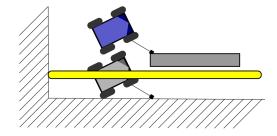


Dynamic Localization

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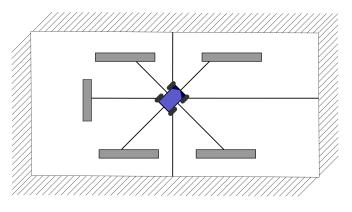
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Dynamic Localization

Motivation

Localization problem.

Consider a pose estimation problem based on rangefinder readings.



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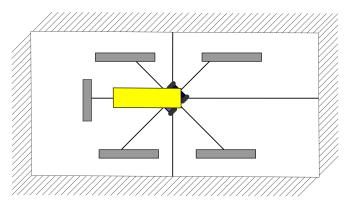
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Dynamic Localization

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Outilne

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- Map Representation
- Polygon-Based Localization
- Experiments

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Conclusion

Static Localization

$h(x, y_i) \in \mathbb{M}$

where:

- $\mathbf{x} = (x, y, \theta)$ is the robot's pose to be estimated
- $\mathbf{y}_i = (\rho_i, \beta_i)$, given by a distance ρ_i and bearing β_i
- M is the known map.

Dynamic Localization

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Conclusion

Static Localization

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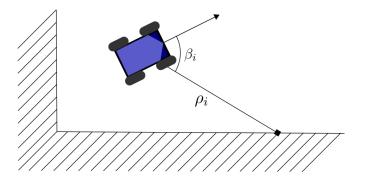
$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}_i) = \boldsymbol{h}_i(\boldsymbol{x})$$

Dynamic Localization

Conclusion

Static Localization

$$\boldsymbol{h}_{i}(\boldsymbol{x}) = \left[\begin{array}{c} \boldsymbol{x} + \rho_{i} \cdot \cos(\beta_{i} + \theta) \\ \boldsymbol{y} + \rho_{i} \cdot \sin(\beta_{i} + \theta) \end{array}\right]$$

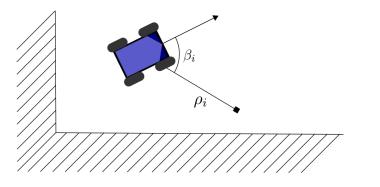


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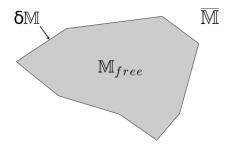
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Conclusion

Map Representation - Static Localization



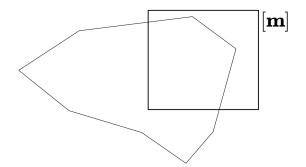
 $\mathbb{M} = \delta \mathbb{M} \cup \mathbb{M}_{\textit{free}}$

Static Localization

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Map Representation - Segment Contractor



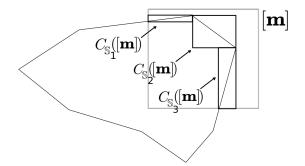
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Static Localization

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Map Representation - Segment Contractor



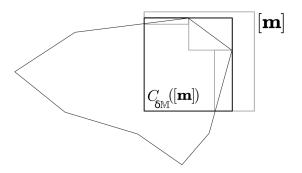
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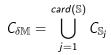
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Map Representation - Segment Contractor





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Conclusion

Map Representation - Segment Constraints

$$c_{1}: det(\boldsymbol{b}_{j} - \boldsymbol{a}_{j}, \boldsymbol{a}_{j} - \boldsymbol{m}) = \begin{vmatrix} b_{j,0} - a_{j,0}, & a_{j,0} - m_{0} \\ b_{j,1} - a_{j,1}, & a_{j,1} - m_{1} \end{vmatrix} = 0, \\ c_{2}: min(\boldsymbol{a}_{j}, \boldsymbol{b}_{j}) \leq \boldsymbol{m} \leq max(\boldsymbol{a}_{j}, \boldsymbol{b}_{j})$$

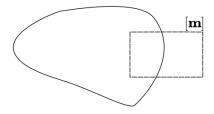
where:

• (a_j, b_j) are the extremities of a segment \mathbb{S}_j • $m = (m_0, m_1) \in \delta \mathbb{M}$.

Dynamic Localization

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Map Representation



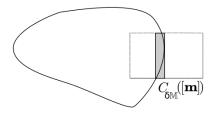
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Map Representation

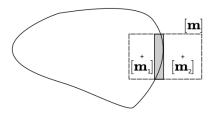


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Map Representation



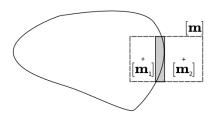
$$egin{cases} [m{m}_k] \subset \mathbb{M}_{free}, & \mathcal{T}(m{m}_k) = true \ [m{m}_k] \subset \overline{\mathbb{M}}, & Otherwise \end{cases}$$

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Map Representation



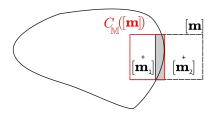
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Static Localization

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Conclusion

Polygon-Based Localization

$$\mathbb{X}_i = oldsymbol{h}_i^{-1}(\mathbb{M}) = \{oldsymbol{x} \in \mathbb{R}^3 : oldsymbol{h}_i(oldsymbol{x}) \in \mathbb{M}\}$$

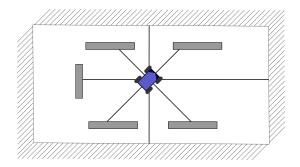
where:

• X_i is a set of possible robot's poses

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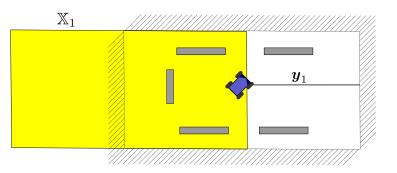
Polygon-Based Localization



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Polygon-Based Localization

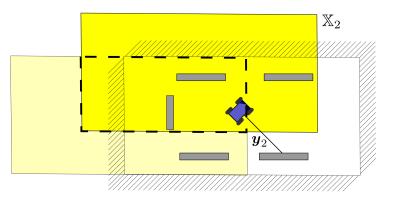


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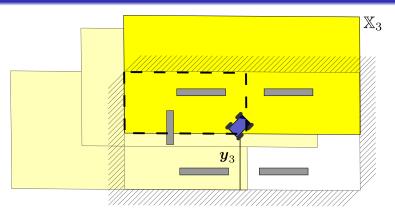
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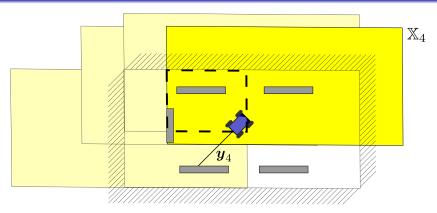
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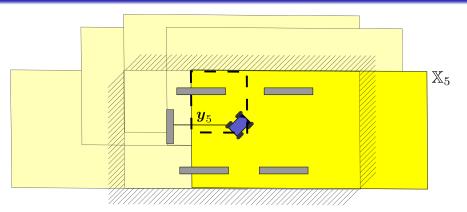
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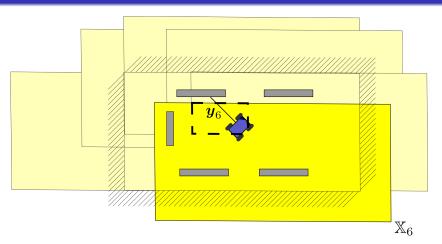
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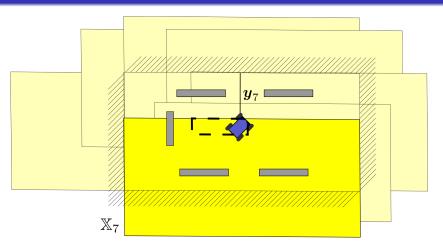
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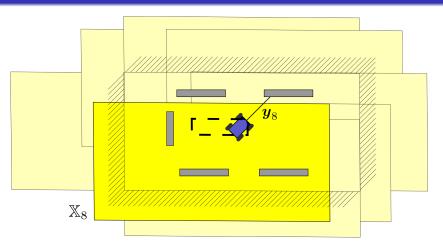
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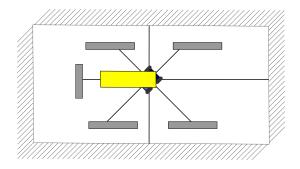
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Polygon-Based Localization





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Experiments - Simulated Scenario

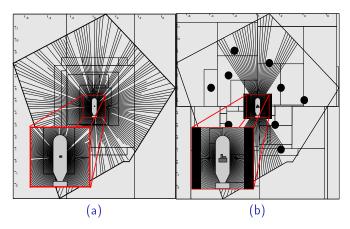


Figure: Static localization performed in the simulated scenarios, without (a) and with (b) unknown obstacles. The pavings were computed with $\epsilon_{\theta} = 10^{\circ}$ for the θ space and $\epsilon = 50$ cm for coordinates x and y.

Dynamic Localization

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Experiments - Real Scenario

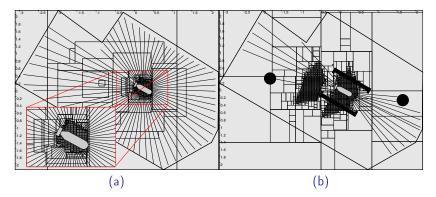


Figure: Static localization performed in the real scenarios, without (a) and with (b) unknown obstacles. The pavings were computed with $\epsilon_{\theta} = 10^{\circ}$ for the θ space and $\epsilon = 5$ cm for coordinates x and y.

Dynamic Localization

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Conclusion

Dynamic Localization

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) \\ \mathbf{y}_k = \mathbf{g}(\mathbf{x}_k) \end{cases}$$

where:

- $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector
- $\pmb{u} \in \mathbb{R}^m$ is the input vector
- $\pmb{y} \in \mathbb{R}^p$ are the measurements
- $f: \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ is the *evolution* function
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 and $oldsymbol{u} = (v,\omega)$

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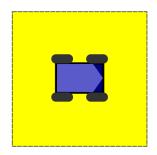




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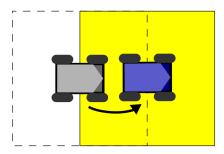
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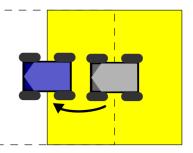
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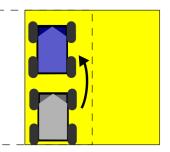
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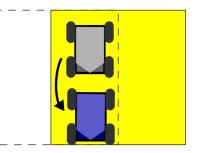
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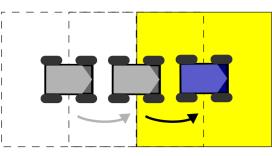
Dynamic Localization

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 $x_3^{}$

 x_{2}

Constraints Network





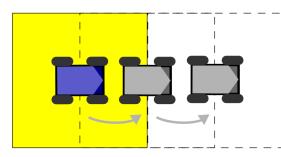


 $x_{_1}$

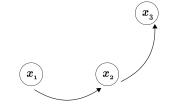
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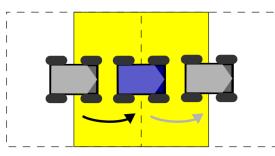
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Static Localization

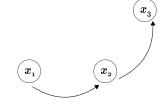
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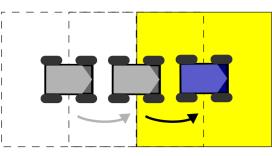
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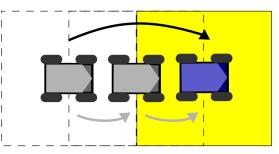


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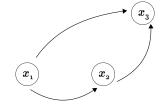
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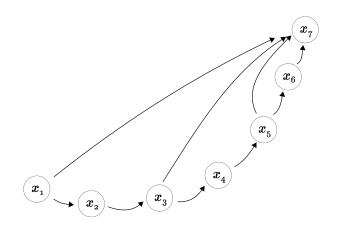


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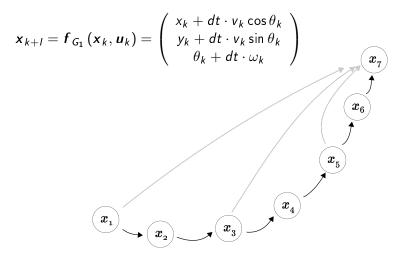
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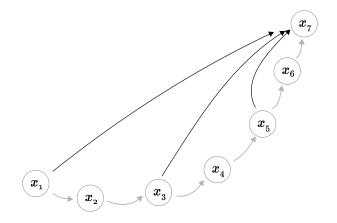
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 $\boldsymbol{x}_{k+l} = \boldsymbol{f}_{G_2}\left(\boldsymbol{x}_k, \boldsymbol{u}_{k:k+l}\right)$



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$$\begin{aligned} \mathbf{x}_{k+l} &= \mathbf{f}_{G_2} \left(\mathbf{x}_k, \mathbf{u}_{k:k+l} \right) = \\ \begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos \left(dt \cdot \sum_{j=k}^i \omega_j \right) \\ v_i \sin \left(dt \cdot \sum_{j=k}^i \omega_j \right) \\ \omega_i \end{pmatrix} \end{aligned}$$



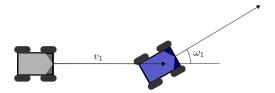
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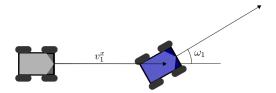
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$$\mathbf{x}_{k+l} = \mathbf{f}_{G_2} \left(\mathbf{x}_k, \mathbf{u}_{k:k+l} \right) = \begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot A \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos\left(dt \cdot \sum_{j=k}^i \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^i \omega_j\right) \\ \omega_i \end{pmatrix} \\ \omega_i \end{pmatrix}$$

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Dynamic Localization

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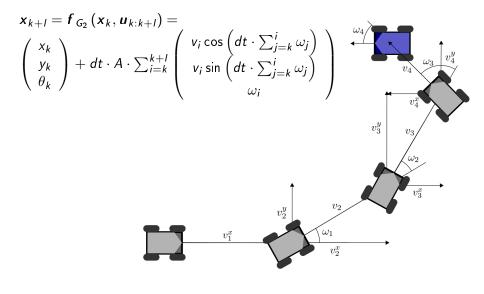
$$\mathbf{x}_{k+l} = \mathbf{f}_{G_2} \left(\mathbf{x}_k, \mathbf{u}_{k:k+l} \right) = \begin{pmatrix} x_i \cos\left(dt \cdot \sum_{j=k}^i \omega_j\right) \\ v_i \sin\left(dt \cdot \sum_{j=k}^i \omega_j\right) \\ \omega_i \end{pmatrix}$$

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Conclusion

Constraints Network



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Static Localization

Dynamic Localization

Conclusion

Constraints Network - Global Movement Model

$$A = \begin{pmatrix} \cos \theta_k & -\sin \theta_k & 0\\ \sin \theta_k & \cos \theta_k & 0\\ 0 & 0 & 1 \end{pmatrix}$$

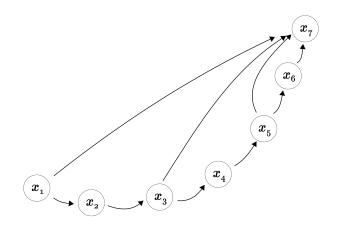
$$\begin{pmatrix} x_k \\ y_k \\ \theta_k \end{pmatrix} + dt \cdot \begin{pmatrix} \cos \theta_k & -\sin \theta_k & 0 \\ \sin \theta_k & \cos \theta_k & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \sum_{i=k}^{k+l} \begin{pmatrix} v_i \cos \left(dt \sum_{j=k}^i \omega_j \right) \\ v_i \sin \left(dt \sum_{j=k}^i \omega_j \right) \\ \omega_i \end{pmatrix}$$

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Dynamic Localization

Conclusion

Constraints Network

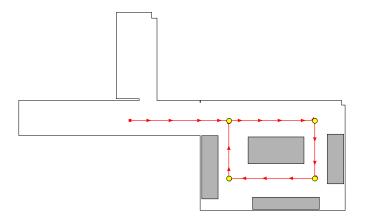


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Experiments



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Dynamic Localization

Conclusion

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Dynamic Localization

Conclusion

Experiments - Simulated Scenario

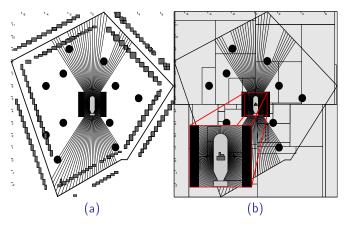


Figure: Static localization performed in the simulated scenario. (a) Segment approach and (b) Polygon-based approach. The pavings were computed with $\epsilon_{\theta} = 10^{\circ}$ for the θ space and $\epsilon = 50$ cm for coordinates x and y.

Dynamic Localization

Conclusion

Experiments - Real Scenario

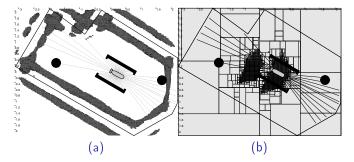


Figure: Static localization performed in a real scenario. (a) Segment approach and (b) Polygon-based approach. The pavings were computed with $\epsilon_{\theta} = 10^{\circ}$ for the θ space and $\epsilon = 50$ cm for coordinates x and y.